University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Fall 2017 – PART I

Tuesday, August 22nd, 2017 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

Assume that the atmosphere near the earth's surface is in approximate hydrostatic equilibrium, where any movement of air parcels is isothermal. Derive an expression for the pressure P of the atmosphere as a function of the height z using the ideal gas law.

Problem 2

You are asked to design a yo-yo that accelerates, as it drops, with a value 1/10 th that of the acceleration due to gravity g. Yo-yo's are formed by two disks of radius *R* and connected by a spindle, of a smaller radius *r*, around which a string is wound (the string goes down vertically from your hand to the spindle). The spindle and the string have negligible mass, and the combined mass of the two disks is *m*. What should be the ratio *R/r* needed to achieve this acceleration as the yo-yo drops from a stationary hand holding onto the yo-yo string?

Problem 3

An electric potential is given by the expression, $V(\vec{r}) = A\, e^{-\alpha r}/r$. Here A and α are constant.

Find

a) the corresponding electric field $\vec{E}(\vec{r})$,

b) the charge density $\rho(\vec{r})$ that generates this potential.

Problem 4

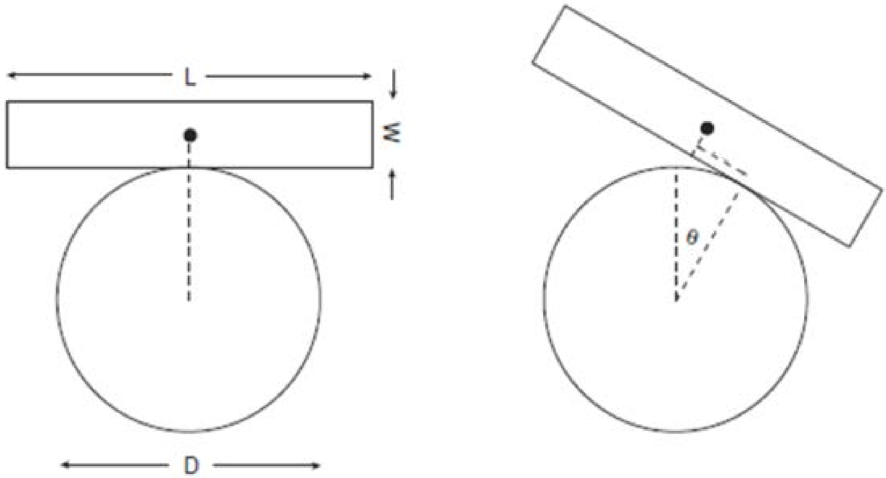
The flux of solar radiation arriving at the earth is 1.4 kW/m^2. Estimate the total number flux of solar neutrinos arriving at the earth.

Problem 5

In the state with given angular momentum ℓ and its projection on the z-axis *m* (i.e., with the wavefunction $\psi_{\ell,m}$) find the average values of ℓ_x^2 *and* ℓ_y^2 .

Problem 6

A plank of length L and height W is in equilibrium, balanced on a cylinder of diameter D. What condition must be satisfied by L, W, and D if this equilibrium is to be stable? Hint: Consider the change in height of the mass center of the plank as it rotates through a small angle θ.



Problem 7

Find the energy eigenvalues and normalized wave functions of the bound state in the potential

$$V(x) = -\alpha\delta(x).$$

Here α is a positive constant and δ is the Dirac delta function. Find the average value of kinetic and potential energies in the bound state.

Problem 8

Let us consider two identical linear harmonic oscillators 1 and 2 separated by R. Each oscillator bears charges $\pm e$ with separation x_1 and x_2, as shown in Fig. The particles oscillate along the x axis. Let p_1 and p_2 denote the momenta. The force constant is C. All particles have mass m. Then the Hamiltonian of the non-interacting system is

$$H_0 = \frac{1}{2m}{p_1}^2 + \frac{1}{2}C{x_1}^2 + \frac{1}{2m}{p_2}^2 + \frac{1}{2}C{x_2}^2$$

a) Write down the Coulomb interaction energy (H_1) between all the charged particles. Then take the approximation, $|x_1|,\ |x_1| \ll R$ and obtain the leading term.

b) By applying the normal mode transformation to the total Hamiltonian $(H_0 + H_1)$, show the characteristic frequencies of these coupled oscillators are

$$\omega = \left\{\left(C \pm \frac{2e^2}{R^3}\right)/m\right\}^{1/2}$$

Problem 9

Consider a system of N non-interacting particles each with a spin $S = 1$ and fixed position in an external magnetic field H. The particles have magnetic moment μ_B. If no magnetic field is present all spin projections S_z of a single particle are degenerate with an energy $E = 0$.

a) Plot the energy for all spin projections as a function of H.
b) Calculate the partition function of a single spin and then of the system as a function of the temperature and magnetic field H.
c) Calculate the average energy $\langle E \rangle$ of the system and find its form for $T \to 0$.

Problem 10

A particle of mass m_o decays at rest in the lab frame, producing particle 1 of mass m_1 and particle 2 of mass m_2. Find the energy of the particle 1 in the lab frame.

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Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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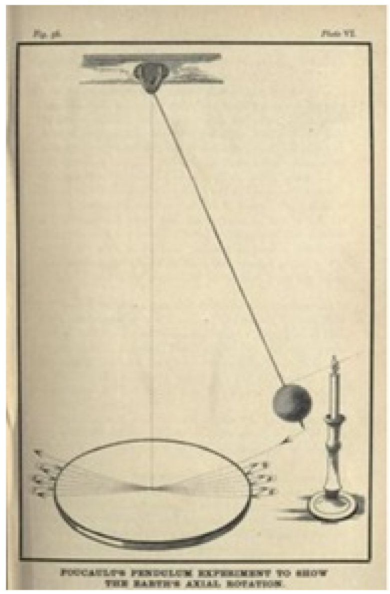
Problem 1

The Coriolis force is a force that acts on objects that are in motion relative to a rotating reference frame. The magnitude of the Coriolis acceleration of the object is a $\vec{a}_C = -2m\vec{\Omega} \times \vec{v}$, where $\vec{a}_C$ is the acceleration of the particle in the rotating system, $\vec{v}$ is the velocity of the particle with respect to the rotating system, and $\vec{\Omega}$ is the angular velocity vector having magnitude equal to the rotation rate ω. The effect of the Coriolis force on the pendulum produces a precession, or rotation with time of the plane of oscillation. Describe the motion of this system, know as a Foucault pendulum. Assume that the oscillations have small amplitude with the horizontal excursions small compared with the length of the pendulum.



Problem 2

Consider a tank that is divided by a barrier into two equal volumes V.

a) One side of the tank is filled with N molecules of an ideal gas. The other side is empty. What is the change in entropy of the entire system if the barrier is removed and the system is equilibrated? Assume that the tank is thermally insulated.

b) The barrier is closed again and the gas on one side is instantaneously heated up to a higher temperature T_i1 while the gas on the other side remains at the initial temperature T_i2. Both sides of the tank are in thermal contact and allowed to equilibrate. What is the final temperature T_f of the gas?

c) Calculate the change in entropy of the entire system in b) as a function of the initial temperatures.

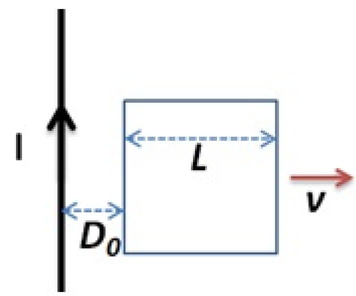
d) Now suppose that the barrier is removed and that the entire equilibrated gas with the heat capacity C_P and C_V undergoes the following quasi-static cycle:

1. a $\rightarrow$ b adiabatic expansion from V_1 to V_2.
2. b $\rightarrow$ c cooling at constant volume from T_1 to T_2.
3. c $\rightarrow$ d adiabatic compression from V_2 to V_1.
4. d $\rightarrow$ a heating at constant volume V_1.

Calculate the thermal efficiency from the ratio of the entire work performed by the gas and the heat taken up in 4).

Problem 3

An infinite straight wire carries a constant current I. A square loop, of total resistance R, has sides of length L, two of which are parallel to the wire. The closest side of the loop is initially at distance D_0 from the wire, and the loop is moving away from the wire, with a constant velocity v, under the influence of an external force.



Compute

1. The flux of the magnetic field through the loop at a generic time t;
2. The induced current in the loop and its direction;
3. The energy dissipated through the loop per unit time;
4. The net magnetic force on the loop;
5. The power input by the external agent.

Problem 4

A particle of spin 1/2 has a magnetic moment $\vec{\mu} = \gamma \vec{s}$, where γ is the gyromagnetic ratio and $\vec{s}$ is the spin operator. The particle is placed in a magnetic field $\vec{B}(t) = B(t)\hat{k}$ with its spin in the $+x$ direction at time $t = 0$. Here $\hat{k}$ is the unit vector in the $+z$ direction.

1. Find the energy eigenvalues of the particle as functions of t.
2. Find the spin wave function of the particle at time $t > 0$.

Problem 5

Spaceships A, B, and C have the same proper length l, and according to an observer on Earth, are moving with same relativistic speed v in the $+x$, $-x$, and $-y$ direction, respectively. Find the length of each spaceship as measured by an observer in the Spaceship A.