University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2000 - PART I

Friday, September 1, 2000 - 9:00 AM - 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed book examination. You may use calculators. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant **PROBLEM NUMBER. BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER,** so that no sheet contains work for more than one problem. Use only one side of the paper: If some problems require more than one sheet for their solution, then be sure to indicate "page 1", "page 2",...etc., under the problem number entered on the sheets. Once completed, all your work should be put, in order, in the manila envelope provided.

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Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2•K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Bohr radius	a_o	5.29×10^{-11} m
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Earth radius		6.37×10^{6} m
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GWE Fall 2000 - PART I

1. A circle of rope of total mass M and radius R is spinning at angular velocity ω about an axis through the center of the circle. What is the tension T in the rope ?

2. A point charge Q is placed at a distance D from the center of an uncharged, solid metal sphere of radius R, thereby polarizing it. What is the potential V of the sphere ?

3. Consider a gas that is nearly ideal, in the following sense. On the one hand, the equation of state can still be taken approximately to be $PV = nRT$. On the other hand, the following experiment reveals that the gas is not quite ideal: A sample of one mole of gas, at initial temperature and pressure respectively of 273 K and 1 atm, is allowed to expand freely into twice its original volume. This free expansion is done entirely under adiabatic conditions, and yet the final temperature of the gas is found to be 263 K. The gas is monatomic, so its specific heat at constant volume is $3R/2$. From the data given, find an approximate value for $(\partial U / \partial V)_T$ (in J/m^3), where U is the internal energy of the gas.

4. Using only the fundamental constants e, h, c, etc. as given on the accompanying sheet in the SI system, form a quantity representing a possible fundamental unit of (a) resistance, and give its value in Ohm, and (b) magnetic flux, and give its value in Tesla$\cdot$m^2.

5. A particle of rest mass m and velocity $v = 3c/5$ collides with another particle of the same mass that is at rest. Assuming that after the collision, the two particles coalesce into one, what are the rest mass M and velocity V of the final state particle ?

6. Someday, perhaps, a thermonuclear reactor will be developed to produce usable energy from nuclear fusion. Imagine that such a reactor could be built (using the reactions that power the Sun) so that one gram of ordinary hydrogen could be fused into helium-4 every second, with very little loss due to escaping neutrinos. What then would be the output power of this reactor, in Watts ? Use $m(^1H) = 1.00783$ u and $m(^4He) = 4.00260$ u (atomic masses), with $m_e = 0.00055$ u and 1 u = 931.5 MeV/c^2.

7. The components (J_x, J_y, J_z) of the quantum mechanical angular momentum operator can be represented by matrices.

(a) For a system of angular momentum $\frac{3}{2}\hbar$, what is the dimensionality of these matrices ?

(b) What does the matrix representing J_z look like when it is chosen to be diagonal ? Same question for the matrix J^2.

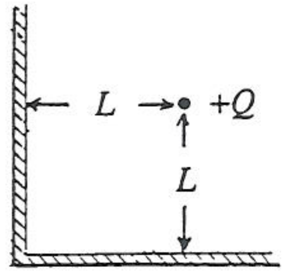
(c) What are the eigenvalues of the matrix $\frac{3}{5}J_x - \frac{4}{5}J_y$?

8. Estimate (a) the average speed (in m/s) and (b) the mean free path (in m) of a nitrogen molecule in this room.

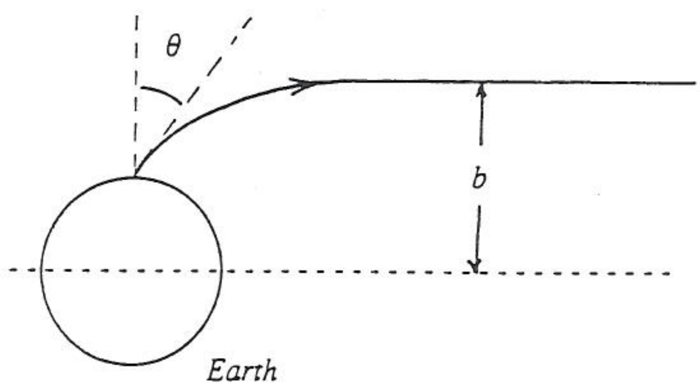
9. Using relativistic mechanics, (a) find the accelerating voltage (in Volts) required for an electron (initially at rest) to reach a velocity that is half of the phase velocity of its associated de Broglie wave, and (b) calculate the de Broglie wavelength (in m) of this electron.

10. Two identical spin-1/2 particles are confined by a one-dimensional potential well such that $V = 0$ for $0 \le x \le L$ and $V = \infty$ elsewhere. The particles do not interact with each other. What are the energies and degeneracies of the first four lowest distinct energy eigenstates of this system ? Be sure to explain your reasoning based on the symmetry properties of space and spin wavefunctions, and summarize your answers in tabular form, giving the energies in terms of E_o, the ground state energy of a single particle bound by this one-dimensional infinite potential well.

11. An infinite region of vacuum is bounded by two mutually perpendicular perfectly conducting, grounded, planes. An electric charge $+Q$ is situated at a distance L from each plane, as shown. Calculate the force **F** on the charge due to the presence of the conducting planes. Express your answer in the SI system, using the unit vectors **i** and **j** in the x- and y-directions (respectively, horizontal and vertical). Is the force attractive or repulsive ?



12. A projectile of mass m starts from the Earth's surface with an initial velocity of magnitude v_o, inclined at an angle θ from the vertical direction. It moves freely under the sole influence of gravity, so that at a large distance from Earth, the kinetic energy of the projectile approaches the value E, while its path turns into a straight line which is at a distance b from the parallel line drawn through the center of the Earth. All of this is shown in the accompanying figure. Denote the mass and radius of the Earth by M and R, respectively, and ignore the Earth's rotation.



Establish that

$$b = \sqrt{\frac{E+C}{E}}R\sin\theta$$

where C is a constant for which you should give an explicit expression in terms of quantities appropriate to this problem.

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GRADUATE WRITTEN EXAMINATION

FALL 2000 - PART II

Saturday, September 2, 2000 - 9:00 AM - 1:00 PM

Part II of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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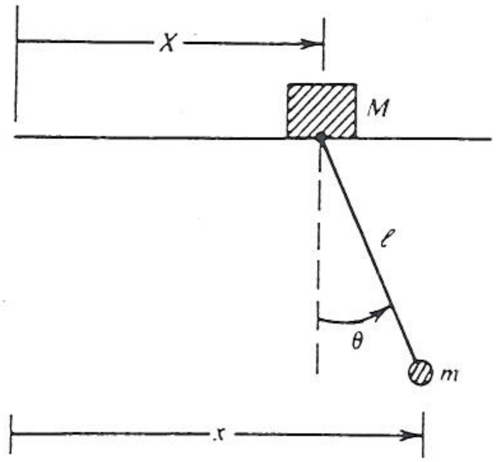
GWE Fall 2000 - Part II



1. A mass M is free to slide on a frictionless air track (which you may take to be along the x-axis, at $y=0$). Suspended to this mass by a pivot and a very light rod of length ℓ is another mass m which swings freely in the plane of the accompanying figure. Both masses are at rest when m is released at some non-zero angle θ_o.

(a) Construct a Lagrangian for this system, and derive the equations of motion. Hint: use X and θ as shown in the figure as generalized coordinates to describe the locations of the two masses.

(b) Determine the angular frequency for small oscillations $|\theta| << 1$. Check your result in the limit $M >> m$.

2. The initial state of a particle confined by infinitely high potential walls to the one-dimensional interval $0 < x < L$, within which it propagates freely at zero potential, is described at $t = 0$ by the wave-function

$$\psi(x,t=0) = A\sin^3\left(\frac{\pi x}{L}\right)$$

(a) Determine the form of the wave-function $\psi(x,t)$ at an arbitrary later time $t > 0$. Hint: the trigonometric identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ will be useful here.

(b) Show that the particle returns to its initial state after a certain time T, and give an expression for this time.

3. A simple model of a metal has n free conduction electrons per unit volume, each electron having spin-1/2 and an associated magnetic dipole moment μ. Now consider a metal at $T = 0$ K, and placed in a weak uniform magnetic field B: the total energy of the conduction electrons in the presence of the magnetic field must then be as small as possible.

(a) Denote the density of spin-up and spin-down electrons (with respect to the magnetic field direction) by n_+ and n_-, respectively, so that $n = n_+ + n_-$. How do the single particle energies of spin-up and spin-down electrons differ ? Calculate the total energy U of the conduction electrons in a sample of fixed volume V of this metal in the presence of the magnetic field: to do so, assume that spin-up and spin-down electrons occupy distinct quantum states, with different Fermi momenta.

(b) Denoting the total number of conduction electrons by $N = nV$, minimize the total energy U at fixed N and V with respect to n_+ to deduce the equilibrium value of $\delta n = n_+ - n_-$ when $\mu B << E_F$, with E_F the Fermi energy of the undisturbed metal. You will need to recall the expansion $(1+\varepsilon)^n = 1 + n\varepsilon + ...$ valid for $\varepsilon << 1$.

4. A rocket ship leaves the Earth at a speed v = 0.6c. When a clock on the rocket says that one hour has elapsed since departure, the rocketship sends a light signal back to Earth.

(a) Suppose that Earth and rocketships were synchronized to zero at the time of departure. According to *Earth* clocks, when was the signal sent ?

(b) Again according to *Earth* clocks, how long after the rocket left did the signal arrive back on Earth ?

(c) Now, according to *rocket* clocks, how long after the rocket left did the signal arrive back on Earth ?

5. A plane electromagnetic wave, with wavelength $\lambda = 3.0\,\text{m}$, travels in free space with its electric field vector $\mathbf{E} = E_o \sin(kx - \omega t)\mathbf{j}$, directed along the $+y$-direction and with amplitude $E_o = 300$ V/m.

(a) What is the frequency ν (in Hz) of this electromagnetic wave ?

(b) What is the direction and amplitude B_o (in Tesla) of the magnetic field **B** associated with this electromagnetic wave ?

(c) What are the values of k (in m^{-1}) and of ω (in rad/s) for this wave ?

(d) What is the time-averaged rate of energy flow per unit area for this wave (in W/m^2) ?

(e) If this wave falls perpendicularly on a perfectly absorbing sheet of area $2.0\ \text{m}^2$, what is the radiation pressure (in N/m^2) exerted on the sheet ?

6. A particle of mass m moves in a circular orbit in a hypothetical atom where the force acting on the particle is a "spring" force $\mathbf{F} = -k\mathbf{r}$ directed towards the center of the atom. Recalling the assumptions made by Bohr in his original derivation of the spectrum of hydrogen, and applying Bohr's method to the present case, derive, in terms of the quantum number n,

(a) the radii r_n of the allowed orbits, and

(b) the corresponding energies of these allowed orbits.