University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2006 – PART I

Tuesday, August 22, 2006 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER (not your name)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m

Stirling's Approximation:

$$\ln(N!) = N\ln(N) - N + \text{(small corrections)}$$

1. As a raindrop falls due to gravity, it increases its mass through deposition of vapor on its surface. Assume the raindrop maintains a spherical shape throughout, and that its radius R(t) increases linearly with time, i.e. that $dR/dt = K = const$; this is equivalent to assuming that the rate of increase of its volume is proportional to its surface area. Neglecting air resistance, find the velocity $v(t)$ as a function of time of a raindrop that starts at rest and with an initial radius R(0)=a, at t=0. Check your answer by verifying that $v(t) \cong gt$ when $Kt << a$,

i.e. a very short time into the raindrop's fall. What becomes of $v(t)$ in the opposite limit, $Kt >> a$?

2. A particle of mass *m* slides without friction on the surface of a sphere of radius *a*. If the particle starts at rest on the top of the sphere, where does the particle leave the surface of the sphere?

3. The frequency *f* of a deep water gravity wave (i.e. an ordinary ocean wave) is given by $f \propto \rho^a g^b \lambda^c$ where ρ, g, λ are the water density, gravitational acceleration, and wavelength of the wave, respectively. What are the values of the exponents *a, b, c* and what is the ratio of the wave group velocity/phase velocity?

4. What is the velocity of recoil of an Fe^{57} nucleus that emits a 100 keV photon, both in units of the speed of light in vacuum and in meters per second?

5. In the course of pumping up a bicycle tire, a liter of air at $T_1 = 300K$ and atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen).

a) What is the final volume V_2 of the air after compression?
b) How much work W is done in compressing the air?
c) What is the temperature T_2 after the compression?

6. The cross-section σ for collision between helium atoms is about $4*10^{-16}$ cm^2. Estimate the mean free path λ of helium atoms in helium gas at atmospheric pressure and temperature.

7. A time-independent magnetic field is given by $\mathbf{B}=2bxy\,\mathbf{i}+ay^2\,\mathbf{j}$, where **i** and **j** are the unit vectors in the x and y direction, correspondingly.
(a) What is the relationship between the constants a and b ?
(b) Determine the steady current density **J** that gives rise to the field.

8. Two unequal capacitors C_1 and C_2 are first charged to the same potential difference V. Next, the charged capacitors are connected in series (that is, "plus" to "minus"). A switch S that is initially open completes the circuit.

(a) What are the final charges Q_1 and Q_2, on each capacitor after the switch is closed?
(b) Calculate the loss in stored electrostatic energy, and discuss (without calculation) what happens to this lost energy.

9. In the design of a camera for use underwater, you have to consider light that has a vacuum wavelength l = 597 nm normally incident from water (refractive index n = 1.35) on a flat glass plate which has on its front surface a coating. The refractive index of the glass is 1.50. What is the refractive index and thickness of the thinnest coating layer that causes destructive interference in the reflected wave, minimizing reflections in the light?

10. The ground state energy of a single, isolated particle in a 3d cubical box (with impenetrable walls) is E_0. What will be the ground state energy of a system of 10 such particles if they are non-interacting and if

(a) they have spin 0?
(b) they have spin ½?

11. In atomic hydrogen, the hyperfine interaction gives rise to a splitting between the total spin (nuclear+electronic) F=1 and F=0 states, which leads to a transition characterized by the astrophysically famous 21cm line. At what temperature of an atomic hydrogen gas cloud will the three F=1 states have a total population equal to that of the F=0 ground state?

12. Carbon-14 is produced by cosmic rays interacting with the nitrogen in the earth's atmosphere. It is eventually incorporated into all living things, and since it has a half-life of 5730 +/- 40 years, it is useful for dating archeological specimens up to several tens of thousands of years old. The radioactivity of a particular specimen of wood containing 3g of carbon was measured with a counter whose efficiency was 18%: a count rate of 12.8 +/- 0.1 counts per minute was obtained. With the sample removed a background count rate of 5.2 +/- 0.1 counts per minute was measured. It is known that in 1g of living wood there are 16.1 carbon-14 radioactive decays per minute. What is the age of this specimen, and its uncertainty? (Errors which are not quoted may be assumed to be negligible).

University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 2006 – PART 2

Wednesday, August 23, 2006 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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Stirling's Approximation:

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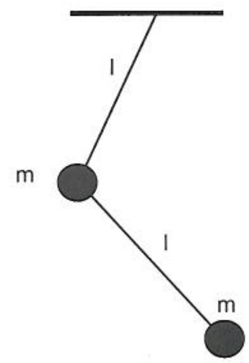
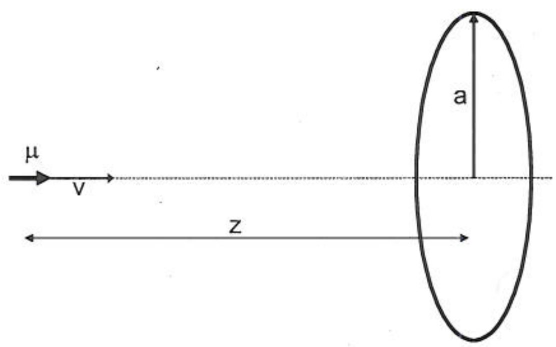
1. Consider a double pendulum, i.e., a system consisting of two simple pendula each of length l, and mass m with one pendulum suspended from the other. Calculate the frequencies of small oscillation for this system, and describe the motion for each of the normal modes. Assume the two pendula oscillate in the same vertical plane.



2. One mole of a monatomic ideal gas undergoes a process for which the relation between pressure and volume is $P = P_0 + \alpha / V$, where both P_0 and α are positive. The gas expands from an initial volume V_1 to a final volume V_2. Find

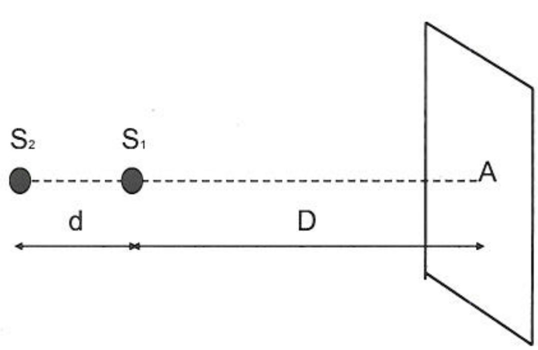
(a) the change in the internal energy ΔU of the gas,
(b) the work W done by the gas on its surroundings, and
(c) the amount of heat Q transferred to the gas by its surroundings.

3. A permanent magnet with the magnetic moment μ moves with speed v along the axis of a circular loop of conducting wire. The loop has radius a and resistance R. If μ points in the direction of motion, what force does the particle experience when it is a distance $z >> a$ from the center of the loop? What is the direction of this force? You may assume that $v << c$ and that the loop has negligible self-inductance.

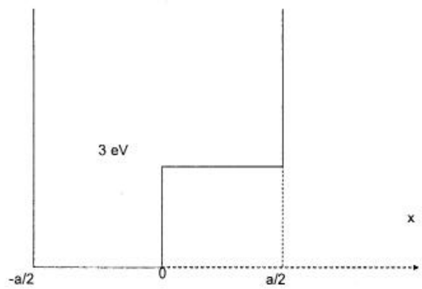


4. Two monochromatic, coherent point light sources of the same wavelength λ, are located on a line perpendicular to a distant screen. The closest source, S_1, is at a distance D from the screen, while the more distant one, S_2, is at a distance D+d; both D and d are much larger than the wavelength of the light sources. *Assuming that there is an interference maximum at the point* A where the line drawn through the sources intersects the screen, find a formula for the radius r of the first bright ring around A on the screen, and get a numerical value assuming

$\lambda = 600nm, D = 3.0m, d = 1.0m$.



5. An electron moves in one dimension in the infinite potential well shown in the diagram. An eigenstate exists at an energy 4eV. Find the minimum value for the width of the potential well a.



6. Janice leaves earth at 9 am on a rocket ship travelling at a speed of v = 0.6c. When the clock on the rocket reads 10 am, she sends a light signal back to Jim on Earth.

(a) When will Jim's sensors on earth detect this signal according to his clock?

(b) After travelling 6 hours (according to her clock of course!) Janice figures she has had enough of space travel and heads back to earth. The inertial dampeners allow her to stop, turn around, and accelerate back up to v=0.6c in a negligible amount of time. When she gets back to Earth, has she aged more or less than Jim? By how much?

(c) Draw a space diagram of the entire round trip as seen in the Earth frame, labelling the x-axis in units of light-hours and the y-axis in units of hours. Include Janice's light signal. Be accurate in your sketch (make it big enough!) - use minor tick marks every ¼ hour.

(d) Since Janice sees Jim's clocks running slow and Jim sees Janice's clocks running slow, each expects the other one to be younger. Explain, using the diagram, why only one is younger.