

Modelling Polarized Dust Emission

Andrei Frolov

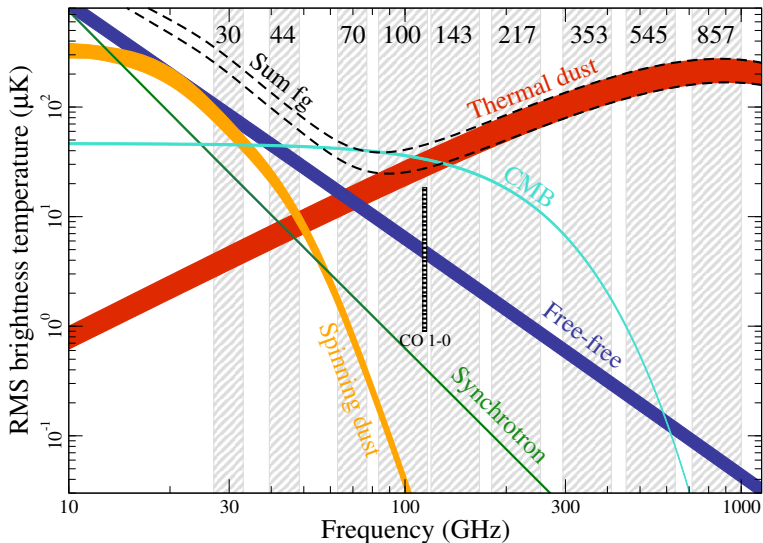
PICO Collaboration Science Meeting

*University of Minnesota
Minneapolis, MN, USA*

1 May 2018

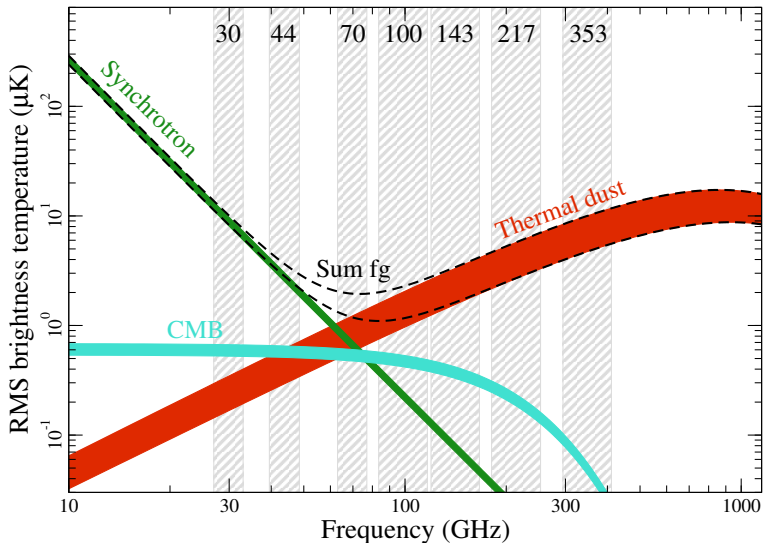
CMB vs. Astrophysical Foregrounds

◦ Intensity ◦ Polarization ◦ Atmospheric Transmission ◦



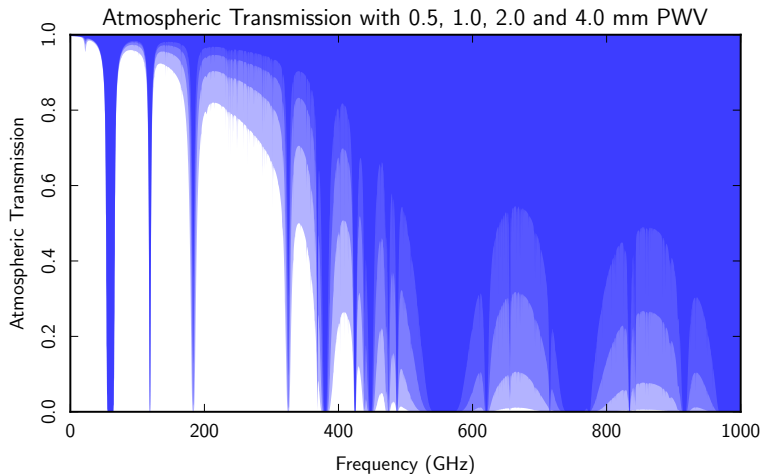
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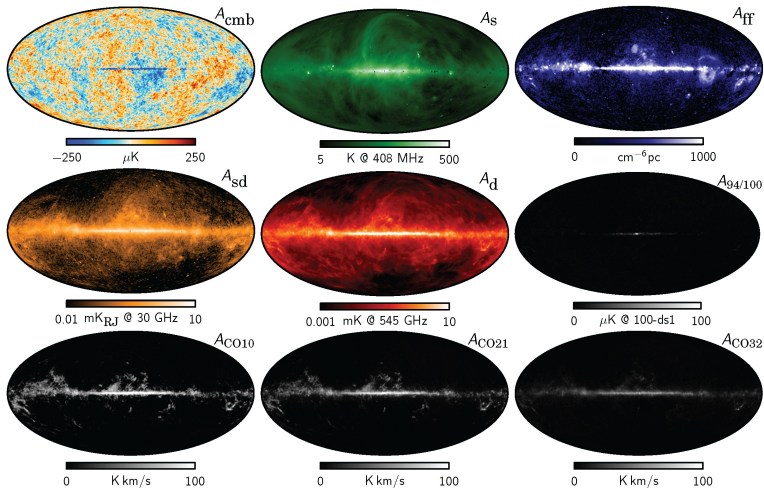


CMB vs. Astrophysical Foregrounds

◦ Intensity ◦ Polarization ◦ Atmospheric Transmission ◦



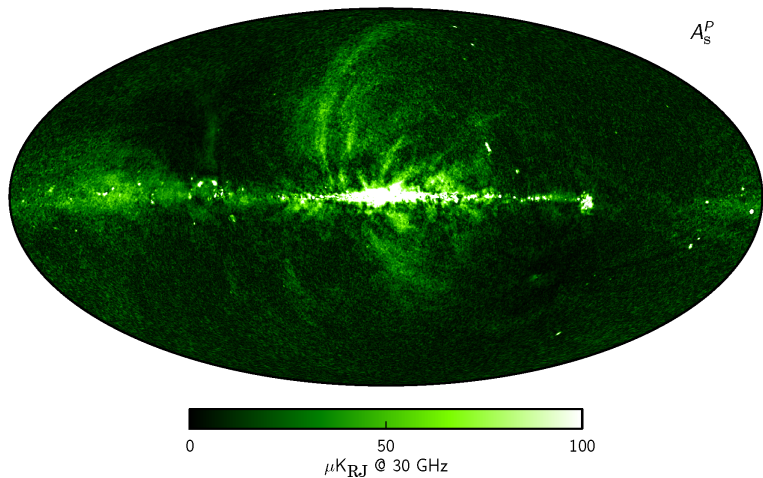
Temperature Component Maps



Polarization Component Maps

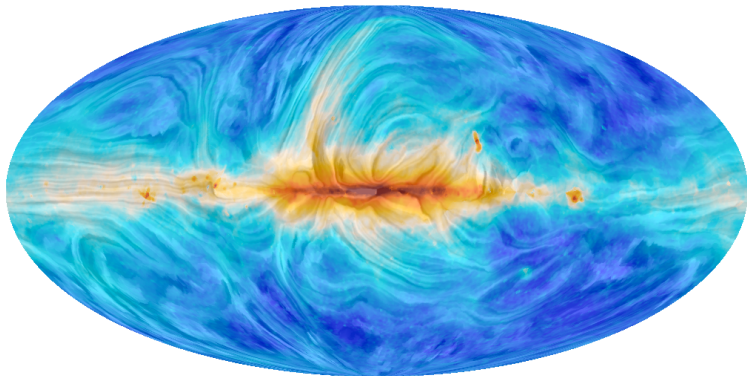
- Two main foregrounds, synchrotron emission and thermal dust
- Amplitude of CMB polarization is less than foregrounds
- Dust emission is highly polarized (polarization fraction is up to 20%)

Synchrotron Polarization Amplitude



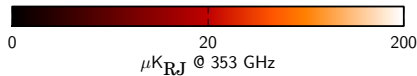
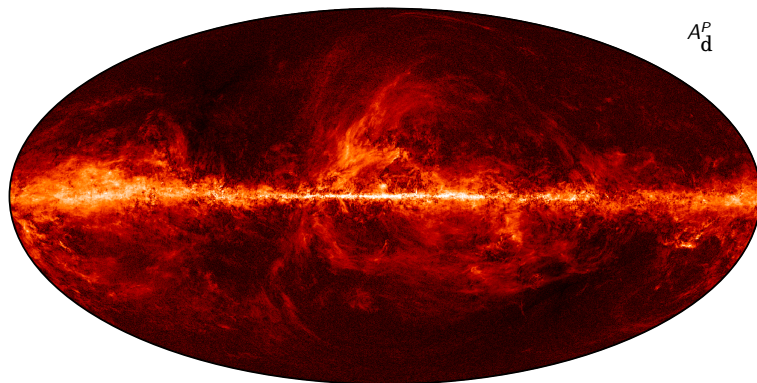
$$P = \sqrt{Q^2 + U^2}, \text{ at 30 GHz, smoothed to } 40'$$

Magnetic Field and Total Intensity



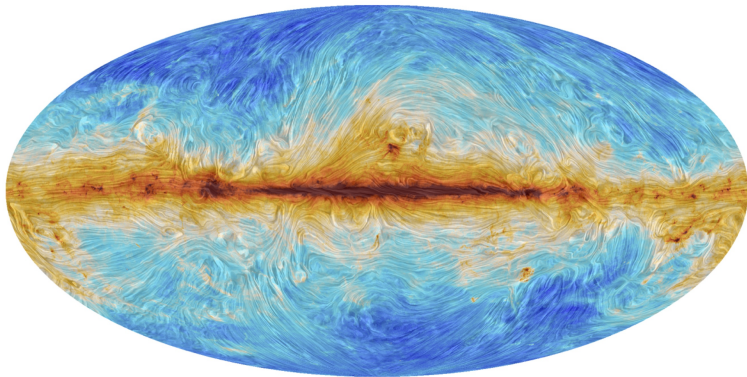
The colours represent intensity. The “drapery” pattern indicates the orientation of magnetic field projected on the plane of the sky, orthogonal to the observed polarization.

Dust Polarization Amplitude



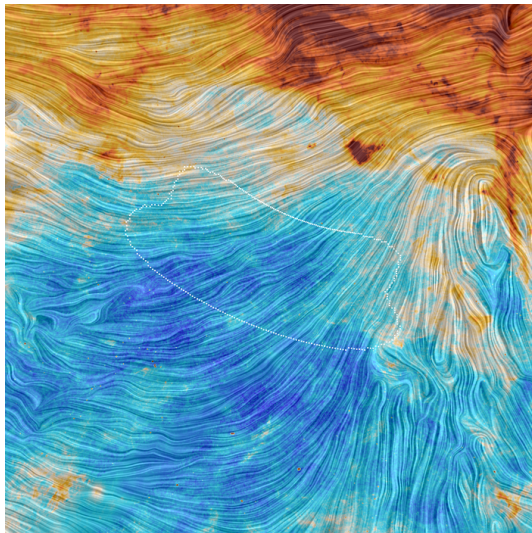
$$P = \sqrt{Q^2 + U^2}, \text{ at 353 GHz, smoothed to } 10'$$

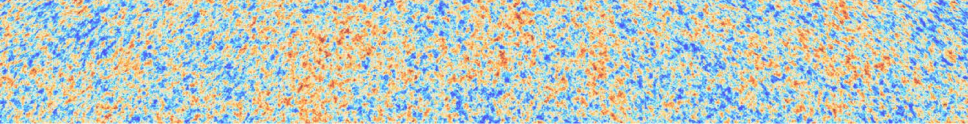
Magnetic Field and Total Intensity



The colours represent intensity. The “drapery” pattern indicates the orientation of magnetic field projected on the plane of the sky, orthogonal to the observed polarization.

Planck View of BICEP2 Field





Modelling Polarized Dust Emission

Polarized Dust Emission

Polarization is caused by magnetic field alignment:

$$I = \int S_{\nu} e^{-\tau_{\nu}} d\tau_{\nu} \left[1 - p_0 \left(\cos^2 \gamma - \frac{2}{3} \right) \right]$$
$$\left\{ \begin{array}{c} Q \\ U \end{array} \right\} = \int S_{\nu} e^{-\tau_{\nu}} d\tau_{\nu} \left\{ \begin{array}{c} \cos 2\phi \\ \sin 2\phi \end{array} \right\} p_0 \cos^2 \gamma$$

(p_0 is intrinsic polarization fraction ~ 0.21)

For a single layer, P/I determines magnetic field orientation:

$$\frac{I - P}{I + P} = 1 - \frac{6p_0}{2p_0 + 3} \cos^2 \gamma$$

Polarization Fraction Tensor

Transform polarization tensor into polarization fraction tensor:

$$\begin{bmatrix} i+q & u \\ u & i-q \end{bmatrix} = \ln \begin{bmatrix} I+Q & U \\ U & I-Q \end{bmatrix}$$

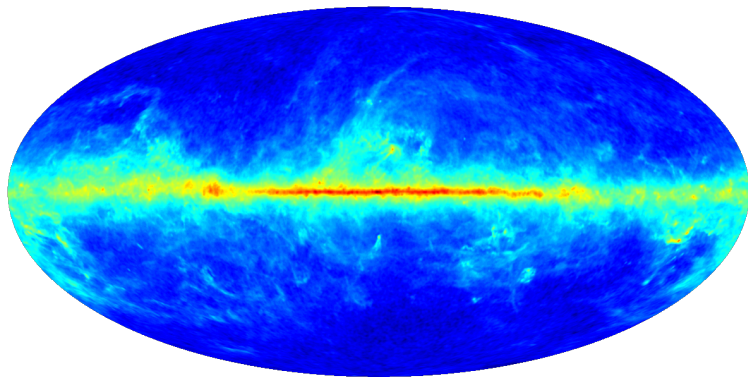
This is an invertible transformation on IQU maps:

$$i = \frac{1}{2} \ln(I^2 - P^2), \quad q = \frac{1}{2} \frac{Q}{P} \ln \frac{I+P}{I-P}, \quad u = \frac{1}{2} \frac{U}{P} \ln \frac{I+P}{I-P}$$

$$I = e^i \cosh p, \quad Q = \frac{q}{p} e^i \sinh p, \quad U = \frac{u}{p} e^i \sinh p$$

Polarization Fraction Tensor

◦ $\ln |E \circ B|$ ◦ $i \circ e \circ b$ ◦

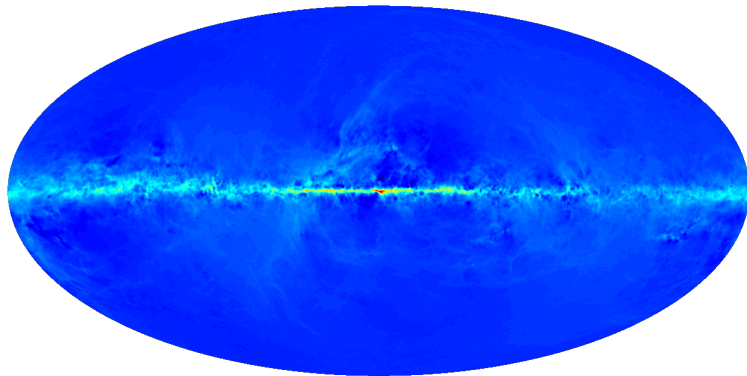


-3.58

-0.277

Polarization Fraction Tensor

◦ $\mathbf{I}$ ◦ $\mathbf{E}$ ◦ $\mathbf{B}$ ◦ $\mathbf{i}$ ◦ $\mathbf{e}$ ◦ $\mathbf{b}$ ◦



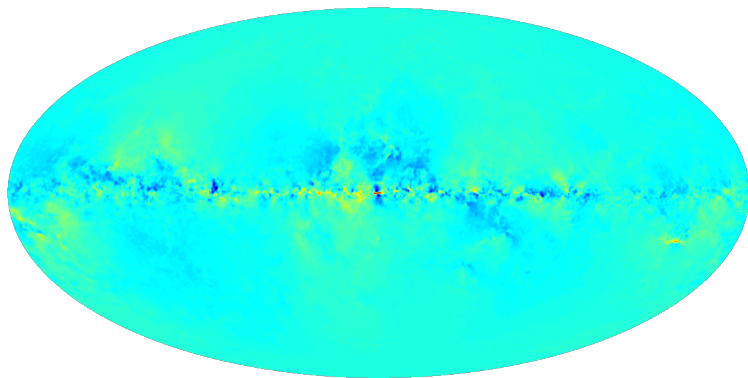
-1.308E-03



+5.305E-03

Polarization Fraction Tensor

◦ $\ln I$ ◦ E ◦ B ◦ i ◦ e ◦ b ◦



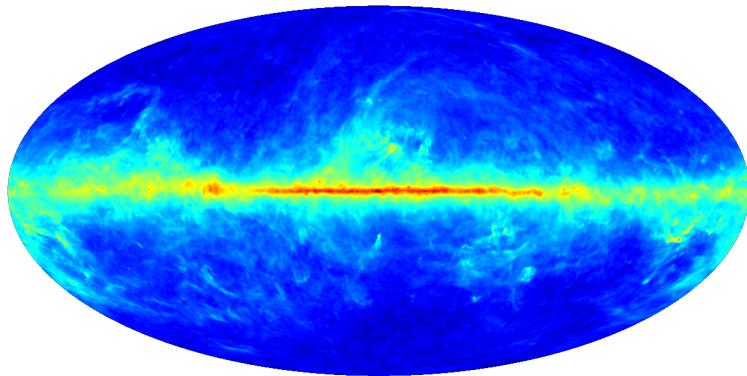
-1.904E-03



+2.685E-03

Polarization Fraction Tensor

◦ $\mathbf{I}$ ◦ $\mathbf{E}$ ◦ $\mathbf{B}$ ◦ $\mathbf{i}$ ◦ $\mathbf{e}$ ◦ $\mathbf{b}$ ◦

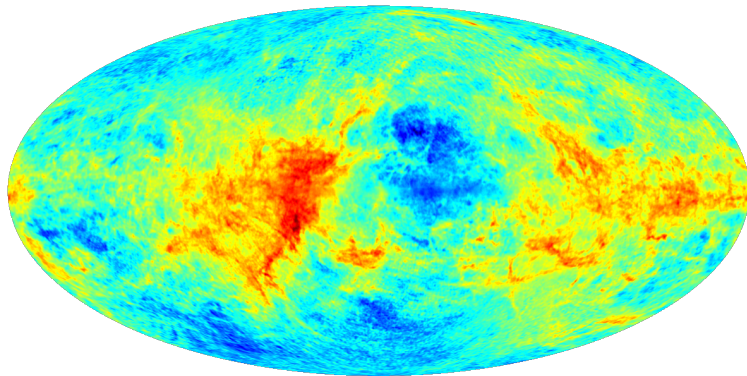


-8.23

-0.638

Polarization Fraction Tensor

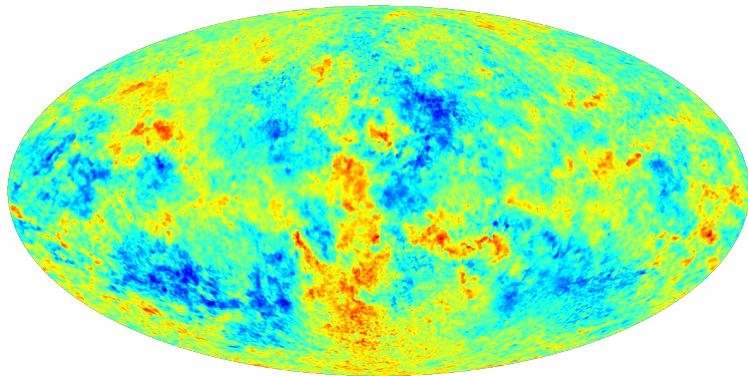
◦ $\mathbf{I}$ ◦ $\mathbf{E}$ ◦ $\mathbf{B}$ ◦ $\mathbf{i}$ ◦ $\mathbf{e}$ ◦ $\mathbf{b}$ ◦



-0.194  +0.191

Polarization Fraction Tensor

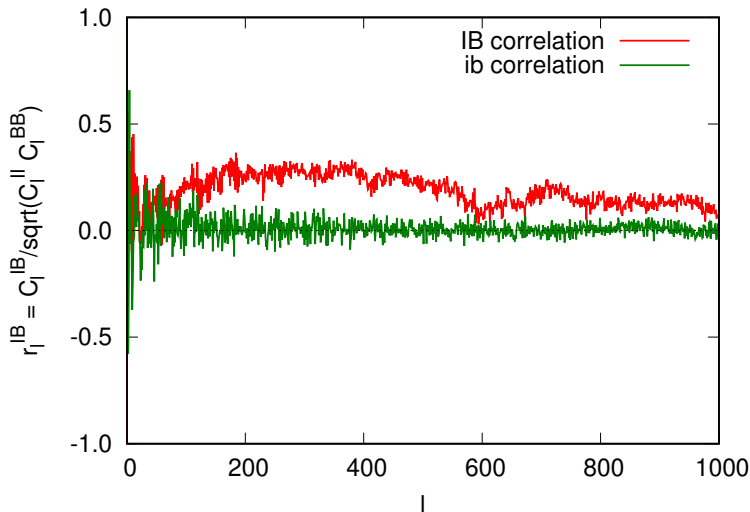
◦ $\mathbf{I}$ ◦ $\mathbf{E}$ ◦ $\mathbf{B}$ ◦ $\mathbf{i}$ ◦ $\mathbf{e}$ ◦ $\mathbf{b}$ ◦



-0.121  +0.117

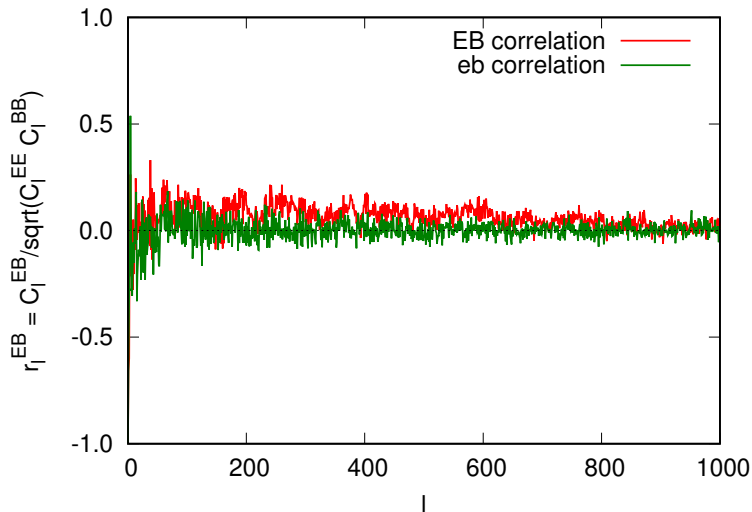
Parity-Violating Correlations Disappear!

◦ $\mathbf{l} \times \mathbf{B}$ ◦ $\mathbf{E} \times \mathbf{B}$ ◦ $\mathbf{l} \times \mathbf{E}$ ◦ $\nabla \times \mathbf{v}$



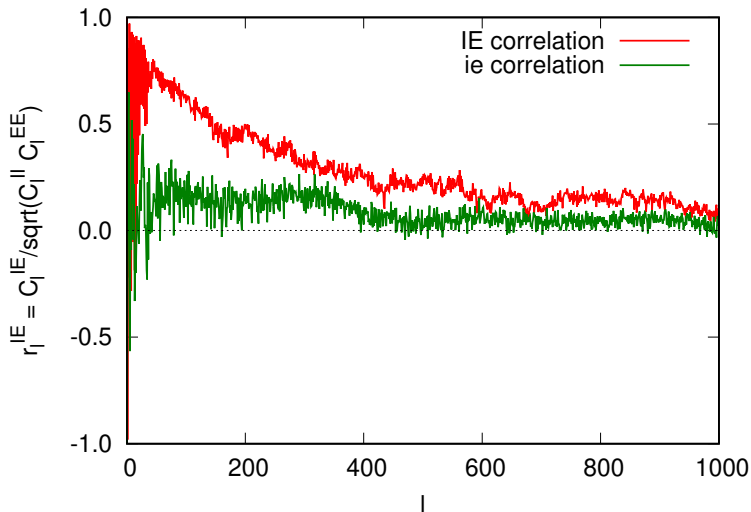
Parity-Violating Correlations Disappear!

◦ $\mathbf{l} \times \mathbf{B}$ ◦ $\mathbf{E} \times \mathbf{B}$ ◦ $\mathbf{l} \times \mathbf{E}$ ◦ $\nabla \times \mathbf{v}$



Parity-Violating Correlations Disappear!

◦ $\mathbf{l} \times \mathbf{B}$ ◦ $\mathbf{E} \times \mathbf{B}$ ◦ $\mathbf{l} \times \mathbf{E}$ ◦ $\nabla \times \mathbf{v}$

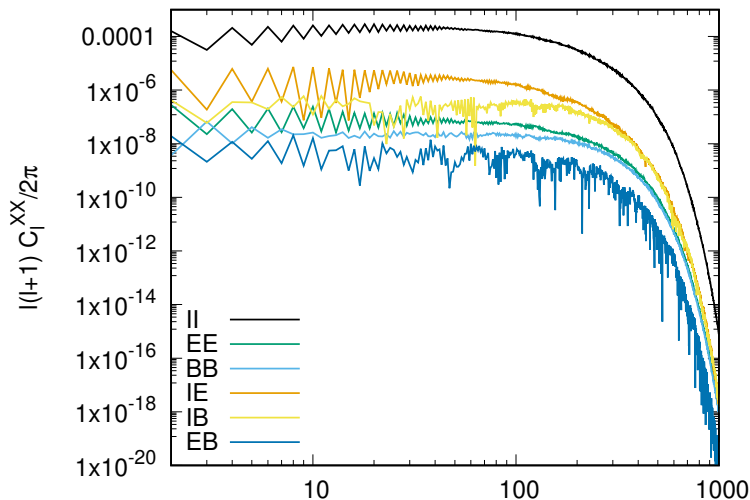


Randomize b to Make It Uncorrelated GRF

$$a_{\ell m}^{(b)} \mapsto \exp[2\pi i \phi_{\ell m}] \cdot a_{\ell m}^{(b)}$$

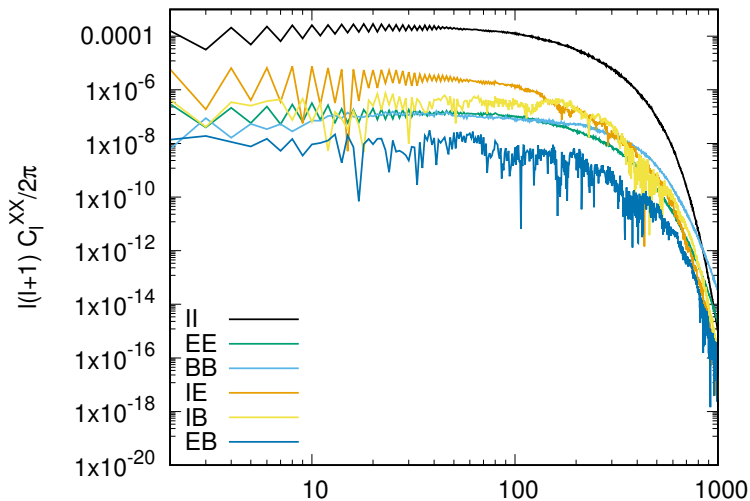
Parity-Violating Correlations Re-appear!

◦ original ◦ randomized b ◦



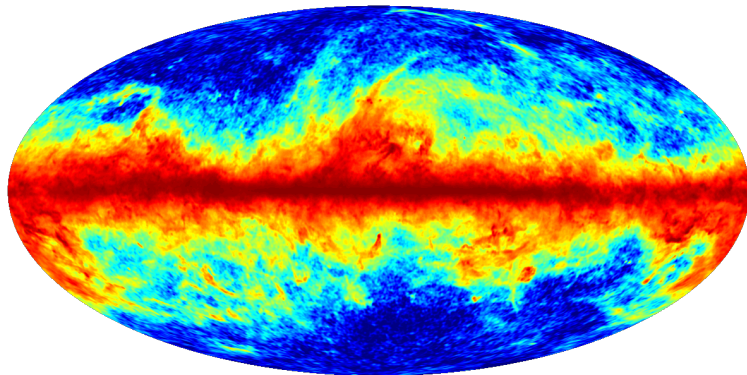
Parity-Violating Correlations Re-appear!

◦ original ◦ randomized b ◦



Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



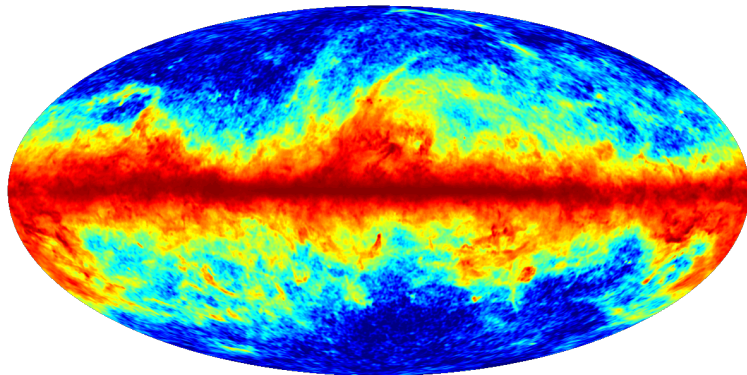
+5.960E-08



+1.00

Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



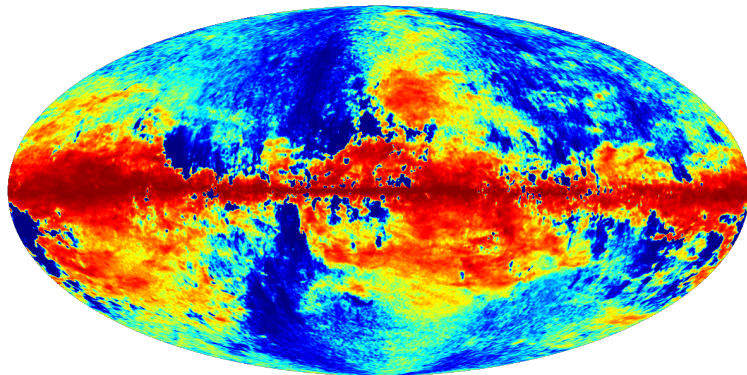
+7.947E-08



+1.00

Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



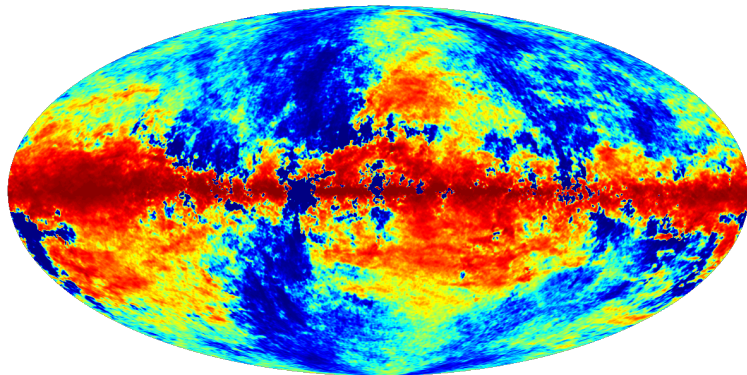
+1.272E-06



+1.00

Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



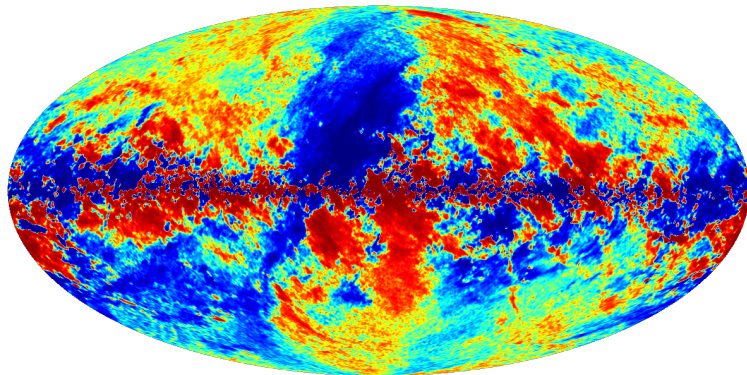
+1.570E-06



+1.00

Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



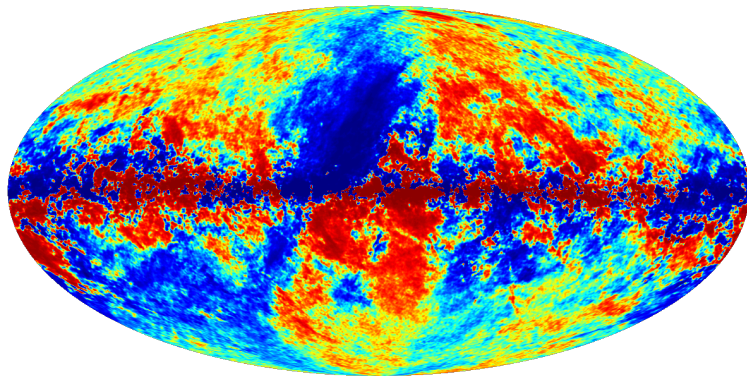
+1.470E-06



+1.00

Randomized Polarization Fraction Tensor

◦ original I ◦ random I ◦ original Q ◦ random Q ◦ original U ◦ random U ◦



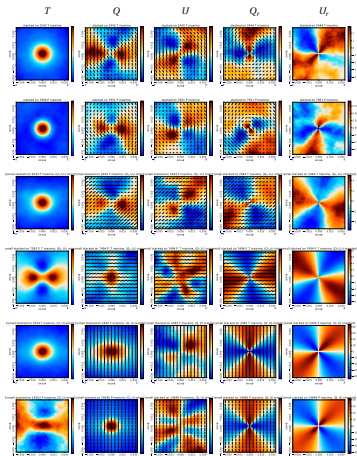
+1.450E-06



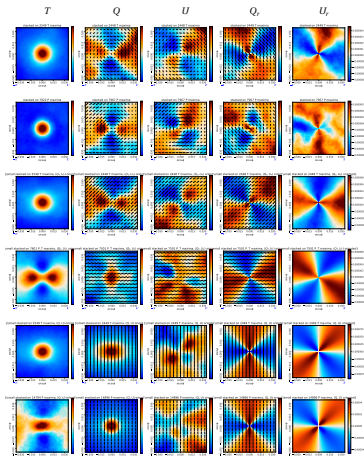
+1.00

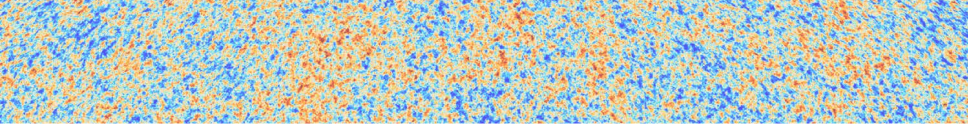
Realizations Reproduce Sky Statistics!

HFI 353GHz, 40' FWHM smoothed, nside = 2048, common mask



Randomized 353GHz, 40' FWHM smoothed, nside = 2048, common mask





Non-Gaussian Map Characterization with Oriented Stacking

The Stacking Family

Three key elements:

- A** What to stack? (cosmic field u)
- B** Where to stack? (selection of patches, e.g., peaks)
- C** How to stack? (patch orientations)

“where” and “how” give constrained parameter(s) q ;

	WMAP & Planck 2013	Planck 2015
What	T, Q, U, Q_r, U_r	$T, Q, U, Q_r, U_r, E, B, Q_T, U_T, \zeta_{\text{dv}}, \dots$
Where	T peaks	$T, E, B, Q^2 + U^2, Q_T^2 + U_T^2, \zeta_{\text{dv}} \dots$ peaks
How	unoriented	oriented and unoriented

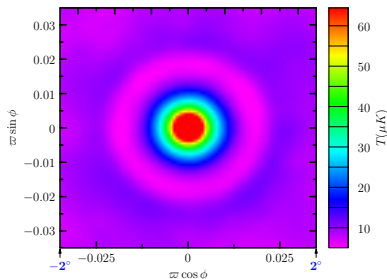
For Gaussian fields,

$$\langle u|q; \text{peak, orientation} \rangle = \langle u q^\dagger \rangle \langle q q^\dagger \rangle^{-1} \langle q | \text{peak, orientation} \rangle.$$

Planck 2015: Stacking Temperature

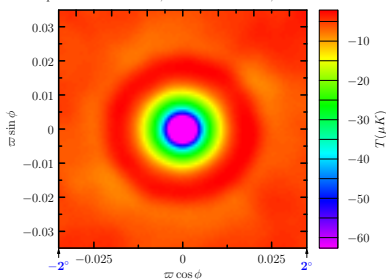
T on hot spots

24645 patches on T maxima, random orientation, threshold $\nu=0$



T on cold spots

24582 patches on T minima, random orientation, threshold $\nu=0$



resolution: FWHM 15 arcmin

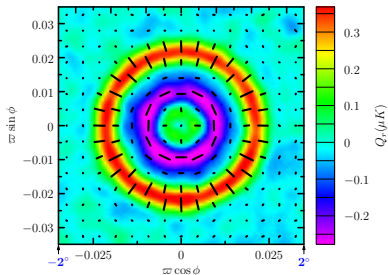
Peaks are selected above a threshold $|T_{\text{peak}}| > \nu \sqrt{\langle T^2 \rangle}$ ($\nu=0$ here).

Full statistics in [Isotropy and Statistics paper!](#)

Planck 2015: Stacking Polarization

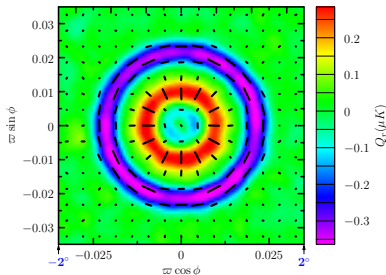
Q_r on hot spots

33214 patches on T maxima, random orientation, threshold $\nu=0$



Q_r on cold spots

33126 patches on T minima, random orientation, threshold $\nu=0$

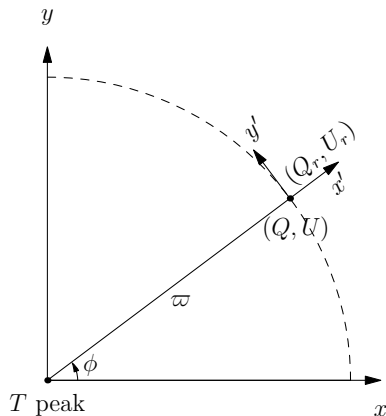


resolution: FWHM 15 arcmin

Peaks are selected above a threshold $|T_{\text{peak}}| > \nu \sqrt{\langle T^2 \rangle}$ ($\nu=0$ here).

Full statistics in [Isotropy and Statistics paper](#)!

How to Rotate the Polarization Field



flat-sky polar coord. (ϖ, ϕ) :

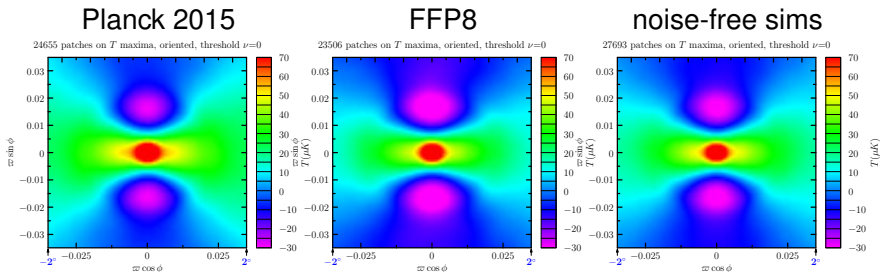
$$\varpi = 2 \sin \frac{\theta}{2}$$

$$Q_r = -Q \cos 2\phi - U \sin 2\phi$$

$$U_r = -U \cos 2\phi + Q \sin 2\phi$$

Oriented Stacking: T on T peaks

peak threshold $\nu = 0$, resolution FWHM 15 arcmin:



Angular dependence ($\cos m\phi$, $m = 0, 2$)

Noise has no noticeable impact.

How to Orient a Patch around a Peak

First derivative vanishes on the peak. Need to use the 2nd derivatives.

Intuitively (flat-sky limit):

$$Q_T \equiv \nabla^{-2}(\partial_y^2 - \partial_x^2)T, \quad U_T \equiv -2\nabla^{-2}(\partial_x \partial_y)T$$

Slightly non-intuitive (on the sphere):

$$Q_T(\mathbf{n}) \pm i U_T(\mathbf{n}) \equiv \sum_{l,m} \left[\int T(\mathbf{n}') Y_{lm}^*(\mathbf{n}') d^2\mathbf{n}' \right]_{\pm 2} Y_{lm}(\mathbf{n})$$

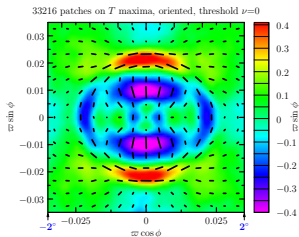
Orient the patch such that U_T **vanishes in the centre**.

$\langle u|q; \text{peak, orientation} \rangle(\varpi, \phi)$ decomposes to $\cos m\phi$, $m = 0, 2, 4$.

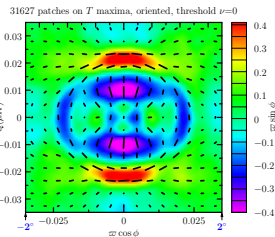
Oriented Stacking: Q on T peaks

peak threshold $\nu = 0$, resolution FWHM 15 arcmin:

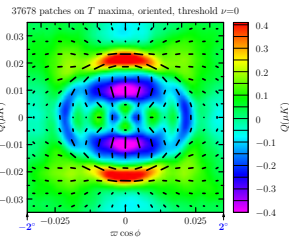
Planck 2015



FFP8



noise-free sims



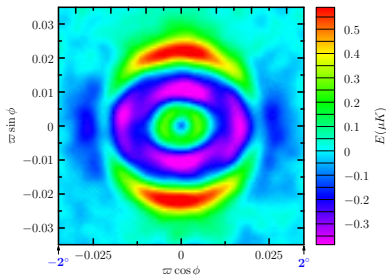
Angular dependence ($\cos m\phi$, $m = 0, 2, 4$)

Again noise has no noticeable impact.

Oriented Stacking of Polarization

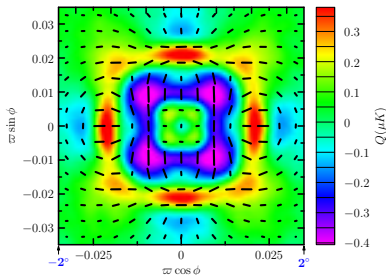
E on oriented T peaks

33216 patches on T maxima, oriented, threshold $\nu=0$



Q on oriented $Q_T^2 + U_T^2$ peaks

58099 patches on P_T maxima, oriented, threshold $\nu=0$

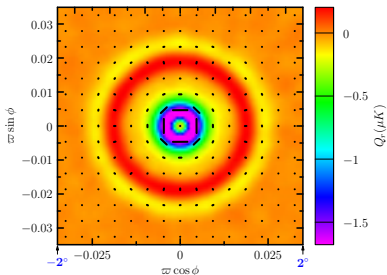


Planck 2015 (peak threshold $\nu = 0$; resolution FWHM 15 arcmin)

Stacking on Polarization Peaks

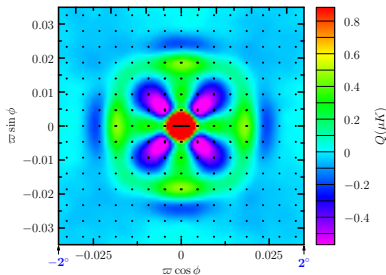
Q_r on unoriented E peaks

99529 patches on E maxima, random orientation, threshold $\nu=0$



Q on oriented $Q^2 + U^2$ peaks

196910 patches on P maxima, oriented, threshold $\nu=0$



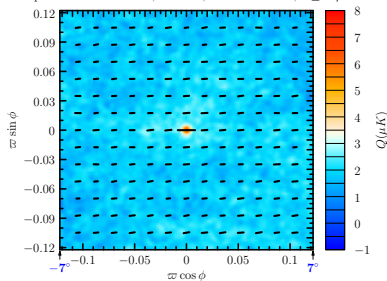
Planck 2015 (peak threshold $\nu=0$; resolution FWHM 15 arcmin)

Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

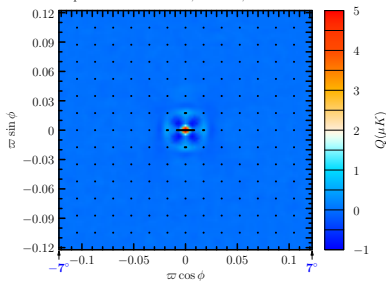
Dust Component, $T < 25\mu K$

43 patches on P maxima, oriented, threshold $\nu = 1$, $I \leq 25\mu K$



CMB Component

33536 patches on P maxima, oriented, threshold $\nu = 1$



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

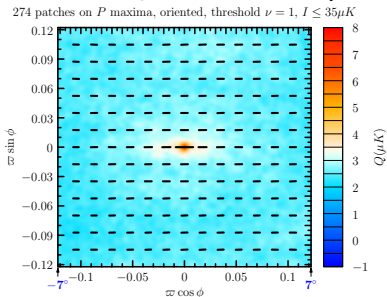
Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

T map FWHM 2° ; Q, U maps FWHM 15 arcmin.

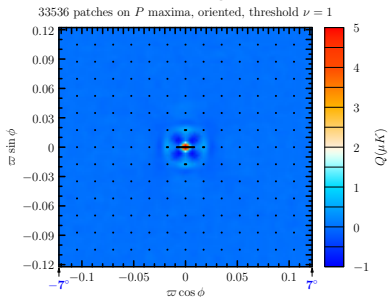
Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

Dust Component, $T < 35\mu K$



CMB Component



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

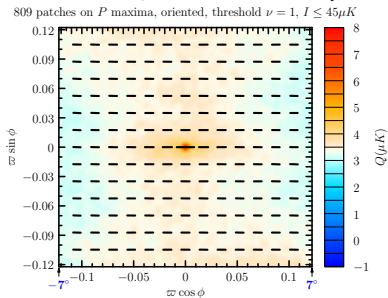
Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

T map FWHM 2° ; Q, U maps FWHM 15 arcmin.

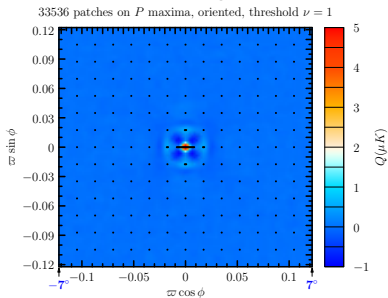
Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

Dust Component, $T < 45\mu K$



CMB Component



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

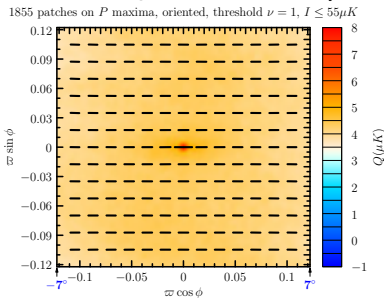
Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

T map FWHM 2° ; Q, U maps FWHM 15 arcmin.

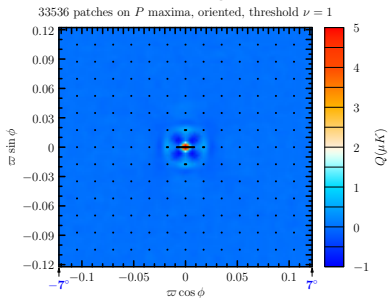
Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

Dust Component, $T < 55\mu K$



CMB Component



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

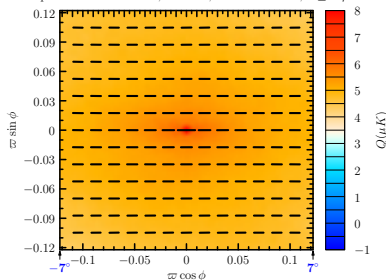
T map FWHM 2° ; Q, U maps FWHM 15 arcmin.

Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

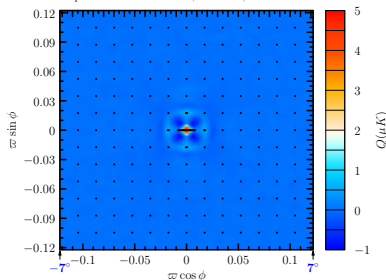
Dust Component, $T < 95\mu K$

6673 patches on P maxima, oriented, threshold $\nu = 1$, $I \leq 95\mu K$



CMB Component

33536 patches on P maxima, oriented, threshold $\nu = 1$



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

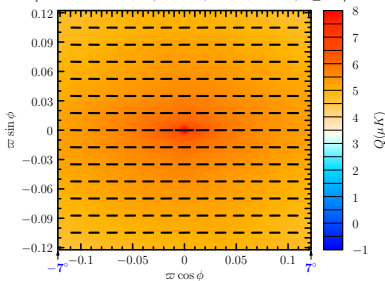
T map FWHM 2° ; Q, U maps FWHM 15 arcmin.

Stacking Polarized Dust

Planck 2015 Component Separated Commander Dust Map

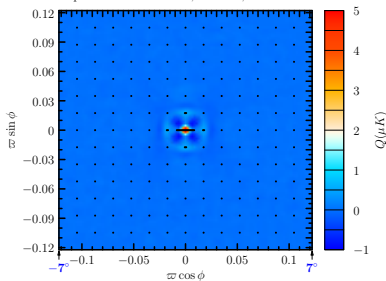
Dust Component, $T < 115\mu K$

8531 patches on P maxima, oriented, threshold $\nu = 1$, $I \leq 115\mu K$



CMB Component

33536 patches on P maxima, oriented, threshold $\nu = 1$



Q stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. U vanishes in the centre).

Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

T map FWHM 2° ; Q, U maps FWHM 15 arcmin.