Graduate Written Exam Part I (Fall 2011)

1. An elevator operator in a skyscraper, being a very meticulous person, put a pendulum clock on the wall of the elevator to make sure that he spends exactly 8 hours a day at his work place. Over the course of his work day, he records that the time during which the elevator has acceleration a is exactly equal to the time during which it has acceleration $-a$. Does the elevator operator work, in actual time, (1) more than 8 hours, (2) exactly 8 hours, or (3) less than 8 hours? Why?

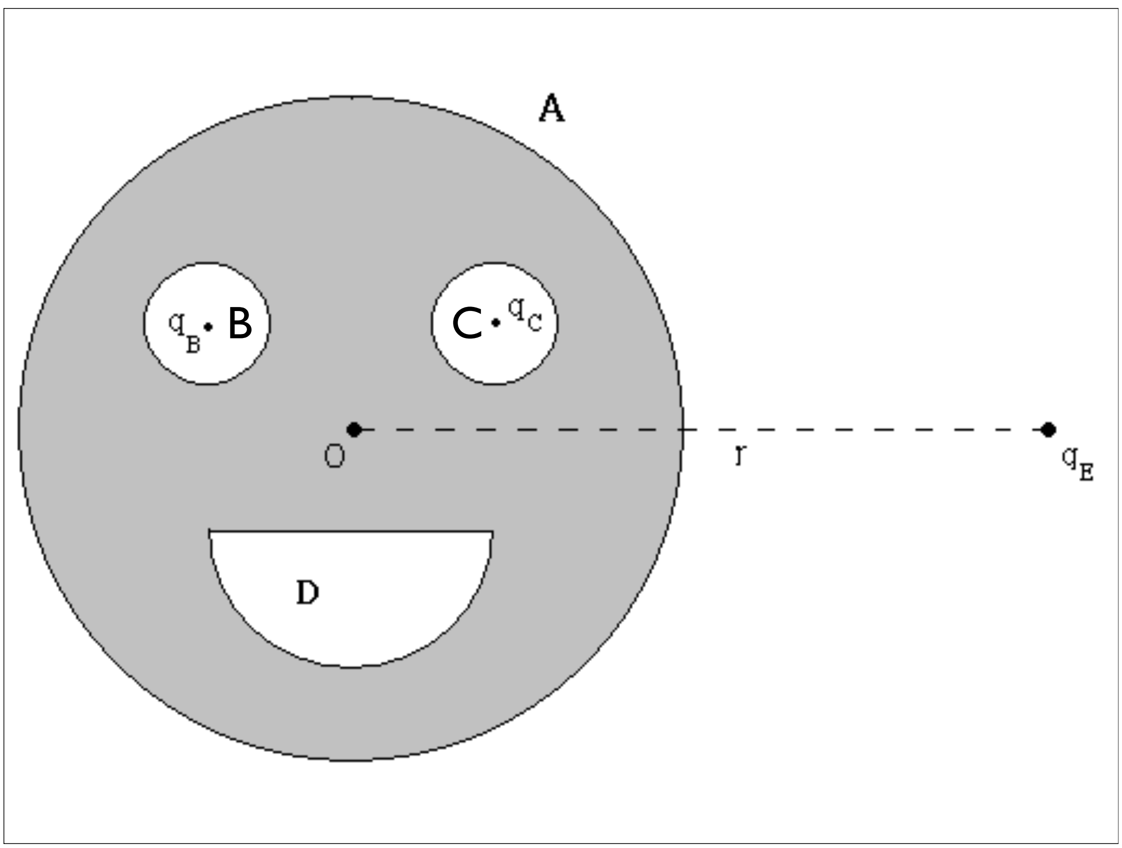
2. A classical particle is subject to an attractive central force proportional to r^α, where r is the radius and α is a constant. Show by perturbation analysis what is required of α in order for the particle to have a stable circular orbit.

3. A neutral conductor A with a spherical outer surface of radius R contains three cavities B, C, and D, but is solid otherwise. B and C are spherical, and D is hemispherical. Without touching A, positive charges q_B and q_C are introduced at the centers of B and C, respectively (see Figure below).

(1) Give the amount and the distribution of the induced charges on the surfaces of A, B, C, and D.

(2) Now another positive charge q_E is introduced at a distance $r > R$ from the center of A (see Figure below). Describe qualitatively the distribution of the induced charges on the surfaces of A, B, C, and D.

(3) Give the amount of the induced charges on the surfaces of A, B, C, and D for the situation in (2).



4. The dielectric strength of air at standard temperature and pressure is 3×10^6 V/m. What is the maximum intensity in units of W/m^2 for a monochromatic laser that can be used in the laboratory?

5. What is the minimum energy of the projectile proton required to induce the reaction $p + p \rightarrow p + p + p + \bar{p}$ if the target proton is at rest?

6. A subatomic particle has spin 1 and negative parity. It decays at rest into an e^+e^- pair, which is produced in the s and d waves. From these data determine (1) the total spin of the e^+e^- pair and (2) the intrinsic parity of e^+ relative to e^-.

7. There is a uniform, vertical gravitational field with a downward acceleration of gravity g above a horizontal, perfectly elastic surface. A particle of mass m can only move above the surface. Give a rough estimate of the energy eigenvalue for the ground state of the particle.

8. Assume that the atmosphere near the earth's surface is in approximate hydrostatic equilibrium, where any movement of air parcels is gentle and adiabatic. Find an expression for the pressure P of the atmosphere as a function of the height z.

9. Consider a gas of atoms in a magnetic field of 10 Tesla. The nucleus of the atom has spin 1/2, magnetic moment $\mu \approx 10^{-26}$ J/Tesla, and mass $m \approx 5 \times 10^{-27}$ kg. The electrons in the atom have zero total angular momentum. What is the maximum number density of the gas for which the nuclei are completely polarized by the magnetic field at zero temperature?

10. A new long-lived particle X is observed to decay via $X \rightarrow K^+ + K^-$. The mass of X is about 1.2 GeV/c^2. You wish to determine this mass to within 1% using the momenta of the K^+ and K^- from decay of X at rest. What should be the maximum relative error on your momentum measurements if you use only a single decay event? You know that the mass of K^+ or K^- is 493.677 ± 0.013 MeV/c^2.

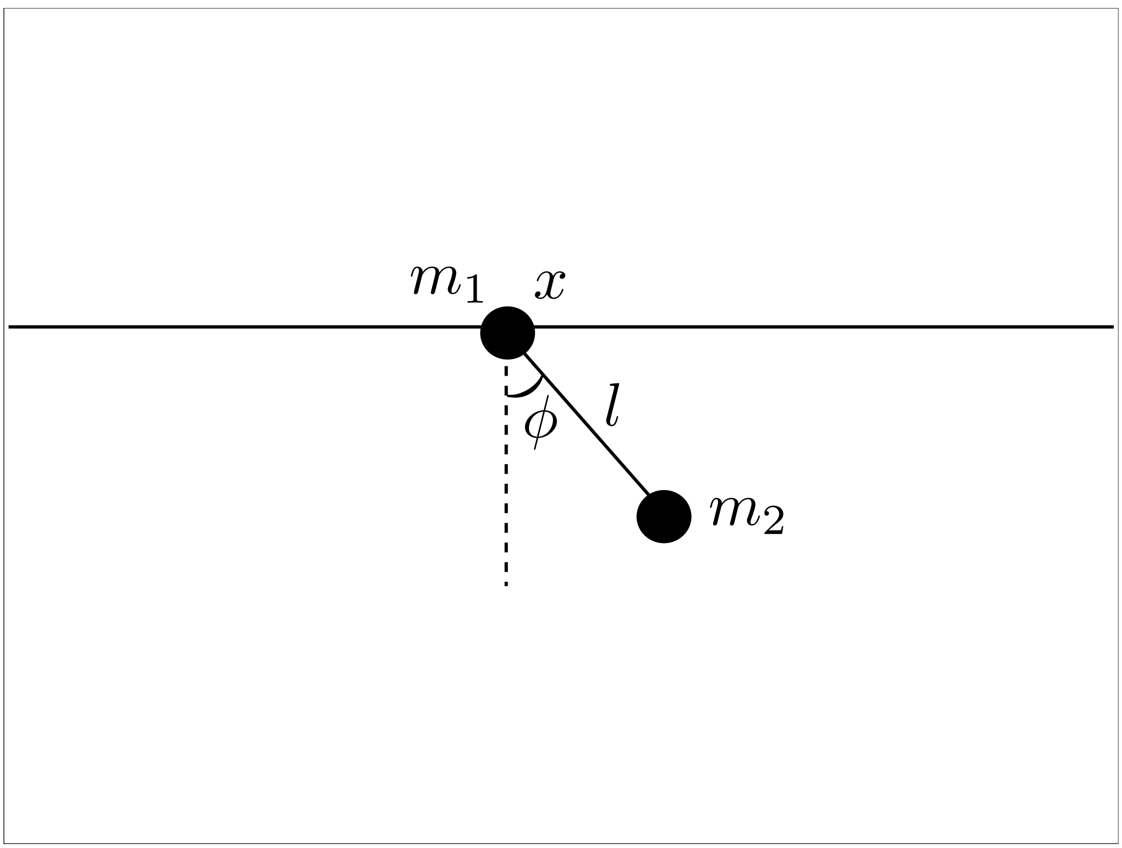
Graduate Written Exam Part II (Fall 2011)

1. Mass m_1 moves freely along a fixed, long, horizontal rod. The position of m_1 on the rod is x. A massless string of length l is attached to m_1 at one end and to mass m_2 at the other. Mass m_2 executes pendulum motion in the vertical plane containing the rod (see Figure below).

(1) Find the Lagrangian of the system.

(2) Derive the equations of motion and the corresponding conservation laws.

(3) Assume that $x = x_0$, $\dot{x} = 0$, $\phi = \phi_0$ ($|\phi_0| \ll 1$), and $\dot{\phi} = 0$ at $t = 0$. Find x and ϕ for $t > 0$.



2. A light bulb has a tungsten filament formed into a coil of 60 turns. The coil is a straight column of 3 mm in diameter and 20 mm in length. The bulb is rated 75 W for an AC source of 110 V with a frequency of 60 Hz. Find the current in the bulb as a function of time t after it is connected to the AC source at $t = 0$. Assume that the voltage of the source is of the form $V(t) = V_0 \cos \omega t$ and the resistance of the filament stays constant.

3. A particle of mass m is in the potential

$$V(x) = \begin{cases} -\Omega_0 \delta(x), & -a/2 < x < a/2, \\ \infty, & \text{otherwise,} \end{cases}$$

where $\delta(x)$ is the Dirac delta function and Ω_0 and a are positive parameters.

(1) For the energy eigenstates that have wave functions with odd parity, find these wave functions and the corresponding eigenvalues.

(2) For the rest of the energy eigenstates, find the approximate eigenvalues by treating $-\Omega_0 \delta(x)$ as a perturbation. What is required of Ω_0 for the approximation to be valid?

(3) For a special value of Ω_0, the energy of the ground state is exactly zero. Find this special value of Ω_0.

4. For the simple harmonic oscillator with the Hamiltonian

$$H_{\text{sho}} = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2,$$

where P and X are the momentum and position operators, the wave functions for the energy eigenstates are $\psi_n(x/a)$, where n is a non-negative integer, a is a constant with the dimension of length, and $\int_{-\infty}^{\infty} |\psi_n(x/a)|^2 dx = 1$. Now consider two distinguishable particles of mass m with the total Hamiltonian

$$H = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{1}{2}m\omega^2[X_1^2 + X_2^2 + (X_1 - X_2)^2].$$

(1) Find the normalized wave functions for the eigenstates of H in terms of $\psi_{n'}(\eta)$ and $\psi_{n''}(\xi)$ by choosing two appropriate dimensionless position variables η and ξ.

(2) Find the eigenvalues of H.

(3) How would the answers to (1) and (2) change if the two particles are identical and have spin 0?

5. Consider only the rotational motion of a diatomic molecule with moment of inertia I.

(1) What is the specific heat for a classical system of N such molecules?

(2) In the quantum mechanical case, the energy levels for an individual molecule are

$$E_{lm} = \frac{l(l+1)\hbar^2}{2I},$$

where $l = 0, 1, 2, \cdots$ and for each l, $m = -l, -l+1, \cdots, l$. For a system of N such molecules, express the partition function Z and the energy E as sums of well-defined quantities.

(3) Calculate the specific heat in the low-temperature and high-temperature limits for the quantum mechanical case.

(4) For what range of temperature is the classical result in (1) valid?