Graduate Written Exam Part I (Winter 2011)

1. A monoenergetic muon beam is produced at point A and directed to point B, which is 20 km away. Suppose that only 2/3 of the muons reach point B and the remainder decay in flight. Find the energy of the muons. In the rest frame of a muon, its mass and lifetime are 105.7 MeV/c^2 and 2.2×10^{-6} s, respectively.

2. A wheel consists of a thin cylindrical steel shell of radius R, mass M, and density ρ mounted on light but very strong spokes. The wheel rotates around a vertical axle, which coincides with its axis of symmetry, and is used to store energy.

(1) Find the stress in the steel shell when the angular speed of rotation is ω.

(2) If steel breaks apart when the stress reaches 3×10^8 N/m^2, what is the maximum energy that can be stored per unit volume?

3. Suppose that Chicago and Minneapolis are connected by an underground train that is confined to run on a straight track of 600 km between the two cities. The train is released from rest at either city and powered solely by gravity (ignore all dissipative forces). Ignore the rotation of the earth and assume that it is a uniform sphere of 6×10^{24} kg in mass and 6400 km in radius. How long does a one-way trip last and what is the maximum speed of the train?

4. A long cylindrical capacitor has an inner cylinder of adjustable radius and a thin outer cylindrical shell of fixed radius b. The inner cylinder is at a fixed potential V and the outer shell is at a fixed zero potential. You need to adjust the radius of the inner cylinder so as to minimize the electric field at the surface of the inner cylinder. Find this radius R_{in} of the inner cylinder.

5. A parallel-plate capacitor has two plates each of area A and is connected to a battery of voltage V. The initial separation between the two plates is d.

(1) If one plate is fixed and the other is moved by an external force to double the separation, what is the work done by the external force?

(2) Show that the work in (1) equals the net change of energy in the battery-capacitor system.

6. A non-relativistic particle of mass m is in the potential $V(x) = -\alpha\delta(x)$ with $\alpha > 0$. How many bound states are there? Find the energy eigenvalue and the expectation value of x^2 for each bound state.

7. The sun radiates photons at a total power of 4×10^{26} Watts, which is supplied by a series of reactions effectively converting four protons into a ^{4}He nucleus.

(1) Write down the effective net reaction of converting four protons into a ^{4}He nucleus by considering conservation laws.

(2) By using the values of the particle masses provided on the cover sheet, estimate the total number of neutrinos emitted by the sun per second.

8. The speed of sound in a gas is $v = \sqrt{(\partial P/\partial \rho)_S}$, where P, ρ, and S are the pressure, mass density, and entropy of the gas, respectively. A standing sound wave forms in a tube filled with iodine vapor (an ideal gas) at 400 K. The frequency of the wave is 1000 Hz and the distance between the adjacent nodes is 6.77 cm. Determine whether iodine vapor is a monoatomic or diatomic gas. The atomic weight of iodine is 127.

9. Derive the relation between the pressure and energy density of black-body radiation. You may do this by considering the collisions of photons with the wall of a cavity enclosing such radiation, or by any other way that you may choose.

10. A windmill has blades of 15 meters in diameter. If the wind is blowing into the windmill at a speed of 12 m/s and the downstream air moves at a speed of 10 m/s, estimate the power generated by the windmill. The density of air blown into the windmill is 1.3 kg/m^3.

Graduate Written Exam Part II (Winter 2011)

1. A pendulum, which consists of a light rod of length l and a bob of mass m, is attached to the ceiling. A second identical pendulum is attached to the bob of the first. Denote the angle each rod makes with respect to the vertical as θ_1 and θ_2, respectively (see Figure 1). Assume that both angles are small for all time t. Find the relative amplitudes and phases of $\theta_1(t)$ and $\theta_2(t)$ when the system is oscillating in the normal modes, as well as the corresponding frequencies.

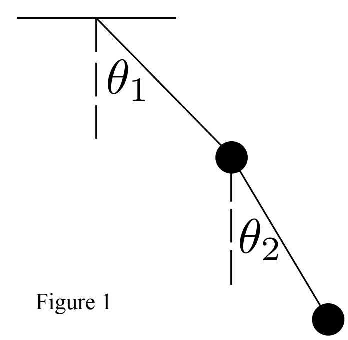

Figure 1

2. In a region with a uniform vertically upward magnetic field of magnitude B, two long straight conducting rails with negligible resistance are set up in a horizontal plane with distance l between them (see Figure 2). Their left ends are connected to a capacitor of capacitance C. A metal rod of mass m and resistance R can move on the rails without friction. At time $t = 0$, there is no charge on the capacitor and the rod is released with a velocity v_0 to the right.

(1) Find the charge on the capacitor as a function of t.

(2) Find the terminal velocity of the rod.

(3) Show that energy is conserved by considering changes between the initial and terminal states.

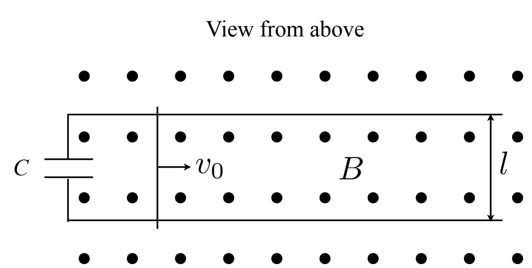

Figure 2

3. (1) A non-relativistic particle of mass m is in the ground state of the potential

$$V(x) = \begin{cases} 0, & 0 < x < a, \\ \infty, & \text{elsewhere.} \end{cases}$$

At some point of time, the potential suddenly changes to

$$V(x) = \begin{cases} 0, & 0 < x < 2a, \\ \infty, & \text{elsewhere.} \end{cases}$$

Some time later, a measurement of the energy of the particle is made. Find the probability that the particle will be measured to have the ground-state energy for the new potential.

(2) Do the same for the three-dimensional case where the initial potential is

$$V(r) = \begin{cases} 0, & 0 < r < a, \\ \infty, & \text{elsewhere,} \end{cases}$$

and the new potential is

$$V(r) = \begin{cases} 0, & 0 < r < 2a, \\ \infty, & \text{elsewhere.} \end{cases}$$

It is suggested that you (a) derive or write down the radial part of the Schrödinger equation that needs to be satisfied, and (b) note that the equation only needs to be solved for the ground state.

4. Consider a single electron in the Coulomb potential of a nucleus with proton number Z. Assume that the electric charge of the nucleus is distributed uniformly within a sphere of radius R.

(1) Calculate the Coulomb potential energy $V(r)$ for the electron as a function of radius r (with the origin at the center of the nucleus).

(2) The ground-state wave function for the electron in a hydrogen-like atom with a point-like nucleus is $\psi(\vec{r}) \propto \exp(-r/a)$, where a is a constant. Use this information to make a leading-order estimate of the shift in the ground-state energy due to the Coulomb interaction between the electron and a nucleus of finite R (relative to the case of a point-like nucleus).

(3) Evaluate the energy shift in (2) for $Z = 81$ and $R = 7$ fm.

5. An ideal gas of N spinless atoms has volume V, temperature T, and partition function Z_0.

(1) By assuming that the atoms obey Maxwell-Boltzman statistics, find an expression for Z_0.

(2) Now consider that each atom has two internal energy levels with energy ϵ and $\epsilon + \Delta$, respectively. Find the new partition function.

(3) Calculate the specific heat at constant volume for the case in (2).