We also note that to provide a foreground-cleaned CMB map at a given resolution, all the channels used for foreground cleaning must have at least that resolution. For primordial B-mode science at the level of $r \simeq 10^{-3}$, both the synchrotron and the dust should be mapped with an angular resolution of about $30'$ or better. For primordial B-mode science with $r \lesssim 0.01$, signal dominated primordial B-mode maps at $\ell \gtrsim 10$ also require delensing, by a factor of about 3 in power for $r = 0.003$, and about 10 for $r = 0.001$. In addition, for $r = 0.001$, figure 4 shows that the cosmic variance of the lensing signal in bins $\Delta\ell/\ell = 0.3$ is at about the level of the B-mode recombination bump, so that delensing is required to achieve a detection. For a noise level of $2\,\mu$K.arcmin, the required delensing by a factor of 3 in power can be achieved with CMB polarisation if the survey has an angular resolution of about $4'$ (see figure 5).

Alternatively, delensing can be achieved with precise maps of the CIB, and the capability to delens with both small-scale CMB polarisation and the CIB would provide a useful cross-check, as well as better delensing overall. Hence, in addition to the above CMB surveys, for enabling "ultimate" CMB polarisation science, possibly in combination with ground-based observations reaching an angular resolution of about $2'$, the space survey must also provide maps of high-frequency foregrounds at a matching angular resolution. This leads to the following additional requirements:

- dust polarisation maps with an angular resolution of about $2'$, for component separation on small scales;
- CIB intensity maps for an alternative way of B-mode delensing, essential for detecting primordial B modes below $r \simeq 0.01$.

These high-frequency observations must be done from space. On the other hand, small-scale synchrotron polarisation maps, which are too challenging to obtain from space because of the required telescope size, can be obtained with large ground-based radio telescopes if necessary.

4.5 Frequency channels

We distinguish requirements for the main CMB science (addressed with the observation of CMB polarisation E and B modes) and for other science goals.

CMB polarisation: The minimum of foreground emission relative to the CMB is located between 60 and 100 GHz. One might then think that the best frequency to observe CMB polarisation would be located precisely at this minimum. However, there are practical advantages to observing at higher frequencies.

- Since the beam size of a diffraction-limited telescope scales as the inverse of the frequency, the angular resolution that can be achieved at 120 and 180 GHz is, respectively, 2 and 3 times better than at 60 GHz.
- When CMB observations are made at 150 GHz or above, most of the complex low-frequency foreground emission signals are low enough that it suffices to reduce their contamination by a factor $\lesssim 10$. Only dust contamination needs to be accurately subtracted (at the sub-percent level) from the CMB polarisation observations.
- This dominant astrophysical foreground can be monitored at $\nu > 200$ GHz with better angular resolution than that of the CMB channels, while for a single dish multi-frequency instrument, low-frequency foregrounds are observed at lower angular resolution, and hence cannot be monitored over the full range of useful angular scales.

- Finally, as shown in Appendix A, the CMB sensitivity per focal-plane area in space is maximal in the 150–250 GHz frequency range.

Taking this into account, we should observe the CMB mostly in bands centred between about 130 and 200 GHz, avoiding the $\nu = 115\,\mathrm{GHz}$ and $\nu = 230\,\mathrm{GHz}$ CO $(J = 1 \rightarrow 0)$ and $(J = 2 \rightarrow 1)$ lines. We also need channels below and above this frequency range for monitoring foreground emission. A factor of around 2 in frequency provides proper leverage for distinguishing the various emission processes, i.e., we need frequency bands extending from about 65 to 400 GHz. Ground-based telescopes can provide synchrotron observations over large patches of sky at $\nu \lesssim 40\,\mathrm{GHz}$, so observing those frequencies with a space-based mission is not a priority. High frequencies, however, must be observed from space.

As discussed above in section 4.1.2, at least 10 different channels in that frequency range, and preferably more, are required to separate the different foreground emission signals from the CMB over a substantial fraction of sky. Additional bands are required to extend the fraction of sky that can be used for CMB science. Spreading frequency channels logarithmically in the 65–400 GHz frequency range with frequency ratios such that $\nu_{n+1} \simeq 1.15\,\nu_n$ (a sampling in frequency well matched with a bandwidth $\Delta\nu/\nu$ of approximately 30% per channel) yields a set of 15 frequency channels that is adequate for CMB science, with some safety margin. Actual *CORE* channels are obtained by a similar process (starting at $\nu_0 = 60\,\mathrm{GHz}$ instead of 65 GHz, and making the channels a bit closer in the main CMB frequency range).

Further optimization depends on assumptions about polarised foreground emission properties. To be on the safe side, it is preferable to pick a baseline with more channels than strictly required, in case polarised foreground emission turns out to be more complicated than current models might suggest.

Other science: We now consider options to further optimize the legacy value of the survey, in particular in combination with complementary ground-based CMB observations with CMB-S4.

Accurate science with galaxy clusters requires a frequency range covering both the minimum and the maximum of the thermal SZ distortion, i.e., ranging from around 120 GHz to 400 GHz. A few channels in that frequency range (at least three, but preferably more for redundancy) are required to separate the thermal SZ from the kinetic SZ and also to monitor temperature effects, which, for hot clusters, can modify the thermal SZ spectrum by 5–10%. These requirements are fulfilled with the channels selected on the basis of CMB polarisation science.

Additional channels above and below this frequency range are required to monitor contamination by radio and/or infrared sources in clusters, and to avoid confusion with small-scale foreground emission, in particular Galactic dust and the CIB. Again, a factor of around 2 in frequency below 120 GHz and above 400 GHz seems adequate, with at least four channels on both sides. This suggests extending the frequency range to span 60–800 GHz, with four channels above 400 GHz and four channels below 120 GHz. A 220-GHz channel, close to the zero of the thermal SZ effect, separates CMB anisotropies and the kinetic SZ from the thermal SZ (assuming that the CO lines are controlled in some way, e.g., with notch filters). For the best synergy with the future CMB-S4 programme, the angular resolution of the high-frequency channels above $\nu \simeq 300\,\mathrm{GHz}$ on the space mission must match that of the ground-based survey at 150 and 220 GHz.

CIB anisotropies are useful as a delensing tool for primary CMB science. As demonstrated with *Planck*, mapping the CIB anisotropies is relatively easy in patches with low dust emission, using channels at 350, 550 and 850 GHz. At higher frequency, low-redshift infrared galaxies, observable by other means, start to dominate the CIB emission, and dust emission from our own galaxy is relatively stronger. With at least five channels above 300 GHz (but preferably more for increased accuracy), it will be possible to use a generalised internal linear combination method (67) to separate CIB emission from thermal dust over a significant fraction of sky, as demonstrated using *Planck* and *IRAS* observations (68). The CIB map obtained in this way, in addition to its intrinsic scientific interest, can be used to delens CMB B modes and improve constraints on the primordial CMB B-mode spectrum, independently of what can be done with CMB delensing only. This is an essential tool for cross-checking and validating delensing based on CMB data only.

A principal component analysis can be used on multi-channel CIB observations to infer the star-formation rate (SFR) as a function of redshift. A study of how well different future experiments would perform (56), showed that the constraints steadily improve with the number of channels between 220 and 850 GHz, provided sufficient angular resolution is available. From their figure 3, a factor of 30 is gained on the SFR figure of merit they define by increasing the number of channels above 200 GHz from 5 to 10, and yet another similar factor is gained with 25 channels above 200 GHz. Although not designed for CIB science, with eight channels above 200 GHz, *CORE* outperforms other experiments such as LiteBIRD (which in addition suffers from reduced angular resolution) and CMB-S4 (with frequency bands limited to atmospheric windows). *CORE* could be further optimised for CIB science; this could be considered at a later stage if compatible with budgetary and programmatic constraints of the implementation of the mission.

4.6 Systematic effects

Polarising bolometric detectors integrate the electromagnetic power along one polarisation axis. They measure a combination of intensity and linear polarisation Stokes parameters. Assuming an infinitely small beam and frequency band, the signal on the detector, once it is calibrated in units of CMB temperature fluctuations, can be modelled as

$$x = I + \eta(Q\cos(2\psi) + U\sin(2\psi)) + n, \tag{4.1}$$

where I is the sky brightness and Q and U are the two Stokes parameters describing linear polarisation in the observed sky direction and at the relevant frequency, η is polarisation efficiency (ideally equal to unity), n is the noise term, and ψ is an angle of observation with respect to a set of reference axes. E and B polarisation are obtained by non-local linear combinations of Q and U (69; 70). In order to measure Q and U, it is necessary to invert a linear system of measurements at different orientations of the form

$$x_i = I + \eta(Q\cos(2\psi_i) + U\sin(2\psi_i)) + n_i, \tag{4.2}$$

which can be rewritten in vector-matrix format

$$\boldsymbol{x} = \mathsf{A}\boldsymbol{s} + \boldsymbol{n}, \tag{4.3}$$

where $\boldsymbol{s} = [I, Q, U]$ is the vector of sky Stokes parameters, and A is an operator that integrates the pixel and frequency-dependent sky signal $\boldsymbol{s}(p, \nu)$, and can be thought of as a generalised mixing or pointing matrix that depends on detector responses (calibration, polarisation efficiency, beams, and frequency bands) and on the scanning and observing strategy