University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2002 - PART 1

Tuesday, August 20, 2002 - 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

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Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ϵ_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m
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		0.511 MeV/c^2
Proton rest mass	m_p	1.6726×10^{-27} kg
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Neutron rest mass	m_n	1.6749×10^{-27} kg
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Avogadro's number	N_A	6.02×10^{23} /mol
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Faraday constant	F	9.65×10^{4} C/mol
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Rydberg constant	R	1.10×10^{7} m^{-1}
Bohr radius	a_0	5.29×10^{-11} m
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
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Mass of Earth		5.98×10^{24} kg
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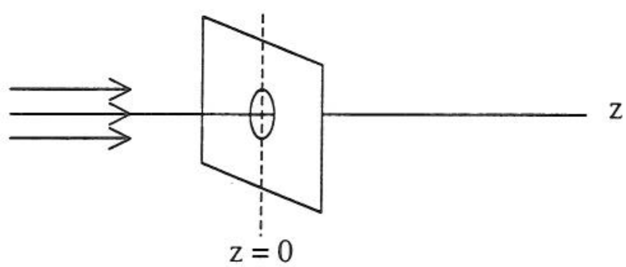
1. A cylindrical bucket is placed on the ground and filled with water to a height of 150 cm. How high from the ground should one punch a hole in the side of the bucket to make a stream of water that strikes the ground at the greatest distance from the bucket? What is that distance?

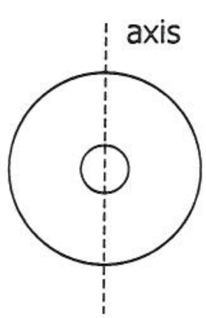
2. A wave packet describing the state of a free particle moving in 1–dimension is given by $\psi(x,0) = \cos kx$.

a) What is the expectation value of the particle's momentum?
b) Find the function $\psi(x,t)$ at any later time t.

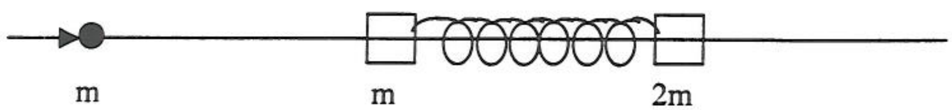
3. A plane wave of light with wavelength λ is normally incident on a screen containing a small circular hole of radius a. How does the light intensity vary at points on the z-axis beyond the screen?



4. The diagram shows the equilibrium configuration of two circular, concentric wire rings: one large <u>fixed</u> ring with radius R and one small with radius $r \ll R$. The small ring has total mass m. Each ring carries the same electric current i. What is the frequency of small oscillations of the smaller ring, if it oscillates about an axis through their diameters?



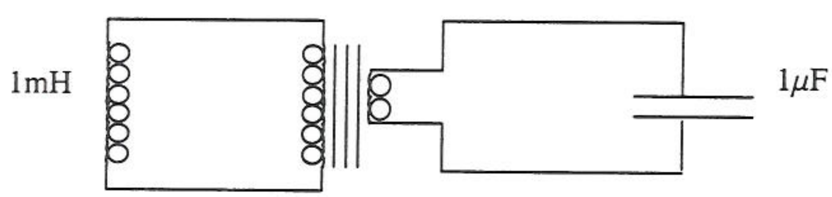
5. Blocks of mass m and $2m$ are free to slide without friction on a horizontal wire. They are connected by a massless spring of equilibrium length L and force constant k. A projectile of mass m is fired with velocity v into the block with mass m and sticks to it. If the blocks are initially at rest, what is the maximum displacement between them in the subsequent motion?



6. Water vapor can be removed from a gas by cryogenic pumping. Suppose you have a container of 10 cm radius which is held at a fixed temperature of 300K and which contains water vapor with a partial pressure of 0.1 mm Hg. Now suppose you cool a 1 cm^2 area of the container wall to a temperature of 77K. Assuming that each water molecule that hits the cold area condenses and sticks to the wall, how long will it take to reduce the partial pressure to 10^{-6} mm Hg?

7. In the state $\psi_{l,m}$ with angular momentum l and its projection m, determine the average values of l_x^2 and l_y^2.

8. A 1 millihenry inductor and a 1 microfarad capacitor are couple together by means of an <u>ideal</u> transformer with a turns ratio of 3:1 as shown. Calculate the resonant frequency of the circuit in Hz.



9. Estimate how many trees are needed to provide one year's newspapers in the USA, and what area of land is needed to provide this wood.

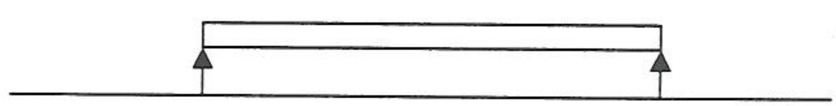
10. Ice on a pond is 10 cm thick and the water temperature just below the ice is 0°C. If the air temperature is –20°C, by how much will the ice thickness increase in 1 hour? Assuming that the air temperature stays the same over a long period, how will the ice thickness increase with time? Comment on any approximation that you make in your calculation.

Density of ice = 0.9 g/cm^3
Thermal conductivity of ice = 0.0005 calorie/cm/s/°C
Latent heat of fusion of water = 80 calorie/g

11. Carbon-14 is produced by cosmic rays interacting with the nitrogen in the earth's atmosphere. It is eventually incorporated into all living things, and since it has a half-life of 5730 ± 40 years, it is useful for dating archaeological specimens up to several tens of thousands of years old. The radioactivity of a particular specimen of wood containing 3 g of carbon was measured with a counter whose efficiency was 18%; a count rate of 12.8 ± 0.1 counts per minute was obtained. With the sample removed a background count rate of 5.2 ± 0.1 counts per minute was measured. It is known that in 1 g of living wood there are 16.1 carbon-14 radioactive decays per minute. What is the age of this specimen, and its uncertainty?

(Where are errors are not quoted, they can be assumed to be negligible).

12. A uniform (thin) rod is supported at its ends by two vertical props that hold it horizontal, as shown. If one of the supports is suddenly removed, what is the instantaneous force on the other support?



University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2002 - PART 2

Wednesday, August 21, 2002 - 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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1. The neutron has a lifetime of 887 s.

a) If neutrons from a cosmic ray interaction 1 light year from Earth reach Earth with a probability e^{-1}, what must their energy be?

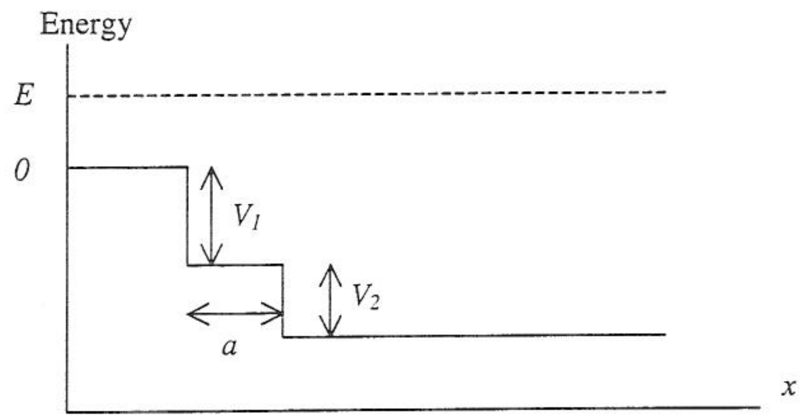
b) If these neutrons decay, what is the maximum angle from the flight path that their decay electrons are emitted?

c) And what is the maximum angle for neutrinos?

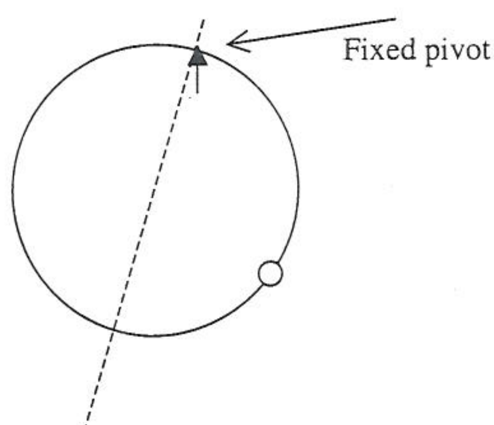
d) What is the maximum energy of the neutrinos emitted at this maximum angle?

(In your calculations you may assume that neutrinos have negligible mass)

2. A particle with mass m and energy E travels in the positive-x direction and crosses the stepped potential shown in the diagram below. What are the conditions for complete transmission?

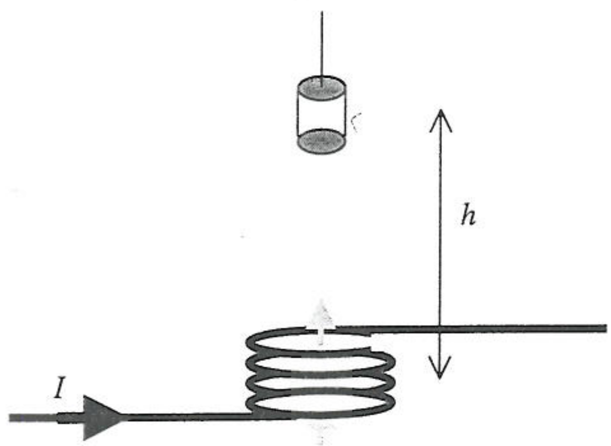


3. A thin hoop of radius R and mass M is supported at one point on its periphery and is able to oscillate under the influence of gravity in its own (vertical) plane. A bead of mass M is attached to the hoop and can move freely (with negligible friction) around the hoop. For small oscillations of both hoop and bead, what are the frequencies of the normal modes of oscillation? What initial conditions would result in the system oscillating in these modes? Make a sketch to illustrate.



4. A string is used to suspend a small copper cylinder (radius a, height b) at a height h directly above a solenoid (radius R, total number of turns N) as shown in the diagram. When a steady current I flows through the solenoid, the tension in the string is reduced slightly, independent of the direction of I.

a) Approximately, what is the magnetic field at the location of the copper cylinder? (Assume that h is much larger than the solenoid dimensions).
b) What does the reduction in the string's tension imply about the magnetic properties of copper? Give some explanation of the phenomenon.
c) Approximately, what is the magnetic dipole moment of the copper cylinder in terms of the magnetic field B_{sol} due to the solenoid and the magnetic susceptibility χ_m of copper?



5. A two-dimensional oscillator has the Hamiltonian

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}\left(1 + \delta xy\right)\left(x^2 + y^2\right)$$

where $\hbar = m = \omega = 1$ and $\delta \ll 1$.

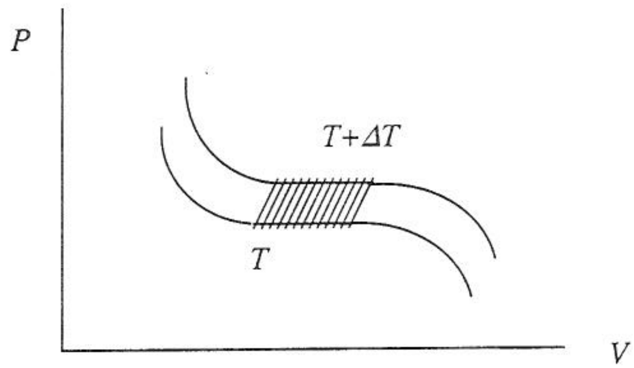
Give the wave function for the three lowest energy levels for $\delta = 0$; evaluate the first-order perturbation of these levels for $\delta \neq 0$. You may find the following integral useful:

$$\int_{-\infty}^{\infty} e^{-x^2} x^{2n} dx = \frac{1.3.5....(2n-1)}{2^n}\sqrt{\pi}$$

6. The pressure-volume diagram below shows two neighboring isotherms in the region of a liquid-gas phase transition. By considering a Carnot cycle between temperatures T and $T+dT$ in the region shaded in the diagram, derive the Clausius-Clapeyron equation relating vapor pressure and temperature:

$$\frac{dP}{dT} = \frac{L}{TdV}$$

where L is the latent heat of vaporization per mole and ΔV is the volume change between gas and vapor phase.



b) Liquid helium boils at temperature T_0 (4.2K) when its vapor pressure is $P_0 = 1$ atmosphere. Suppose you pump on the vapor and reduce its pressure to a much smaller value P_m. Assuming that the latent heat L does not change significantly with temperature and that the helium vapor pressure is much less than that of the liquid, calculate the approximate temperature T_m of the liquid in equilibrium with its vapor pressure P_m. Express your answer on terms of L, T_0, P_0, P_m, and any other required constants. You may assume that helium vapor obeys the ideal gas law.