Part I. 10 best solutions out of 12 problems

1. When a diver is standing still on the edge of a long diving springboard, the edge of the board is deflected down by 30 cm from its unloaded position. Assuming that the deflection of the edge of the springboard is proportional to the vertical force and neglecting the mass of the board, find the period (in seconds) of small oscillations of the board with the diver standing on its edge.

2. Electrons with kinetic energy E are incident normally from vacuum on a metal surface. The work function of the metal (that is, the minimal energy required to extract one electron from the metal) is F. Calculate the fraction of electrons reflected from the surface. Neglect any temperature effects.

3. The B meson factory at the Stanford Linear Accelerator Center is designed to produce pairs of B mesons in the reaction $e^+e^- \to B\bar{B}$ by colliding electron and positron beams and to observe the decay of the B mesons in flight. At the collision point the positrons have a total energy of $9.0\,\text{GeV}$ (per particle). The mass of the B meson is $5.28\,\text{GeV}/c^2$. At what minimal energy of the electrons in the electron beam the pairs of B mesons start being produced? If the energy in the electron beam is just above this threshold, what is the mean path length of the B meson, given that its mean lifetime (in its rest frame) is $1.6\,\text{ps}$?

4. A charged metal object of an arbitrary shape loses 90% of its charge in 10 minutes while being suspended from a thin insulating thread. The relative dielectric constant ϵ of the air is very close to one. Find the electrical conductiviy σ of the surrounding air in $\Omega^{-1}m^{-1}$.

5. A fisherman's boat is at rest floating in the still water of a lake. Initially the fisherman is sitting near the bow. He then moves to a seat near the stern. After everything returns to standstill, by what distance has the position of the boat relative to the shore been changed? Assume that the frictional force of the boat on the water is proportional to the velocity as $F = -K\,v$. (The mass of the fisherman is m, the mass of the boat is M and the distance between the aft and the rear seats is L.)

6. Two sodium atoms, each with a total angular momentum number 1/2, are placed in a uniform magnetic field $\vec{B}$. They interact with each other via an exchange interaction. This leads to a Hamiltonian

$$H = -g\left[(\vec{j}_1 \cdot \vec{B}) + (\vec{j}_2 \cdot \vec{B})\right] + K\,(\vec{j}_1 \cdot \vec{j}_2) \quad ,$$

where g and K are constants and $\vec{j}_1$ and $\vec{j}_2$ are the total angular momentum operators for the atoms. Find the energy levels of this system.

7. A pinhole camera has no lens, just only a small hole to form an image. If the hole is too large, the image is fuzzy because of the finite hole size. If it is too small, the image is fuzzy because of diffraction. Determine the size of the hole for the sharpest image for green light with wavelength 560 nm and the distance from the hole to the screen of 20 cm.

8. An engine using 1 mole of ideal gas performs a cycle consisting of three steps:
(1) an adiabatic expansion from initial pressure of 2.64 atm and volume 10 ℓ to a final pressure of 1 atm and volume 20 ℓ;
(2) a compression at constant pressure to its original volume of 10 ℓ;
and
(3) heating at constant volume to its original pressure of 2.64 atm.

Find the efficiency of the cycle, i.e. the ratio of the total mechanical work done by the engine to the amount of heat supplied to the engine.

9. An electric bulb is rated at 100 W when used with a DC voltage of 110 V. What total power is dissipated if this voltage is applied to two such bulbs connected in series? It can be assumed that each bulb dissipates heat by radiation from its filament similar to a black body, and that the resistance of the filament is proportional to its absolute temperature.

10. A rock is found to contain 4.20 mg of ^{238}U and 2.00 mg of ^{206}Pb. Assume that the rock contained no lead at the time of its formation, so that all the lead now present is due to decay of the uranium originally present in the rock. Find the age of the rock given that the half-life of ^{238}U is 4.47×10^9 yr. The decay times of all intermediate elements are negligibly short.

11. Two vertical parallel rectangular glass plates are partially submerged in water. The distance between the plates is $d = 0.1\,\text{mm}$, and their width is $l = 20\,\text{cm}$. Assuming that the water between the plates does not reach their upper edges and that the wetting of the plates is complete, find the force of attraction between the plates. The surface tension of water is $\alpha = 72 \times 10^{-3}\,\text{N/m}$.

12. A foil of ^{57}Fe contains some nuclei in an excited state. They decay by emitting photons of energy 14.4 keV. Ignore the recoil of the nuclei. The foil is at temperature 300K, at which temperature its molar heat capacity is 3R. Calculate the approximate R.M.S. velocity of the iron nuclei and calculate how the center of the observed frequency range of the emitted photons varies as the temperature T increases, $(d\nu/dT)$ (in Hz/K). (This quantity is also sometimes called the relativistic temperature coefficient of the frequency.)

Part II. 5 best soultions out of 6 problems

1. A spherical satellite with mass 2460 kg and diameter 2.3 m is in a circular orbit around the Earth at 200 km above the surface. The orbit of the satellite 'decays' due to friction of the atmosphere. The force of friction at such altitudes can be approximated by the formula $F = -\rho\, A\, v^2$, where v is the velocity of the spacecraft, A is its cross sectional area and $\rho \approx 10^{-9}\,\mathrm{kg/m^3}$ is a coefficient.

(a) Calculate the rate of change of the *total* energy of the satellite dE_{total}/dt. What is its sign?

(b) Calculate the rate of change of the *kinetic* energy of the satellite dE_{kin}/dT. What is its sign?

(c)Estimate the time it will take the satellite to descend to the altitude 100 km.

(At 100 km the density of the atmosphere sharply rises, so that the satellite will then rapidly slow down and reenter the atmosphere. The first manned spaceflights carried extra life support supplies to last for the duration that you calculate in this problem, which supplies were to be used in case the reentry engine would not fire.)

2. You are given a short length L of a copper wire to make a flat loop antenna for receiving a linearly polarized radio wave from a distant radio station with wavelength λ, such that $\lambda \gg L$. The loop antenna is connected to the input connector of a receiver, which is sensitive to the voltage difference across the contacts of its input connector (the distance between the contacts is very small and can be neglected).

(a) How should the loop be oriented with respect to the direction of the source of the radio wave and with respect to the direction of the linear polarization (i.e. of the electric field in the wave) for maximal signal?

(b) What shape of the loop antenna maximizes its sensitivity?

(c) What is the maximal amplitude of the voltage at the receiver input that you can achieve, if $L = 1\,\mathrm{m}$, $\lambda = 100\,\mathrm{m}$, and the amplitude of the electric field in the incoming wave is $E_{max} = 1 \times 10^{-3}\,\mathrm{V/m}$?

3. The heat capacity C_p at constant (normal) pressure of the nitrogen gas N_2 at the temperature $T = 300\,\mathrm{K}$ is $29.124\,\mathrm{J\,mol^{-1}\,K^{-1}}$.
(a) What heat capacity would you expect if the vibrational motion of the N_2 molecule is totally frozen at such temperature?
(b) Assume that the excess in the measured value of C_p is due to a weak excitation of the vibrational mode and estimate (in eV) the energy of the first excited state of this mode.
(If needed, you can approximate the solution of the equation $x^2\,e^{-x} = \epsilon$ by $x \approx \ln(1/\epsilon) + 2\ln[\ln(1/\epsilon)]$, in a situation where ϵ is a small number. The value of the molar gas constant is $R = 8.314472\,\mathrm{J\,mol^{-1}\,K^{-1}}$.)

4. Consider electrically neutral hydrogen at a very high mass density ρ in the range from $\rho \approx 10^9\,kg/m^3$ to $\rho \approx 10^{11}\,kg/m^3$. Assume that the temperature is low, so that it can be set at zero: $T = 0$. Consider also that electrons and protons can produce neutrons through the reaction

$$e + p \to n + \nu_e\ ,$$

with the neutrino freely leaving the plasma. No neutrons are produced in hydrogen at normal conditions because the neutron is heavier than the proton by $\Delta = m_n - m_p = 1.3\,\mathrm{MeV}/c^2$.
(a) Argue that even when the above reaction takes place, the total number density of the nucleons (i.e. of protons plus neutrons) is conserved and that the number density of electrons is equal to that of the protons.
(b) Assume that electrons, protons, and neutrons are noninteracting particles and find the Fermi momentum for each of them at a given number density. Decide which of the particles can be considered as nonrelativistic and which cannot at the mass density of plasma in the considered range.
(c) Find the critical mass density of plasma ρ_0 (in $\mathrm{kg/m^3}$) at which neutrons start being produced through the indicated reaction.
(d) Describe (approximately) the behavior of the number density of electrons n_e as the total mass density changes from $\rho = 0.5\,\rho_0$ to $\rho = 2\,\rho_0$.

5. A point electric charge q is placed at the distance r from the center of a hollow thin spherical metal shell of radius R with $r > R$. The sphere has total electric charge Q.
(a) Find the force acting on the charge q.
(b) Find the total electrostatic energy of the system.
(c) Find the force acting on the charge q if the sphere is grounded (instead of having fixed charge Q), so that its potential is maintained at the same value as at infinity. Any effect of the grounding wire can be neglected.
(Hint: prove that the electric field outside the sphere is equivalent to that of the system, where the sphere is replaced by two 'image' point charges, one at the center of the sphere and one elsewhere on the line between the center and the charge q. You will have to find the position of this second image charge and the values of both image charges.)

6. A particle of mass m is moving in a one-dimensional potential $V(x)$ such that

$$V(x) = \begin{cases} \frac{m\omega^2}{2} x^2 & \text{if } x > 0 \\ +\infty & \text{if } x \leq 0 \end{cases}$$

(a) Consider the motion classically. What is the period of motion in such potential and the corresponding cyclic frequency?
(b) Consider the motion in quantum mechanics and show that the wave functions of the levels in this potential should coincide with some of the levels of a simple oscillator with the potential $m\omega^2 x^2/2$ at all (positive and negative) x.
(c) Find the spectrum of the levels in the potential $V(x)$. How is the spacing between consequtive energy levels related to the frequency of the classical motion?