University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Fall 2016 – PART I

Tuesday, August 23rd, 2016 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
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Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
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Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
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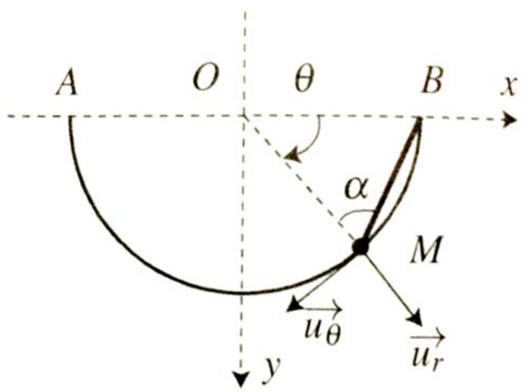
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

A ball of mass m move with no friction along half a circle of center O and radius R (figure below). AB is the diameter of the circle, Ox is the horizontal axis and Oy the vertical axis pointing down. We call θ the angle between Ox and OM. The ball is attached to a spring of elastic constant k, attached to B on the other side. The ball moves along the arc of circle.

1. Determine the potential energy $E_p^* = E_p/E_0$ (with $E_0 = mgR$) as a function of θ and p, where p is function of k, R, g and m.
2. Determine the equilibrium positions(s) θ_e of the system. Draw the plot $E_p^*(\theta)$ as a function of θ for p = 2. Determine θ_e for p = 2.



Problem 2

A hollow cylinder of mass m and radius a rolls without slipping down a movable wedge of mass M. The angle of the wedge relative to the horizontal surface is α, and the wedge is free to slide on this smooth horizontal surface. The contact between the cylinder and the wedge is perfectly rough. Find the acceleration of the wedge.

Problem 3

A cylinder with adiabatic walls (i.e. thermally insulated walls) is closed at both ends is initially divided into two equal volumes by a frictionless piston that is also thermally insulating. Initially the volume, pressure and temperature of the ideal gas in each side of the cylinder are V_0 , p_0 , and T_0 respectively. A heater in the right-hand volume is used to slowly heat the gas on that side until the pressure there reaches $64 p_0/27$. If the heat capacity C_v of the gas is independent of temperature, and $\frac{C_p}{C_v} = \gamma = 1.5$, find the following in terms of V_0 , p_0 , and T_0:

(a) the entropy change of the gas on the left;
(b) the final left-hand volume; and
(c) the final left-hand temperature.

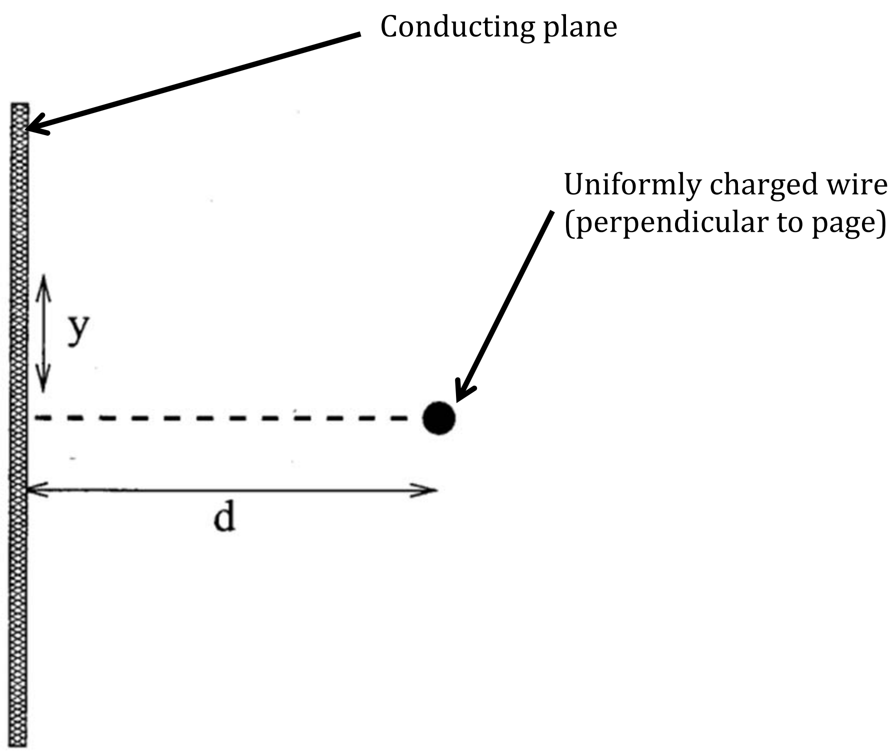
Problem 4

In many cases, graphene can be modeled as a two-dimensional gas of non-interacting electrons with energy $\varepsilon(k) = \hbar v|k|$, where $\hbar k$ is the momentum and v is the effective velocity. Each state is fourfold degenerate due to the spin and the so-called valley degrees of freedom. Consider a positive chemical potential corresponding to an electronic density n. Calculate the ratio between the average energy per particle and the Fermi energy of the system at zero temperature.

Problem 5

A very long wire of radius a is suspended a distance d above an infinite conducting plane. The wire is uniformly charged with uniform charge density λ. In the case that $d >> a$, find approximate expression for:

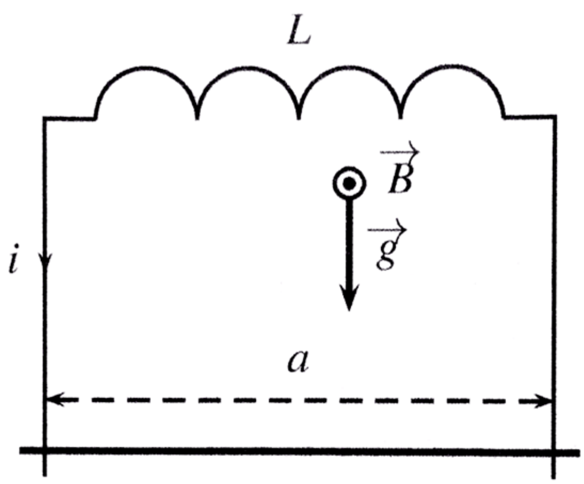
1. The capacitance per unit length of the wire, conducting plane system.
2. The surface charge density on the conducting plane as a function of y, the distance along the plane lateral to the wire.



Problem 6

A thin rod of length a, mass m and resistance R moves vertically with no friction and closes an electrical circuit with an inductance L. In this problem, we consider that the total resistance of the circuit is R and the total inductance of the circuit is L. A magnetic field **B** is applied perpendicularly to the circuit (out of page, the figure below). The rod is dropped at t = 0 with no initial velocity.

1. Determine the differential equation of the current in the rod.
2. If R = 0, determine the current intensity i(t) and the speed of the rod v(t).



Problem 7

Consider a thin divergent lens with a focal distance f’ = -30 cm.

1. Determine the distance between the lens and the virtual image of a point A located 30 cm in front of the lens.
2. If a vertical object AB of height 1 mm is placed at point A, what is the height of its virtual image?

Problem 8

A particle is confined between two planes at x = 0 and x = L in an infinite well. The wave function is:

$$\psi(x,t) = A\sin(kx)e^{-i\omega t}$$

1. Determine the possible values for k as a function of L and a positive integer n.
2. Find A as a function of L.
3. Draw $|\psi(x,t)|^2$ as a function of x in the case of n = 1 and n = 2.

Problem 9

A particle of mass m is in the ground state in one-dimensional box of length L with impenetrable walls. Find the distribution of the probability of the momentum p. (This means finding the function ρ(p) such that the probability dP of measuring the momentum in the differential interval dp is given by dP = ρ(p) dp).

Problem 10

τ^- lepton decays into muon (μ^-), muon antineutrino ($\tilde{\upsilon}_\mu$) and τ neutrino (υ_τ). What is the maximal possible momentum of the muon (in MeV/c^2) in the rest frame of the τ lepton. The mass of τ^- is M = 1777 MeV/c^2, the mass of the muon m = 106 MeV/c^2 and both neutrinos are very light, so that their masses can be neglected.

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GRADUATE WRITTEN EXAMINATION

Fall 2016 – PART II

Wednesday, August 24th, 2016 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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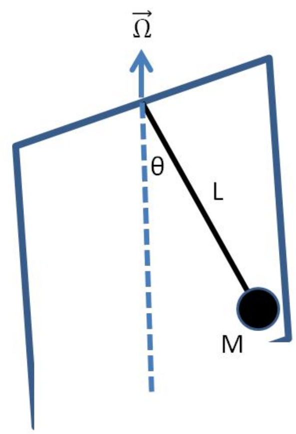
Problem 1

A simple pendulum of length L and mass M is suspended from the middle of a horizontal, rigid rod in such a way that the pendulum can swing only in the plane perpendicular to the plane of the support frame, as shown in the figure. The pendulum is a massless, rigid rod with the mass M at the end. The horizontal rod is rotated counterclockwise (when viewed from above) at rotation rate $\vec{\Omega}$. Let θ be the angle between the pendulum and the vertical.



a) Find the angles of equilibrium (whether stable or unstable). Under what conditions are the equilibrium angles stable points?
b) Using the reference frame that rotates with the support (i.e., in which the support frame is stationary), find the oscillation frequency for small amplitude oscillations about $\theta = 0$. For what rotation rates $\vec{\Omega}$ is $\theta = 0$ a stable equilibrium point?
c) If there are other stable equilibrium angles(s) θ, find the frequency of small amplitude oscillations about these equilibrium point(s).

Problem 2

A Helmholtz coil consists of two parallel circular current loops of identical radius R and separated by a distance d from each other. Each loop carries a uniform current I along the same direction. Define the z axis to be the common axis that crosses the centers of the two coils, such that $z = 0$ is at the point midway between them.

(a) What is the value of d for which the amplitude of the magnetic field $\boldsymbol{B}$ produced by the Helmholtz coil is such that $\partial B/\partial z = 0$ and $\partial^2 B/\partial z^2 = 0$ at $z = 0$?
(b) For the value of d that you obtained above, calculate $\boldsymbol{B}(\boldsymbol{z} = \boldsymbol{0})$.

Problem 3

A futuristic starship with the mass one million metric tons departs from a base in the outer space. The cruising speed of the starship corresponds to the time dilation (as observed from the base) 10 times. The starship accelerates in a straight line and reaches the cruising speed and then decelerates (also in straight line) reaching its destination near a star in a distant galaxy, which moves very slowly with respect to the ship's home base. On the way back the ship again accelerates to its cruising speed and then decelerates returning to the base. Assuming that all the starships in the future are propelled by converting their fuel into light with 100% efficiency and perfectly directing all the generated light in the direction opposite to the thrust, find the mass of the starship after it returns to the base. Ignore gravitational effects.

Problem 4

A one-dimensional simple harmonic oscillator of angular frequency ω is acted upon by a spatially uniform but time-dependent perturbation force:

$$F(t) = \frac{F_0 \tau/\omega}{(\tau^2 + t^2)} \ , \qquad -\infty < t < +\infty$$

At $t = -\infty$, the oscillator is known to be in the ground state. Calculate, to leading order in the perturbation potential derived from the force above, the probability that the oscillator is in the first excited state at $t = +\infty$. You may find useful the following relationships between the position and momentum of the harmonic oscillator and the creation and annihilation operators:

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$$

$$p = i\sqrt{\frac{m\omega\hbar}{2}}(-a + a^\dagger)$$

Problem 5

Consider a classical statistical mechanics system consisting of N subsystems labeled by i = 1 . . .N, each of which can exist in two states $s_i = \pm 1$. Call n_+ the number of + values and n_- the number of – values. Let the total energy of the system be given by:

$$E = -J\sum_{i=1}^{N} s_i$$

a. Let the energy of the system be fixed at $E_0 = N\varepsilon$ where $-J < \varepsilon < 0$ (microcanonical ensemble). Express the number of configurations in terms of N and $x = n_+/N$. Express x in terms of E_0, N and J.

b. Focus on a particular subsystem, i = 1. Compute the probability r (as a function of x) that $s_1 = +1$ divided by the probability that $s_1 = -1$, directly in the microcanonical ensemble, again assuming that N is large.

c. Now suppose that the entire system is used as a heat bath for the subsystem considered in part (b). What temperature does the entire system have as a function of J and x?

d. Repeat part (b) above, treating subsystems s_2 …. s_N as a heat bath for system s_1 and then working in the canonical ensemble. Are your answers consistent?

e. Give a qualitative sketch of the temperature as a function of energy. Note a peculiarity in the system for $E_0 > 0$.

You may find Sterling's approximation useful: $\log n! \cong n \log n - n$