University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2001 - PART 1

Friday, August 24, 2001 - 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

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Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ϵ_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m
Electron rest mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Proton rest mass	m_p	1.67×10^{-27} kg
		0.938 GeV/c^2
Neutron rest mass	m_n	1.68×10^{-27} kg
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Planck constant	h	6.63×10^{-34} J.s
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Molar gas constant	R	8.31 J/mol.K
Avogadro's number	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
		8.62×10^{-5} eV/k
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2.K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Bohr radius	a_0	5.29×10^{-11} m
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg
Mass of Sun		1.99×10^{30} kg
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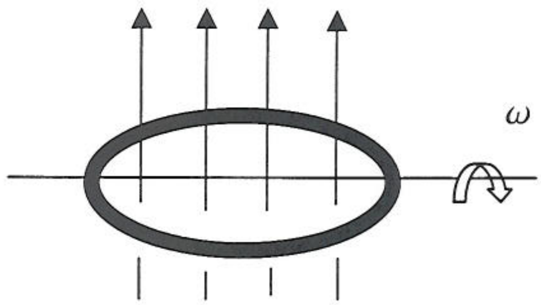
1. A thin copper ring of radius r and resistance R rotates about a horizontal diameter in a uniform, vertical magnetic field B. If the ring rotates at a frequency ω, at what rate is energy being dissipated in it?



2. A point particle of mass m and initial velocity v is scattered by a spherical mass M of radius R via an inverse square repulsive force (k/r^2). If $m << M$ what impact parameters will allow the particle to make contact with the sphere? Interpret your result physically as v increases from zero.

3. Suppose a complete set of states for a quantum mechanical system consists of only two states $|\phi_1>$ and $|\phi_2>$. The Hamiltonian H for this system has the following matrix elements:

$$<\phi_1|\, H \,|\phi_1> = 0; \;\; <\phi_2|\, H \,|\phi_2> = 0$$
$$<\phi_1|\, H \,|\phi_2> = \; <\phi_2|\, H \,|\phi_1> = a(\text{real})$$

Calculate the eigenvalues and eigenvectors of H.

4. The propagation of a shock wave is determined by the total energy E in the shock and the density ρ of the medium. Use dimensional analysis to determine how the distance $R(t)$ traveled by the shock in time t depends on the relevant parameters.

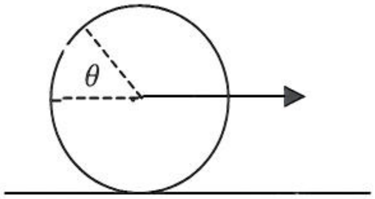
5. The Earth's atmosphere may be treated as a classical diatomic gas that is in hydrostatic equilibrium and has an average molecular weight of 0.029 kg/mole.

a) If the atmosphere were isothermal, at what altitude would its pressure be one half its sea-level value?

b) The atmosphere is not isothermal. What is the rate at which the temperature decreases with increasing altitude, assuming that the atmosphere is adiabatic? Is your result reasonable?

6. Mud is thrown tangentially from various points on the rim of a circular wheel (radius R) of a bicycle traveling along a horizontal road at a uniform speed v ($> \sqrt{gR}$). The wheel rolls without slipping. Neglecting any dissipative effects, show that none of the mud can rise to a height greater than

$$R + \frac{v^2}{2g} + \frac{gR^2}{2v^2}$$



7. Consider a quantum-mechanical particle of mass m moving in one-dimension x.

a) The particle is trapped in a potential well given by

$$V_0(x) = 0; \quad |x| \leq a, \qquad = +\infty; \quad |x| > a$$

Find the normalized wave-functions and their energy levels.

b) Suppose that the potential is actually $V_0(x) + V_1(x)$ where

$$V_1(x) = \left(\frac{G\hbar^2}{ma^2}\right)\sin\left(\frac{\pi x}{2a}\right), \text{ with } |G| \ll 1.$$

(G is merely a numerical constant). Using first-order perturbation theory, find the approximate ground-state wave-function and the corresponding energy perturbation.

8. Consider a degenerate gas of spin-1/2 non-relativistic particles of mass m.

a) Using the observation that for such a gas the inter-particle spacing is about one half of the de Broglie wavelength corresponding to the Fermi energy, obtain an approximate expression for the Fermi energy in terms of the fermion density.

b) If a neutron star of mass 4×10^{30} kg is 20 km in diameter, what is its Fermi energy?

9. A cook has a spherically shaped soup spoon. On looking into the concave side he sees his inverted image 4 cm from the spoon. Without changing his distance to the spoon, he turns it over and sees an erect image of himself 3 cm from the spoon. What is the radius of curvature of the spoon?

10. Shortly after the discovery of the cosmic microwave background (CMBR), Greisen, Zatsepin, and Kuzmin pointed out that pion photoproduction due to cosmic ray proton interactions with the CMBR put a maximum energy limit on the cosmic ray protons: the so-called GZK limit. The relevant reaction is:

$$p + \gamma_{CMBR} \rightarrow p + \pi^0$$

where the mass of the π^0 is 135 MeV. Assuming that all of the CMBR photons have the median energy of a 2.7K blackbody, what is the proton threshold energy for this reaction?

11. A particular star is 10 pc away and a planet has been observed in a circular orbit about this star with an orbital period of 16 Earth-years. The planet's orbit as seen from Earth has a maximum diameter of 4 arcsec and has the star at its center. Assuming that the planet's mass is much smaller than that of the star, what can you say about the mass of the planet? And what about the mass of the star?
(By definition, at a distance of 1 parsec (pc), a projected linear distance of one astronomical unit, i.e. the mean radius of the Earth's orbit around the Sun, subtends an angle of 1 arcsec).

12. A large lens is used to focus light from the sun on to a solar panel that absorbs all the incident radiation, emits like a black body and is otherwise isolated from its surroundings.

a) If the lens subtends 0.12 steradian as seen from the panel, what is the maximum temperature that the panel can reach?

b) Assuming that the panel is made flat and thin, how large can its surface be, given a lens of diameter 10 meters?

(The sun's temperature is about 6000K and its angular diameter is about 30').

University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2001 - PART 2

Saturday, August 25, 2001 - 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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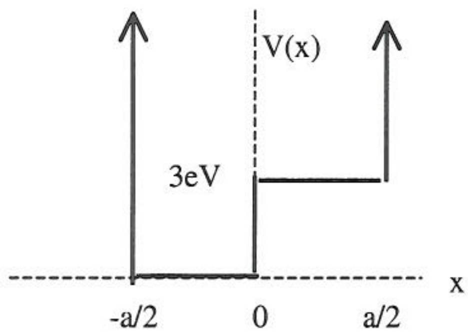
1.

The drawing shows a half-cylinder of radius R and mass M resting on a rough horizontal surface. Calculate the frequency of small amplitude, rolling oscillations about the equilibrium position.

2.



An electron moves in one dimension in the infinite potential well shown in the diagram. An eigenstate exists at an energy 4eV. Find the minimum value for the width of the potential well a.

3. A tent of volume V contains a large number N of flies. All the flies have the same speed v but their directions of motion are random. A hole of area A is made in one wall of the tent, connecting it to a second identical tent containing no flies. Derive an expression for the time dependence of N assuming that the flies are non-interacting and that their collisions with the tent walls are elastic.

4. A thin dielectric rod of cross-section A extends along the x-axis from $x=0$ to $x=L$. The polarization of the rod is along its length and is given by $P_x = ax^3 + b$.

a) Find the volume density of polarization charge and the surface polarization charge on all the surfaces of the rod.

b) Show that the total polarization charge vanishes.

c) What is the total dipole moment?

d) What is the electric field along the axis of the rod, at a distances $\gg L$?

5. Consider a large sphere of gas with total mass 2×10^{30} kg, radius = 7×10^{5} km, that shines with total luminosity 4×10^{26} watt. How long (in years) could the sphere shine at this constant total luminosity if:

a) The gravitational energy gained from its collapse from infinity to its present size is all that is available. (Since we do not specify the internal mass distribution, your answer will be approximate).

b) The sphere is composed entirely of hydrogen and the conditions allow nuclear fusion reactions converting hydrogen to helium, the hydrogen is all converted, and the conversion yields 7 MeV per hydrogen nucleus.

c) What would be the radius of the sphere with the same mass and luminosity for which the time scales for parts a) and b) are the same?

6. In a crystal, when an atom moves from its regular lattice-site position to an interstitial site, a Frenkel defect is formed. Consider a monatomic crystal with N atoms (i.e., N regular sites) and M possible interstitial sites. Assume that an energy E_I is required to move an atom from its regular position to an interstitial position. Let the number of atoms in interstitial positions be n.

Assuming that $n \ll M$ and $n \ll N$, calculate:

a) The energy $E(n)$ relative to the ground state of the crystal, and also the entropy $S(n)$.

b) The average value of n, assuming that the crystal is in thermal equilibrium at temperature T.

(Some possibly useful information: $\ln N! \cong N.\ln N - N$ for large N)