University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Spring 2017 – PART I

Thursday, January 12th, 2017 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

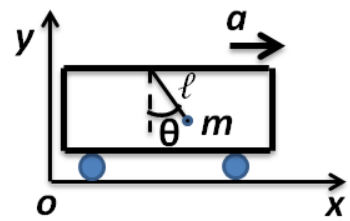
USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

A pendulum (having mass, m, and string length, ℓ,) hangs from the ceiling of a train (see the figure). The train is moving with a constant acceleration, a. Find the Lagrangian of the pendulum and equation of motion for θ. In the limit of a small angle θ, what is the period of the oscillation?

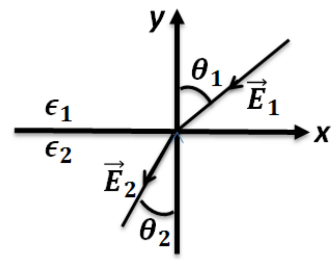


Problem 2

Suppose we have a system of N non interacting particles (spin = 0) at temperature T. Each particle can be in one of three energy states. The energy in the ground state is 0 and the energies in both excited states are ε and ε'. Calculate the entropy and Helmholtz free energy of the system using its partition function. What is the average energy of the system at temperature T?

Problem 3

At the interface between one linear dielectric and another, the electric field lines bend at the interface as shown. Assuming there is no free charge at the interface, find out the relation between angles θ_1 *and* θ_2 as a function of ϵ_1 *and* ϵ_2. *Warning: two arrows indicate the directions of two static electric field not the light rays in optics.*



Problem 4

Consider an ideal classical gas with N atoms. The gas expands quasi-statically and reversibly from a volume of V_0 to $2V_0$ at constant temperature T_0. Calculate the work done by the gas, the heat taken up by the gas and the change in entropy as a function of T_0 and V_0.

Problem 5

Electron in the hydrogen atom in the ground state is described by the wave function

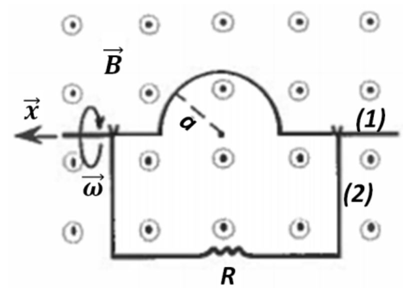
$$\psi(x,y,z) = Aexp(-r/r_1)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, A *and* r_1 *are constants.*

Use the Schrodinger equation to find r_1 and the energy eigenvalue E in terms of electron's mass and electric charge.

Problem 6

A long perfect conducting wire (1) consists of a portion of a half circle having radius a. Another perfect conducting wire (2) has a U-shape and is connected to a resistor having resistance R. The wire (1) rotates around the x-axis with angular velocity ω while it maintains in good electrical contact with wire (2) which is stationary. The whole circuit is placed in a uniform magnetic field as shown in Fig. a) Work out the induced emf. b) Find out the average power dissipated.



Problem 7

A particle moves in the one-dimensional infinite square well, $V(x) = 0,\ if\ 0 < x < \ell$ and $V(x) = \infty,\ if\ x < 0\ or\ x > \ell$. The probability of finding the particle in the interval $(\ell/3, 2\ell/3)\ is\ W_0$, where the subscript 0 marks the ground state. Find W_0.

Find the probability W_n for the excited states, $n = 1, 2, 3,...$ as a function of n. What is the limit of $W_n\ at\ n\ \rightarrow\ \infty$?

Problem 8

Consider the water wave (ϕ) propagates at the air-water interface along a canal of constant depth h. After considering both the effect of gravity and surface tension, the time variation of ϕ can be written as $d^2\phi/dt^2 + \{[gk + (\sigma k^3/\rho)]tanh(kh)\}\phi = 0$. Here g is the gravity acceleration; σ, the surface tension; k, the wave vector and ρ , the density of water.

a) What is the expression of the frequency describing this wave motion?
b) What is the properly approximation for the case of deep water ($h \gg \lambda$) waves?
c) What are the phase and group velocities for the deep water wave?
d) Will the surface tension effect be more important in the short or long wave length? Give your physical reasoning.

Problem 9

A circular table top of radius 1 m and mass 3 kg is supported by three equally spaced legs on the circumference (see the Figure). When a vase is placed on the table, the legs support 1, 2 and 3 kg, respectively. How heavy is the vase? (*This part, 2 points*) Where is it located on the table?

Problem 10

A muon has a half-life time of 1.5 X 10^{-6} second. It is found that one out of a million fast muons survives after traveling a distance of 1.3 X 10^{7}m as measured in the lab frame. Find the Lorentz factor of the fast muons in the lab frame.

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GRADUATE WRITTEN EXAMINATION

Spring 2017 – PART II

Friday, January 13th, 2017 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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Problem 1

The followings are four Maxwell's equations.

$$\nabla \cdot \vec{E} = \rho/\epsilon_o, \quad \vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t, \quad \nabla \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \partial \vec{E}/\partial t.$$

a) In vacuum, derive the wave equation for the B-field. (4 points)

The functions,

$$\vec{E} = \tilde{E}_0 exp[j(kz - \omega t)]\hat{x}; \ and \ \vec{B} = \tilde{B}_0 exp[j(k'z - \omega t + \varphi)]\hat{n}$$

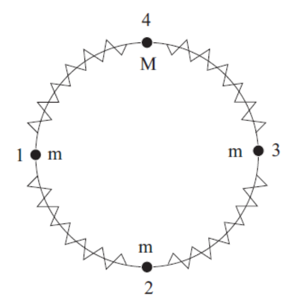
are the general solutions of wave equation of E and B field, separately. Here $\tilde{E}_0$ and $\tilde{B}_0$ are complex constants. $\hat{n}$ is a unit vector.

b) Why do we only need to consider sinusoidal functions for the solution of wave equations? (2 points)

c) In order to represent an EM wave, give your physical arguments for the relations between k and k', the direction for $\hat{n}$ and the value for φ. Give you physical reasoning using the relevant wave equations or Maxwell's equations. (10 points)

d) Obtain the Poynting vector as a function of the parameters given in this problem. (4 points)

Problem 2



Consider four masses that can only move along a circle with fixed radius r. The masses are connected by four identical springs with spring constant k. The masses are given by $m_1 = m_2 = m_3 = m,$ and $m_4 = M$. Find the eigen-frequencies of the system's oscillations.

Hint: first write down the appropriate equations with the displacement η_i of $each$ $mass, then$ $make$ the $following$ $change$ of $variables$:

$$\eta_1 = (\xi_1 + \xi_2)/2\,; \ \eta_2 = (\xi_3 + \xi_4)/2\,; \ \eta_3 = (\xi_1 - \xi_2)/2\,; \ \eta_4 = (\xi_3 - \xi_4)/2$$

Problem 3

Consider the quartic anharmonic oscillator with the Hamiltonian,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \beta x^4,$$

where β is the anharmoinicity coefficient. Find the correction of the first order in β to the ground state energy of the harmonic oscillator. Also find the correction terms for the first and second excited levels.

Problem 4

Rubber has interesting thermodynamic and mechanical properties. First, we look at the description of a single, one dimensional polymer chain made of N rigid segments with length *l*. Each segment can be orientated in positive or negative direction with equal probability.

a) Calculate the number of configurations of the entire chain as a function of the end-to-end distance *x* and *N*.

b) The probability for a certain end-to-end distance x can be approximated by

$$P(x) = \frac{2}{\sqrt{2\pi N}} \exp\left(-\frac{x^2}{2N}\right)$$

Calculate from this the total number of configurations and then the entropy of the polymer as a function of $x, T \text{ and } N$. Here *N* is a large number.

c) Calculate the force and spring constant of the polymer as a function of $x \text{ and } T$. What will happen if a weight is held by the rubber band and the band is heated up?

Problem 5

Consider the scattering of particle 1 that is massless by particle 2 of mass *m* that is at rest in the lab frame. Let $E \text{ and } E'$ be the lab-frame energy of particle 1 before and after scattering, respectively. Find the probability distribution of E' assuming that in the center-of-mass frame, particle 1 is scattered in all directions with equal probability.