Foreground models

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Commander is a code to do Bayesian component separation with parametric fitting using a Gibbs Sampler. Following is a mathematical description of the Foreground models used when doing component separation with Commander. The input maps used to generate the simulated sky maps using PySM are also plotted.

1 CMB

The Cosmic Microwave Background (CMB) radiation is described with the following model

$$\boldsymbol{s}_{\mathrm{CMB}} = \boldsymbol{a}_{\mathrm{CMB}} \frac{x^2 e^x}{(e^x - 1)^2} \tag{1}$$

where $x = h\boldsymbol{\nu}/kT_0$ where k and h is Boltzmanns constant and Plancks constant and T_0 is the CMB temperature. $\boldsymbol{a}_{\mathrm{CMB}}$ is the amplitude of the signal at given frequencies $\boldsymbol{\nu}$.

1.1 Model 90.91 and 90.92

Both 90.91 and 90.92 use the same model and input for CMB.

2 Thermal Dust

Thermal dust dominates the highest frequencies. It is described with a modified black body given by

$$\boldsymbol{s}_{\mathrm{d}} = \boldsymbol{a}_{\mathrm{d}} \left(\frac{\boldsymbol{\nu}}{\nu_0 \mathrm{d}} \right)^{\beta_{\mathrm{d}}+1} \frac{e^{h\nu_0^{\mathrm{d}}/kT_{\mathrm{d}}} - 1}{e^{h\boldsymbol{\nu}/kT_{\mathrm{d}}} - 1} \tag{2}$$

where $\boldsymbol{a}_{\mathrm{d}}$ is the amplitude of the signal at given frequencies $\boldsymbol{\nu}$. ν_0^{d} is the pivot frequency, or reference frequency, normally chosen to be the highest frequency where dust has the highest signal-to-noise ratio. β_{d} is the spectral index and T_{d} is the temperature of the thermal dust.

2.1 Model 90.91

Model 90.91 has a single dust component with spatially varying spectral index between 1.1 and 1.8, and spatially varying dust temperature with a mean of 21 K. The amplitude maps can be seen in Figure 1 and both the temperature map and the spectral index map can be seen in Figure 2

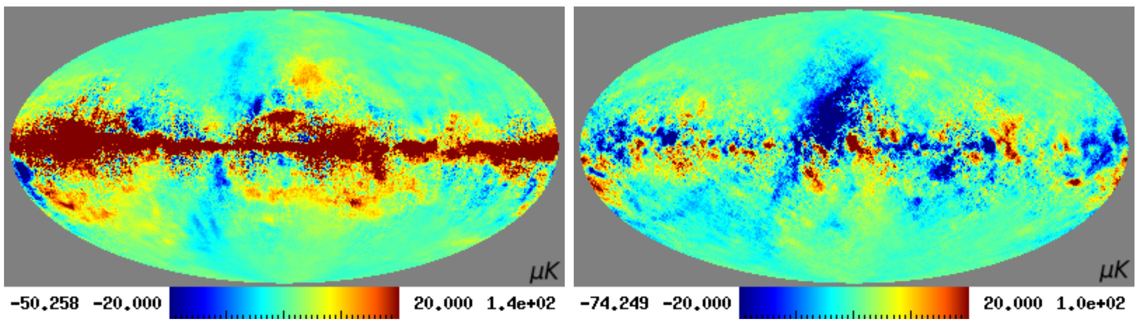

Figure 1: Dust amplitude maps for model 90.91. Stokes Q to the left and stokes U to the right. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

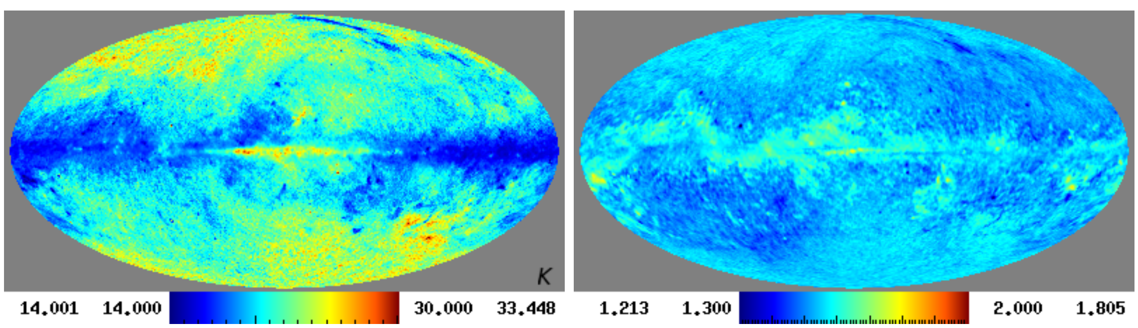

Figure 2: Dust temperature map to the left and dust beta map to the right. Both for model 90.91. Stokes Q and U have identical temperatures and spectral indexes. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

2.2 Model 90.92

Model 90.92 has two dust components, both with spatially varying temperature maps. Dust model 1 has a mean temperature of 9.7 K and a homogenous spectral index of 1.6. Dust model 2 has a mean temperature of 15.7 K and homogenous spectral index of 2.8.

The dust maps can be seen in Figure 3 4. -

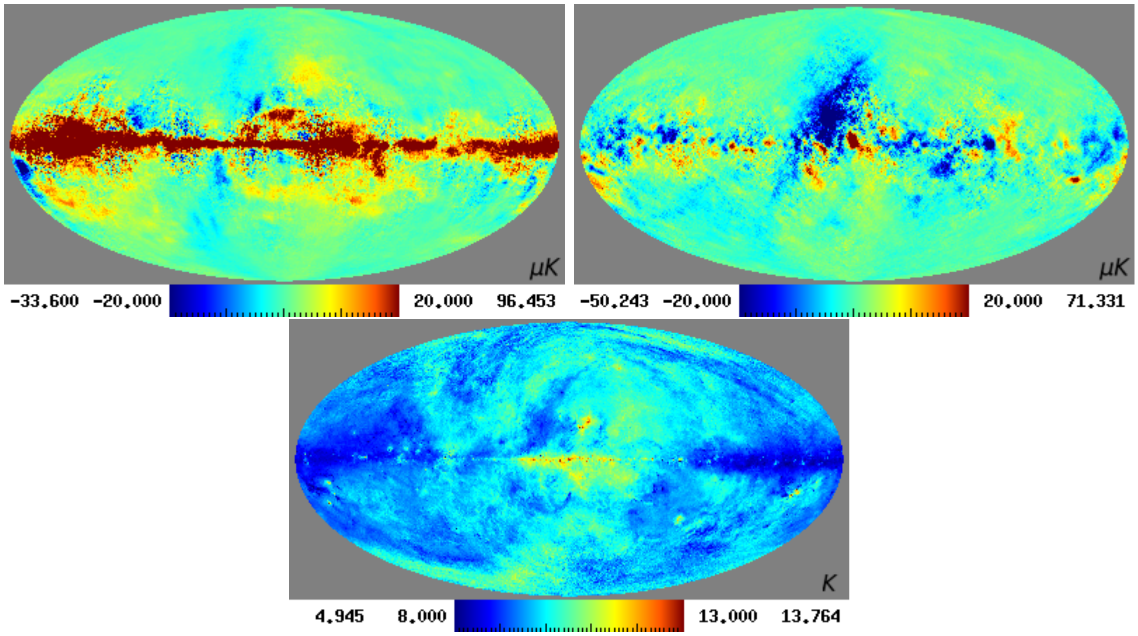

Figure 3: Dust model 1 for model 90.92. The dust amplitude maps at the top with stokes Q to the left and stokes U to the right. The dust temperature map with a mean of 9.7 K at the bottom. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

3 Synchrotron

The synchrotron radiation dominates the lowest frequencies. It is described by a power law given by the following model

$$\boldsymbol{s}_{\mathrm{s}} = \boldsymbol{a}_{\mathrm{s}} \left(\frac{\boldsymbol{\nu}}{\nu_0^{\mathrm{s}}} \right)^{\beta_{\mathrm{s}} + C \log \left(\frac{\boldsymbol{\nu}}{\nu_0{}^{\mathrm{s}}} \right)} \tag{3}$$

where $\boldsymbol{a}_{\mathrm{s}}$ is the amplitude of the signal at given frequencies $\boldsymbol{\nu}$. ν_0^{s} is the pivot frequency. This is often chosen to be the lowest frequency where the Synchrotron radiation has the highest signal-to-noise ratio. β_{s} is the spectral index of the synchrotron model. C is the curvature of the power law.

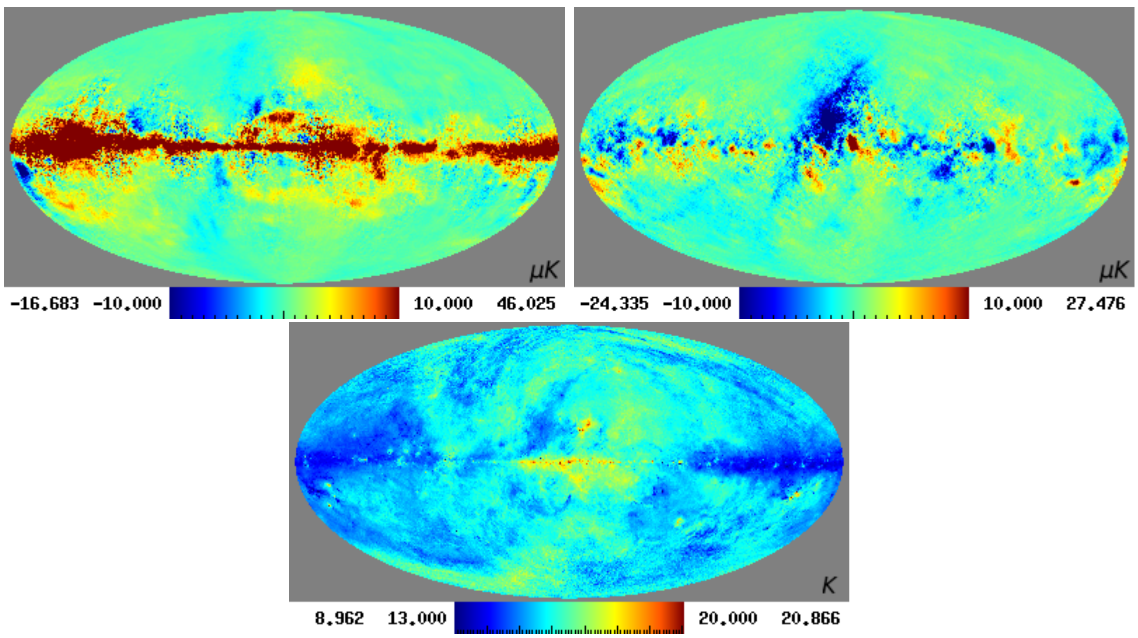

Figure 4: Dust model 2 for model 90.92. The dust amplitude maps at the top with stokes Q to the left and stokes U to the right. The dust temperature map with a mean of 15.7 K at the bottom. Stokes Q and U have identical temperatures and spectral indexes. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

3.1 Model 90.91 and 90.92

Model 90.91 and 90.92 has the same spatially varying spectral index maps for synchrotron with values ranging between -3.3 and -2.74. 90.92 has a curvature amplitude of -0.052 pivoted at 23 GHz, and 90.91 has no curvature. The synchrotron amplitude maps can be seen in Figure 5 and the spectral index map in Figure 6

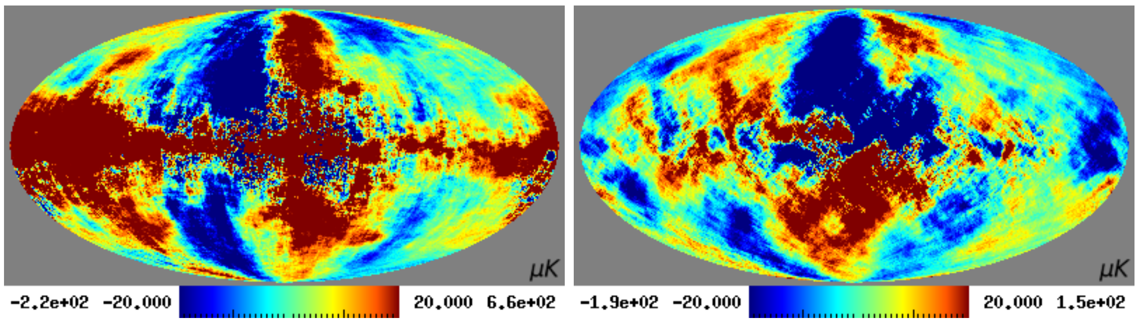

Figure 5: Synchrotron amplitude maps for both model 90.91 and 90.92. Stokes Q to the left and U to the right. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

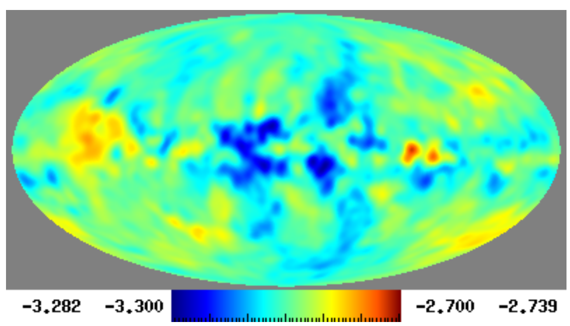

Figure 6: Synchrotron spectral index map for both model 90.91 and 90.92. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map. Stokes Q and U have identical spectral indexes.

4 Anomalous Microwave Emission (AME)

Anomalous Microwave Emission (AME), or spinning dust emission, is modeled with a SED template $pmbs_0$ that peaks at 30 GHz. The model is given by

$$\boldsymbol{s}_{\mathrm{sd}} = \boldsymbol{a}_{\mathrm{sd}}\boldsymbol{\nu}^{-2}\boldsymbol{s}_0^{\mathrm{sd}}\left(\boldsymbol{\nu}\frac{30.0\mathrm{GHz}}{\nu_p}\right) \tag{4}$$

where $\boldsymbol{a}_{\mathrm{sd}}$ is the amplitude at given frequencies $\boldsymbol{\nu}$ and ν_p is a peak position parameter.

4.1 Model 90.92

Model 90.92 has included polarized AME emission. The AME signal is 2 % of that in temperature. Model 90.91 does not have any AME added.

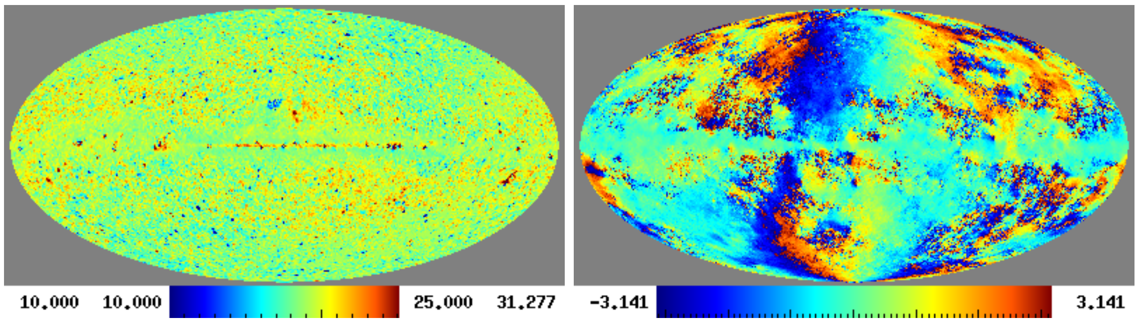

Figure 7: AME maps for model 90.92. The amplitude map at the peak frequency to the left and the polarization angle to the right. Stokes Q and U have identical maps for AME. The numbers next to the color bar are the color range for the plotting while the outer numbers are the extreme values in the map.

5 Summary

All the above equations are given in RJ units.

For 90.91 we get the total model

$$\boldsymbol{s}_{\mathrm{RJ}}^{\mathrm{tot}} = \boldsymbol{s}_{\mathrm{CMB}} + \boldsymbol{s}_{\mathrm{d}} + \boldsymbol{s}_{\mathrm{s}} + n \tag{5}$$

with 7 parameters to be fitted: $\boldsymbol{\theta} = \{\boldsymbol{a}_{\mathrm{CMB}}, \boldsymbol{a}_{\mathrm{d}}, \boldsymbol{a}_{\mathrm{s}}, \beta_{\mathrm{d}}, \beta_{\mathrm{s}}, T_{\mathrm{d}}, C\}$. n is the white noise.

For 90.92 we get the total model

$$\boldsymbol{s}_{\mathrm{RJ}}^{\mathrm{tot}} = \boldsymbol{s}_{\mathrm{CMB}} + \boldsymbol{s}_{\mathrm{d}}^{1} + \boldsymbol{s}_{\mathrm{d}}^{2} + \boldsymbol{s}_{\mathrm{s}} + \boldsymbol{s}_{\mathrm{sd}} + n \tag{6}$$

with 12 parameters to be fitted: $\boldsymbol{\theta} = \left\{\boldsymbol{a}_{\mathrm{CMB}}, \boldsymbol{a}_{\mathrm{d}}^{1}, \boldsymbol{a}_{\mathrm{d}}^{2}, \boldsymbol{a}_{\mathrm{s}}, \boldsymbol{a}_{\mathrm{sd}}, \beta_{\mathrm{d}}^{1}, \beta_{\mathrm{d}}^{2}, \beta_{\mathrm{s}}, T_{\mathrm{d}}^{1}, T_{\mathrm{d}}^{2}, C_{\mathrm{s}}, \nu_p^{\mathrm{sd}}\right\}$

A description fo the PySM models can be found at `https://pysm-public.readthedocs.io/en/latest/models.html`. For a full overview of the Foreground models in Commander, please see BeyondPlanck Collaboration (2020) `https://arxiv.org/pdf/2011.05609.pdf`