University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2012 – PART I

Tuesday, August 21, 2012 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator, but only as an aid to numerical calculations. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×108 m/s
Elementary charge	e	1.60´10-19 C
Electron rest mass	me	9.11´10-31 kg
Electron rest mass energy	mec^2	0.511 MeV
Permeability constant	mo	1.26´10-6 H/m
Permeability constant/4π	mo/4π	10^{-7} H/m
Proton rest mass	mp	1.67´10-27 kg
Proton rest mass energy	mpc^2	938 MeV
Neutron rest mass	mn	1.68´10-27 kg
Neutron rest mass energy	mnc^2	940 MeV
Planck constant	h	6.63´10-34 J·s
Gravitational constant	G	6.67´10-11 m3/s2·kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	NA	6.02´1023 /mol
Boltzmann constant	kB	1.38´10-23 J/K
Molar volume of ideal gas at STP	Vm	2.24´10-2 m3/mol
Earth radius		6.37´106 m
Earth-Sun distance		1.50´1011 m

Stirling's Approximation:

ln(N!) = Nln(N) – N + (small corrections)

$$\text{mass of electron } m_e = 0.511 \text{ MeV}/c^2$$

$$\text{mass of proton } m_p = 938.272 \text{ MeV}/c^2$$

$$\text{mass of } \alpha \text{ particle } m_\alpha = 3727.379 \text{ MeV}/c^2$$

$$\sin\theta_1 \sin\theta_2 = \tfrac{1}{2}[\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]$$

$$\int_0^\infty x^n e^{-x} dx = n!$$

$$\int_0^\infty x^2 e^{-x^2} dx = \tfrac{\sqrt{\pi}}{4}$$

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Graduate Written Exam, Part 1

Tuesday August 21st 2012

1. Assume you have three identical particles and three single particle states $|a\rangle$, $|b\rangle$ and $|c\rangle$ available for them. Count how many different three-particle states there can be if the particles are (a) fermions, (b) bosons.

2. Show that a particle in a one-dimensional infinite square well initially in a state $\Psi(x, 0)$ will always return to that state after a time $T = 4ma^2/\pi\hbar$ where a is the width of the well.

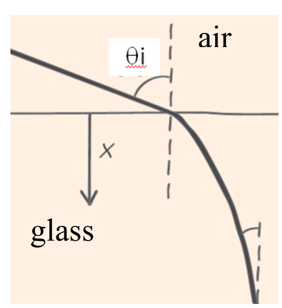
3. Consider a system of N independent classical molecules, each at a fixed position, with magnetic moment $\vec{\mu}$ in an external magnetic field $\vec{B}$. Determine the partition function, and hence find the free energy and the magnetization at temperature T, when the molecules can only be oriented parallel or antiparallel to the external magnetic field.

4. A photon collides with a stationary electron. If the photon scatters at an angle θ, show that the resulting wavelength, λ', is given in terms of the original wavelength, λ, by

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where m is the mass of the electron.

5. The composition of a glass block varies as a function of the distance x from the top surface. Thus the index of refraction $n(x)$ increases as a function of x according to the relationship $n(x) = 1.50 - (0.20)/(x+1)^2$, where x is expressed in centimeters. A beam of light strikes the surface with an angle of incidence θ_i, measured from the vertical, as shown in the figure opposite. What will be the direction of the beam deep inside the block?



6. You want to measure a current when using a voltmeter and a precision resistor. You measure a voltage of 0.85 V across a 10,000 Ω resistor and -0.05 V when the input to the voltmeter is short-circuited. The precision of the voltmeter is 0.001 V and the resistor is rated at 0.1%. What is the value of the current and the precision of the measurement?

7. Find the capacitance per unit length of two coaxial conducting cylindrical tubes, of radii *a* and *b*.

8. Suppose that the radius of the Earth were to gravitationally collapse uniformly by one percent, with its mass remaining the same. What would happen to the Earth's kinetic energy of rotation? If it changes, how does it change and by how much? Assume that the Earth is a uniform sphere.

9. A neutron star has a radius of 10 km, a mass of 3.0×10^{30} kg. Find the nearest distance to the surface that a person 2 m tall could approach the pulsar without being pulled apart. Assume a uniform mass distribution, feet toward the pulsar and that a person starts to come apart when the force that each half of the body exerts on the other exceeds ten times the body weight on Earth. What is the period of revolution in a circular orbit about the pulsar at this distance?

10. Given that the latent heat of fusion for water is 334 kJ/kg and the density of water and ice are 1000 kg/m^3 and 917.0 kg/m^3, respectively. At what pressure will ice melt at -0.1^{o}C? It may be useful to remember that the Gibbs free energy is the same across a phase boundary.

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GRADUATE WRITTEN EXAMINATION

FALL 2012 – PART 2

Wednesday, August 22, 2012 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
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Molar gas constant	R	8.31 J/mol•K
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Graduate Written Exam, Part 2

Wednesday August 22nd 2012

Answer as many questions as you can.

1) In the middle of an infinite square well extending from $x = 0$ to $x = a$ we put a delta function potential $H = \kappa\delta(x-a/2)$ where κ is constant. **(a)** Find the first order perturbation correction to the energies. **(b)** For the ground state, find also the second order correction to the energy.

2) **(a)** Beginning with the Lorentz transformations show that when an object is moving with velocity v_2 in a reference frame that is moving with velocity v_1 with respect to an observer, the velocity v of the object as seen by the observer is:

$$v = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2}$$

(b) A spaceship is initially at rest with respect to frame *S*. At a given instant, it starts to accelerate with a constant acceleration, a, in the instantaneous rest frame of the spaceship. What is the relative speed of the spaceship in frame *S* when the spaceship's clock reads time t? **(c)** The command centre, which is stationary in frame *S*, communicates with the spaceship by using a laser with a wavelength of λ_s. What would be the wavelength of the laser beam observed on board the spaceship at time t?

3) Consider a classical model of a CO_2 molecule where the masses are connected by springs of spring constant k. Assume all motion to be linear along the axis of the molecule. **(a)** Find the relative frequencies for the two vibrational modes. **(b)** Find the eigenvectors for the modes of the molecule, including any zero frequency mode.

4) Consider a free Fermi gas of N electrons in two dimensions confined to a square of area A. **(a)** Find the Fermi energy (ϵ_F) in terms of N and A. **(b)** Derive the formula for the density of states and show that it is independent of energy. **(c)** Use this to find the chemical potential μ as a function of N and the temperature. **(d)** What is the behavior of the system at low temperature?

5) If magnetic charges were found, **(a)** write down the proper set of four Maxwell equations in vacuum to include the electric and magnetic charges as well as electric and magnetic currents. **(b)** Suppose a magnetic monopole of strength, q_m, passes through a zero-resistance loop of wire and moves far away. If the self-inductance of the loop is L, what would be the electric current induced in the loop? **(c)** Set up the equation of motion of an electrically charged particle (charge, q_e, mass, m) about a fixed magnetic monopole of strength, q_m.