University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2005 – PART I

Tuesday, August 23, 2005 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER (not your name)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

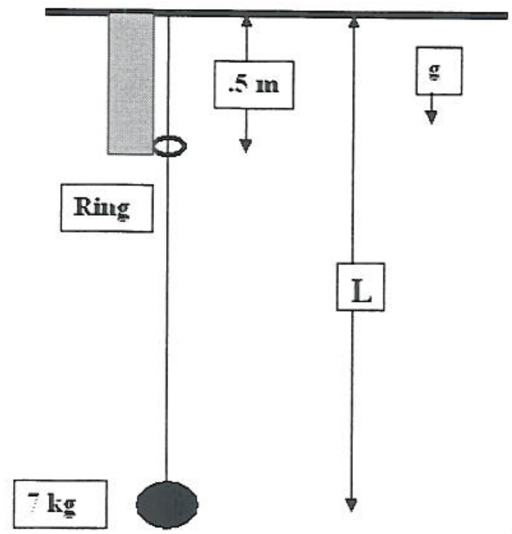
Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m

Stirling's Approximation:

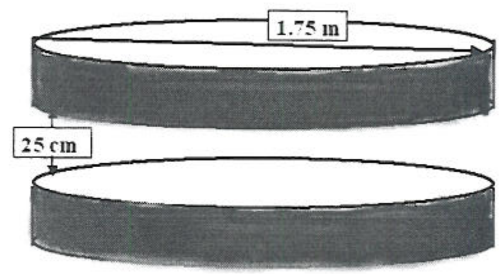
$\ln(N!) = N\ln(N) - N$ + (small corrections)

A. SHORT PROBLEMS

1.
A very light hanging elastic cord passes through a fixed, tight but frictionless ring 0.5m below its attachment point and supports a 7 kg ball at its end. The weight of the ball causes the cord to extend 1m beyond its natural (unloaded) length. A student lightly pokes the ball once (in a random, but lucky direction) and is surprised to see it oscillate in a perfect (small) circle (in a vertical plane). How long (L) was the stretched cord?



2.
Estimate by simple arguments the force between the iron pole tips of a large dipole magnet for a cyclotron. The space between the 1.75m diameter poles is 25cm. The (vertical) magnetic field of the cyclotron is 2T when the iron core is saturated, as it is in this case. Neglect any field that escapes the gap.



3. An electron is in a quantum state:

$$\Psi(\mathbf{r}) = [(1/30)^{1/2}]\,[3u_{100}(\mathbf{r}) - 4u_{211}(\mathbf{r}) + 2u_{321}(\mathbf{r}) - u_{400}(\mathbf{r})].$$

where the $u_{nlm}(\mathbf{r})$ are the normalized eigenstates of the hydrogen atom characterized by the quantum numbers **n, l** and **m**. What is the expectation value of the square of the angular momentum, $\mathbf{L}^2$?

4. You have 10 grams of pure gold, A=197, and 20 grams of pure silver, A=108. What is the approximate entropy increase when these elements are mixed to form a uniform alloy? Give your answer in Joules per degree Kelvin. You may ignore any effects from interactions between individual atoms, e. g.. the heat of fusion.

5. The half-life of a neutron is 614 seconds and its rest mass is 940 MeV/c^2 What kinetic energy must a neutron from the sun have to have exactly a 50% probability of reaching the earth (a distance of 8 light minutes)?

6. A system of 3 identical masses, m, are connected by springs such that the equations of motion are given by:

$$m(d^2x_1/dt^2) = -2kx_1 + kx_2$$
$$m(d^2x_2/dt^2) = kx_1 - 2kx_2 + kx_3$$
$$m(d^2x_3/dt^2) = kx_2 - 2kx_3$$

Find the normal mode frequencies for this system.

7. All N atoms in a two-dimensional crystal can oscillate only in the plane of the crystal. Find the heat capacity of this crystal in the limit of temperatures much higher than the characteristic Debye temperature. Assume only harmonic oscillations of the atoms.

8. ^{62}Cu , an unstable nucleus with a half-life of approximately 10 minutes, is produced in an accelerator at a constant rate of R nuclei per second. Starting with no nuclei, how much time does it take for the number of ^{62}Cu nuclei to equal 80% of the steady-state number (the number at "infinite" time)?

9. A beam of π^- is incident on a stationary proton target. Calculate the minimum energy that the π^- beam must have in order to produce a K^0 and a Λ^0 particle in the reaction

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

(Use $m_\pi = 140$ MeV/c^2, $m_p = 938$ MeV/c^2, $M_K = 498$ MeV/c^2 and $M_\Lambda = 1115$ MeV/c^2.)

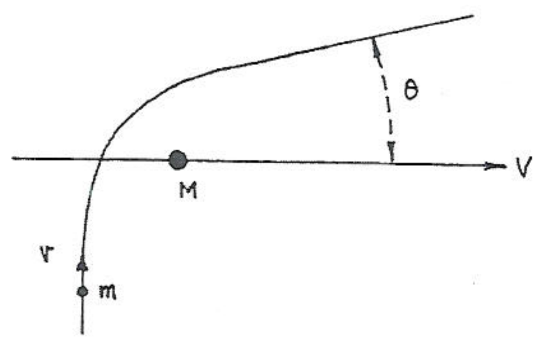
10. The earth's core is mostly iron and extends to about half of the Earth's radius. If all the iron in the core were made into a wire as long as the radius of the visible universe, estimate the radius of the wire. (If you do not know a relevant number, explain the reasoning used to guess that number.)

11. A speaker at a past Colloquium stated that the force per unit area between two neutral conducting plates due to polarization fluctuations of the vacuum (the Casimir force) is of the form

$$F/A = K\,(h)^a\,(c)^b\,(z)^d$$

where K is a dimensionless constant, h is Planck's constant, c is the speed of light and z is the distance between the plates. Find the exponents a, b and d.

12.
Exploration of the remote outer planets has been facilitated by the energy boost that can be obtained from a "slingshot effect" using the planet Jupiter. Consider the special case in which a planet of mass M and velocity V moves in an orbit that for short time intervals can be approximated by a straight line. A space probe of mass m << M travels with an initial velocity v that is perpendicular to the path of the planet. The space probe interacts with the planet and has a final direction that makes an angle θ with respect to the planet's velocity. Find the final speed of the probe, neglecting any change in the direction or speed of the planet. Hint: consider carefully what frame of reference might be easiest.



University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2005 – PART 2

Wednesday, August 24, 2005 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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Stirling's Approximation:

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B. LONG PROBLEMS

1. Consider a spin-1/2 particle of mass M. The Hamiltonian is $H = H_0 + H_{SO}$ where:

$$H_0 = (\mathbf{p}^2/2M) + \tfrac{1}{2}M\omega^2\mathbf{r}^2 \quad \text{and} \quad H_{SO} = (2\beta/\hbar^2)\mathbf{S}\cdot\mathbf{L} \ .$$

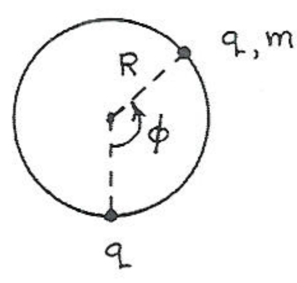
S is the spin operator and **L** is the orbital angular momentum operator. It is known that the spectrum of H_0 is given by $E_{kl} = \hbar\omega(2k + l + 3/2)$ where k = 0,1,2,3...represents the number of nodes in the radial wave function and l = 0,1,2,3... the orbital angular momentum quantum number.

(a) Write down the eigenvalues of the total Hamiltonian H.

(b) Make a sketch of the spectrum including the levels for k=0,1 and l=0,1,2. Show the ordering of the levels, assuming that $\beta = -0.3\ \hbar\omega$. Label each level by the appropriate quantum numbers and list the degeneracy of each of the eigenstates. Do this in terms of $\varepsilon = (E/\hbar\omega)-3/2$ for $\varepsilon < 3$.

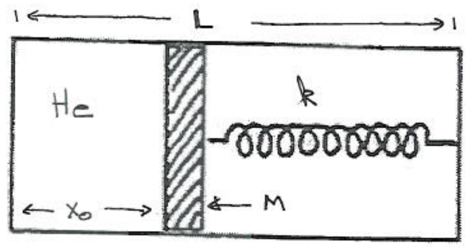
(c) Suppose that the complete Hamiltonian as defined above and in part (b) defines a common potential for a set of independent fermions (so that interactions between particles are neglected, and each particle experiences the potential). If there are 13particles in the system, what will be the total angular momentum of that system?

2.
A particle of charge q and mass m is constrained to move freely along a circle of radius R in a vertical plane at the earth's surface. Another particle of charge q is fixed at the lowest point of the circle. Find the equilibrium position of the first charge and the frequency of small oscillations of that charge about its equilibrium position. Hint: it is useful to introduce the variable $x = \sin(\varphi/2)$.



3.

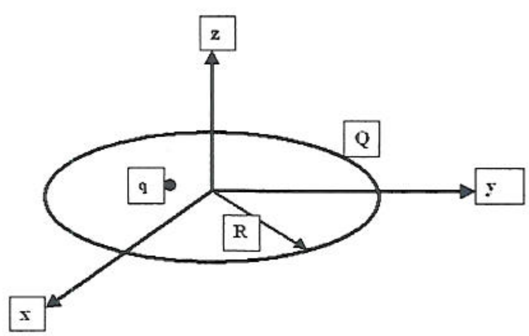
The system is a movable piston of mass M which is connected to a massless spring inside a closed cylinder as shown in the figure. The cross-sectional area of the piston is A, the length of the cylinder is L and the spring constant of the spring is k. The piston can slide within the cylinder without friction and without any leakage of the gas past the piston. When the piston is at the left end of the cylinder, the spring is relaxed. When n moles of helium gas are present in the volume to the left of the piston, the piston is at equilibrium at a distance x_0 from the left end. The volume to the right of the piston remains evacuated.



(a) Find x_0 when the temperature of the gas is T. Assume that the helium behaves as an ideal gas.
(b) Compute the heat capacity of the system. Assume that the heat capacities of the piston and cylinder walls are negligible.
(c) Compare your answer to (b) with the heat capacity of the same amount of helium with its volume held constant.
(d) Compare your answer to (b) with the heat capacity of the same amount of helium when its pressure is held constant.

4.

A charge of +Q is distributed uniformly on a circle of radius R in the x-y plane. The circle is centered at the origin. Calculate the frequency ω of small oscillations about the origin for a particle of mass m and charge +q that is constrained to move in the x-y plane (inside the circle). One way to solve this is to:



(a) First, find the electric field vector (due to the ring of charge +Q) on the z-axis as a function of z.
(b) Using your result from part (a), along with Gauss's law in differential form, determine the electric field vector in the x-y plane near the origin.
(c) Now calculate the oscillation frequency ω as specified above.

(You may find another method to solve this if you wish.)

5. The possible radioactive decay of the proton has been a topic of much interest over the past two decades. A typical experiment to detect proton decay is to construct a very large reservoir of water and put into it devices to detect Cerenkov radiation produced by the products of proton decay. It seems as if 30,000 tons is insufficient!

(a) Suppose that you have built a reservoir with 1,000,000 metric tons of water (1 ton is 1000kg). If the proton mean life τ_p were to be 10^{35} years, how many decays would you expect to observe in one year? Assume that your detector is 100% efficient and that protons bound in nuclei and free protons decay at the same rate.

(b) A possible proton decay is

$$p \rightarrow \pi^0 + e^+ .$$

Find the energy of the positron.

(c) Another possible decay of the proton is

$$p \rightarrow K^0 + \pi^0 + e^+ .$$

Find the maximum energy of the positron for this decay.

(Assume $m_\pi = 135$ MeV/c^2, $m_K = 498$ MeV/c^2 and neglect m_e.)

6. A proton has a radius of 1.2 fm.

(a) What is the electric potential at the nuclear surface?

(b) The normalized radial ground state wave function of the electron in a hydrogen atom is

$$u_{100} = (2/a_0^{3/2})\, e^{-r/a0}$$

Use this to calculate the electron charge distribution in the hydrogen atom. The charge and matter distributions coincide.

(c) Write down an expression for the electric field within the hydrogen atom at distances greater than the proton radius.