University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2003 – PART I

Tuesday, August 19, 2003 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	Values
Speed of light in vacuum	c	3.00×10^{8} m/s
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Permeability constant	μ_0	1.26×10^{-6} H/m
Electron rest mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Proton rest mass	m_p	1.6726×10^{-27} kg
		0.93827 GeV/c^2
Neutron rest mass	m_n	1.6749×10^{-27} kg
		0.93957 GeV/c^2
Planck constant	h	6.63×10^{-34} J.s
		4.14×10^{-15} eV.s
Molar gas constant	R	8.31 J/mol.K
Avogadro's number	N_A	6.02×10^{23} /mol
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Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2.K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Bohr radius	a_0	5.29×10^{-11} m
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg
Mass of Sun		1.99×10^{30} kg
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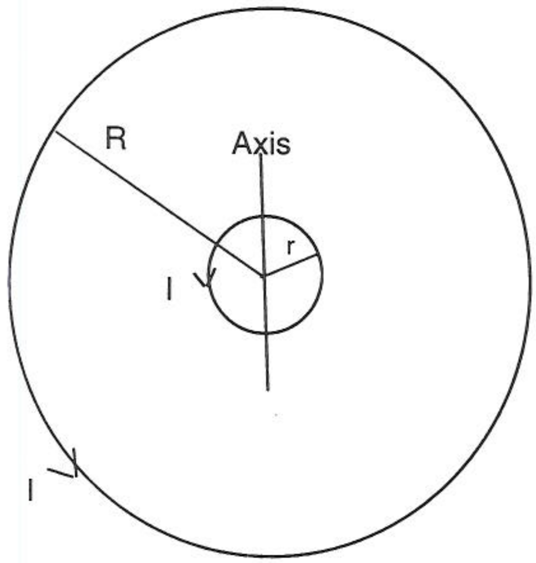
Fall 2003 GWE Short Problems

1) Consider a nucleus of ^{22}Ne consisting of 10 protons and 12 neutrons. Protons are distinguishable from neutrons, so two of each particle can be put into each energy state. (spin up, spin down). Assume that the radius of the ^{22}Ne nucleus is 3.1×10^{-15} m.

a) estimate the Fermi energies for each type

b) estimate the total kinetic energy of the nucleus in neon in MeV.

2) Oscillation of Magnetic Coils: The drawing shows the equilibrium configuration of two circular concentric wire rings. One is a small ring with radius r and mass m. The small ring can rotate freely about its vertical axis. The large ring has radius R and is fixed in space so it can not move. Each ring carries a current I (in the same direction). What is the frequency of small oscillations of the small ring about its vertical axis. (Assume $r << R$)



3) Consider a system of N particles at temperature T which has only two energy states, $E_1 = 0$, $E_2 = \varepsilon$.

a) Compute the average energy $\langle E \rangle$ of the whole system, and find the low and high temperature limits.

b) Calculate the heat capacity C_v of the whole system.

4) Rotating Disk: A conducting disk in the x-y plane with radius r is rotated with an angular frequency of ω about the z axis passing through its center. There is a uniform magnetic field B_0 in the z direction. Find the potential difference between the center and outer edge of the disk. (Assume non-relativistic motion and assume steady state with no current flow).

5) A pair of equal but opposite charges + Q and – Q each with mass m are connected by a spring k. The charges oscillate with initial amplitude A about their equilibrium position. What is the time scale for the loss of the total energy of the system due to radiation of electromagnetic energy?

6) Find the commutator of the operators of coordinate $\hat{x}$ and kinetic energy $K = \frac{\hat{p}^2}{2m}$ and the corresponding uncertainty relation for Δx and ΔK.

7) Calculate the average of the operator of angular momentum $<\hat{\vec{l}}> = <\psi|\hat{\vec{l}}|\psi>$ for:

(a) $|\psi> = |1,1>$

(b) $|\psi> = |1,-1>$

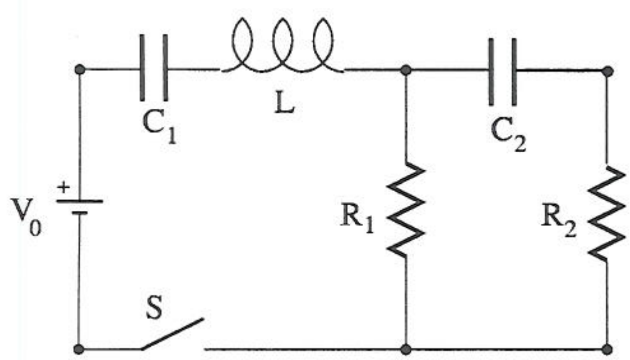
(c) $|\psi> = \frac{1}{\sqrt{2}}(|1,1> + |1,-1>)$, where $|1,\pm1>$ are the state with $l = 1$ and $l_z = \pm 1$.

8) Consider a satellite that is initially in a circular orbit around the Sun at the distance of Earth's orbit (1 AU = 1.50×10^8 km). This satellite has a rocket that can change its velocity by 10 km/s.

(a) If you want this satellite to go as close to the Sun as possible, in which direction would you fire this rocket, and how close to the Sun would the satellite go?

(b) If you wanted the orbit of the satellite to go as far away from the Sun as possible, in which direction would you fire this rocket, and how far from the Sun would the satellite go?

9) Consider the electrical circuit shown below. At time t=0, the switch S is closed. What is the total charge that will flow from the battery, in terms of the quantities shown?



10) The eta meson is a short-lived particle with a mass of 547 MeV/c^2, which is observed to decay into three neutral pions nearly 1/3 of the time. What is the maximum energy that one of the pions can attain if the eta decays at rest? The mass of the neutral pion is 135 MeV/c^2.

11) A stationary (in one inertial frame of reference) long thin wire carries an electric current I. The mobile charge carriers responsible for this current move along the wire with an average velocity v according to an observer in this inertial frame. What is the linear charge density of the mobile charge carriers, from the perspective of someone traveling with a constant velocity u with respect to this frame, in the direction opposite the motion of the charge carriers? Do **not** assume that the velocities in this problem are small compared to the speed of light.

12) Consider a charged, insulating slab with dielectric constant ε that has a thickness $2L$ in the z direction and is very large in the x and y direction that contains a charge per unit volume that varies linearly from $-\rho_0$ to ρ_0 from one side of the slab to the other, i.e., $\rho = \rho_0 z/L$ with z going from $-L$ to L. Find the electric field everywhere inside the slab (magnitude and direction), and the potential difference between the two edges of the slab.

University of Minnesota
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GRADUATE WRITTEN EXAMINATION

FALL 2003 – PART 2

Wednesday, August 20, 2003 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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GWE LONG PROBLEMS

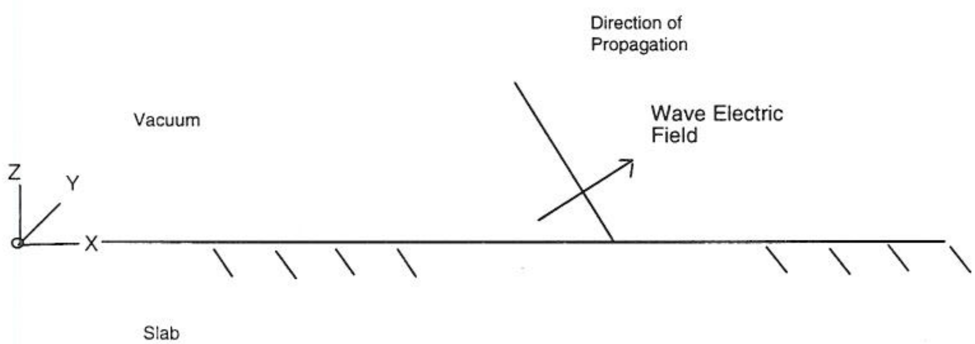
1) Electromagnetic wave incident on semi-infinite slab: A plane electromagnetic wave in vacuum with frequency ω is propagating in the x - z plane at an angle of 30 degrees relative to the z axis. It is incident on a planar slab of material with a constant index of refraction, n_0 for z < 0. The surface normal at the vacuum-slab interface is directed in the z direction. The amplitude of the total wave electric field is E_0 in the vacuum and it is polarized such that the electric field component in the y direction is zero. Start from Maxwell's equations and, (you may assume $\mu_{SLAB} = \mu_0$ non-magnetized medium; also $\frac{k^2c^2}{\omega^2} = n^2 = \varepsilon\mu$)

a) Determine the amplitude of the z component of the wave magnetic field in the slab.

b) Determine the amplitude of the x component of the wave electric field in the slab.

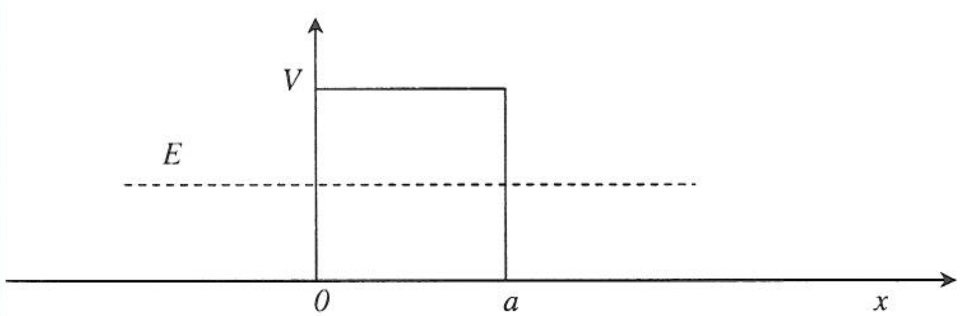
c) Determine the y component of the wave magnetic field in the slab.

d) What is the z component of the Poynting flux carried by the electromagnetic wave in the slab?



2) Calculate the transmission coefficient *T* for the potential barrier of the height *V* and width *a* when the energy *E* is less then the height of the barrier *V*.

a)Simplify the result for the case $\sqrt{\frac{2m(V-E)a^2}{\hbar^2}} >> 1$.



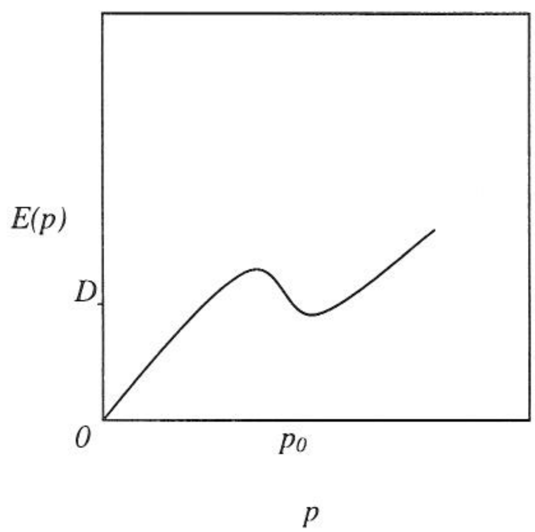
3) In order to explain the experimental data on the low temperature specific heat of super-fluid helium, Landau (1938) conjectured that the low energy spectrum of its Bose excitations, $E(p)$, has a peculiar form. Namely at $p << p_0$ it goes as $E(p) = sp$, reaches a maximum and then goes through a minimum at $p = p_0$. Near the minimum it can be described by $E(p) = D + (p - p_0)^2/2m$. Excitations in the first (linear) part of the spectrum are called phonons. They are similar to acoustic phonons in solids. Excitations in the second (parabolic) are called rotons. Later it became known that the sound velocity, $s = 239$ m/s, $D/k_B = 8.65$K, $p_0 = 1.92(h/2\pi)A^{-1}$ and $m = 0.16\ m_{He}$, where m_{He} is the helium atom mass. The number of excitations is not fixed, so that their chemical potential is zero.

a) Calculate the phonon contribution to the specific heat, with $I = \int_0^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$

b) Calculate the roton contribution to the specific heat,

c) Estimate the critical temperature, τ, where the crossover between the phonon and roton contributions occurs. Use the fact that $\tau << D/k_B$

(It was the observation of this crossover, which lead Landau to his conjecture)



4) In a plasma with equal concentration of free positive and negative ions the Coulomb potential e^2/r of a point charge e is screened and acquires the Yukawa form $(e^2/r)\exp(-\kappa r)$. Here r is the distance from the point charge to the observation point and κ is the inverse screening radius.

a) Calculate the Fourier transform of the Yukawa potential.

b) What is Fourier transform of the Coulomb potential?

5) In semiconductors electron states of some donors are similar to the hydrogen atom because a donor has the same charge as a proton. The only difference is that the Coulomb potential of a donor $V(r) = e^2/\kappa r$ contains a large dielectric constant κ of a semiconductor and electron has an effective mass m^* which is usually much smaller than the free electron mass m.

a) Calculate the ground state energy of a hydrogen-like donor in GaAs

   ($\kappa = 12$ and $m^* = 0.07\ m$).

b) Write the wave function of such a state. Calculate corresponding Bohr radius a.

c) Some donors can be located exclusively at the interface $z = 0$ of the semiconductor (Si), which occupies half space $z > 0$ and its oxide (SiO), which occupies half space $z < 0$. The forbidden gap of SiO is much larger than that of Si so that at $z < 0$ potential energy $V(r) = \infty$ and an electron cannot penetrate there. On the other hand, at $z > 0$ the electron is still subjected to the three-dimensional Coulomb potential $V(r) = e^2/\kappa r$ of donor (the donor is at $\mathbf{r} = 0$). If we direct the polar axis of the spherical system of coordinates along z this means that the wave function vanishes at $\theta = \pi/2$.
   Find the ground state energy of such a surface donor using your knowledge of the properties of the wave functions of excited states of conventional hydrogen atoms.

6) A particle is constrained to move (with gravity but without friction) on a circular wire rotating with constant angular velocity Ω about a vertical diameter. The wire has radius R. Find the equilibrium position of the particle and the frequency of small oscillations about this equilibrium. Show that the behavior of the system is different above and below a critical angular velocity Ω_c.