Notes

We know that we can write down the position of a point on our disk in terms of the distance to the point, r, on the disk and the angle θ that a line joining the point and the center of the disk makes with respect to the x-axis. We can write this down in vector form where $\vec{r}$ is the position vector:

$$\vec{r} = r\left[\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right].$$

In this case θ depends on time. Just like with x in linear motion we can write down θ in terms of angular velocity and angular acceleration:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2.$$

Now, if we want to find velocity we take the derivative of the position vector, remembering to use the chain rule!

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} = r\frac{d\theta}{dt}\left[-\sin(\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2)\hat{\imath} + \cos(\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2)\hat{\jmath}\right] \\ &= r(\omega_0 + \alpha t)\left[-\sin(\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2)\hat{\imath} + \cos(\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2)\hat{\jmath}\right].\end{aligned}$$

If we want to find the magnitude of the velocity remember that we can take the dot product of velocity with itself:

$$|v|^2 = \vec{v}\cdot\vec{v}.$$

This means we multiply the x components of the two vectors togther, multiply the y components of the two vectors together, and add the two results. Remeber that $\cos^2\theta + \sin^2\theta = 1$ even if θ depends on time! Therefore,

$$|v|^2 = r^2(\omega_0 + \alpha t)^2.$$

Remember that just like with linear velocities and positions, $\omega(t) = \omega_0 + \alpha t$ is just like $v(t) = v_0 + at$. and so we have that

$$|v| = r\omega(t)$$

just as was derived in class!

When we want to find the acceleration things get a bit more complicated. We have to use a product rule (and a chain rule again)! Now, we'll think about this intuitively. We have that

$$\vec{v} = r\omega(t)\left[-\sin\theta(t)\hat{\imath} + \cos\theta(t)\hat{\jmath}\right].$$

If we first take the derivative of the $r\omega(t)$ term we are taking the derivative of the magnitude of the velocity of a point on the rim. This represents the *tangential* acceleration of the object. If we take the derivative of the vector portion of the above equation we are calculating acceleration due to the changing vector of the velocity. If we remember from circular motion, this is the *centripetal* acceleration. Carrying out this derivative like in class we have

$$\begin{aligned}\vec{a} &= r\alpha\left[-\sin\theta(t)\hat{\imath} + \cos\theta(t)\hat{\jmath}\right] + r\omega(t)^2\left[-\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}\right] \\ &= \vec{a}_t + \vec{a}_c \\ \vec{a}_t &= r\alpha\left[-\sin\theta(t)\hat{\imath} + \cos\theta(t)\hat{\jmath}\right] \\ \vec{a}_c &= r\omega(t)^2\left[-\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}\right].\end{aligned}$$

Now we see that the centripetal acceleration points opposite the direction of the position vector, and the tangential acceleration is perpendicular to the centripetal acceleration.

In the lab report you are asked to find the magnitude of the tangential acceleration of a point on the disk and a point on the spool. To find the tangential acceleration of a point on the disk, we can simply take the dot product of the tangential acceleration with itself and find that

$$|a_t| = \sqrt{r^2\alpha^2(\sin^2\theta + \cos^2\theta)} = r\alpha.$$

If you are trying to find this from your fits, and you have

$$\begin{aligned}v_x &= A + (Bt + C)\cos\theta(t) \\ v_y &= A + (Bt + C)\sin\theta(t)\end{aligned}$$

then for tangential acceleration of the point on the disk you follow the steps above and *just* take the derivative of the $Bt + C$ term! Then we get

$$\begin{aligned}a_{tx} &= B\cos\theta(t) \\ a_{ty} &= B\sin\theta(t).\end{aligned}$$

For a the tangential acceleration of the radius of the spool, you need to think about how the tangential acceleration of the disk is related to the angular acceleration of both the disk and the spool and then how the tangential acceleration of the spool is related the the angular acceleration of the spool.

Aside

This is NOT asked for in the lab report. However, what we did in class to find the magnitude of total acceleration (which is not asked for in the report) is to take the dot product of the acceleration with itself:

$$\begin{aligned}|a|^2 &= \vec{a}\cdot\vec{a} \\ &= (\vec{a}_t + \vec{a}_c)\cdot(\vec{a}_t + \vec{a}_c) \\ &= \vec{a}_t\cdot\vec{a}_t + 2\vec{a}_t\cdot\vec{a}_c + \vec{a}_c \\ &= |a_t|^2 + |a_c|^2\end{aligned}$$

where in the last line we use that the tangential and centripetal acceleration are perpendicular and so their dot products are zero (you can check this if you'd like). This was how we found that

$$|a_{total}| = \sqrt{r^2\alpha^2 + r^2(\omega_0 + \alpha t)^4}.$$

It's also interesting to note that $|a_c|$ depends on time. This is because a_c is related to the tangential velocity, which is changing in time!