GRADUATE WRITTEN EXAMINATION, August 2009

Long Problems *Best 5 of 6*

1. A long molecular string is composed of N chemical units ($N >> 1$). Each unit can have two states: short (length=a) and long (length =b). The short and long states have energies 0 and ϵ respectively.

   (a) Calculate the average length ($< L >$) of the string as a function of temperature. Check that your answer evaluates to the expected result in the low and high temperature limits.

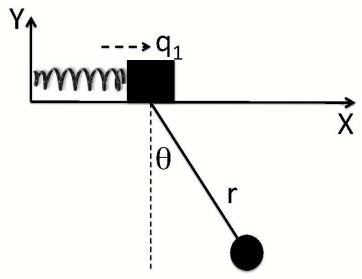
   (b) Calculate the fluctuations of the string's length (RMS).

   (c) This part can be completed independently of parts a) and b). If the length of the string is fixed to be L so that the number of particles in the long state is $N_b = \frac{L-Na}{b-a}$,then what is the energy ($E(N, L)$) and entropy ($S(N, L)$) of the system?

2. A pendulum of mass m and length r is attached to a support of mass m (see figure). The support is free to move along a horizontal track. A spring, of force constant k, attaches the support to a wall. The following relation exists among the various quantities: $\frac{2g}{r} = \frac{k}{m} \equiv \omega_0^2$. q_1 is the displacement of the spring from the equilibrium position.

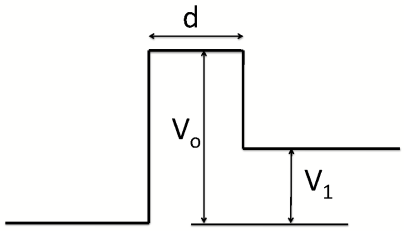
   (a) Write the kinetic and potential energies for this system in terms of the generalized coordinates q_1 and q_2, where q_1 is defined as above and $q_2 = r\theta$.

   (b) For this part you will assume the small angle approximation. Determine the frequencies for the normal mode oscillations in terms of ω_0 by solving the required differential equations. You can arbitrarily choose the amplitude of the solution for one of the generalized coordinates to be unity.



3. Show that the magnetic field near the center of a set of Helmholtz coils is uniform to second order in the displacements from the midpoint along the line connecting the center of the coils. Helmholtz coils are two parallel and coaxial coils of radius R which are separated by a distance R. The current flows in the same direction in each coil.

4. A particle of mass m traveling to the right impinges on a the potential barrier shown in the figure. The particle has energy E ($V_1 < E < V_0$). Find the transmission probability. To avoid laborious algebra you can express your answer as a ratio of determinants. Make sure that all the symbols in the determinants are defined.



5. Kaons decay to two pions ($K \rightarrow \pi\pi$). Assume we have a thousand kaons at rest at the center of a 10 meter (inner radius) spherical shell detector. The mean lifetime of a pion in it's rest frame is 25 nanoseconds. In how many of the thousand kaon decays would you expect there to be at least one pion that reaches the detector. You will need the rest-masses of the kaon ($M_K = 496 MeV/c^2$) and pion ($M_\pi = 139 MeV/c^2$)

6. A double star system is composed of two identical stars, each of mass M and separated by a distance r.

   (a) What is the gravitational potential energy and the effective radial potential energy of the stars?

   (b) Find the frequency of small radial oscillations about a circular orbit in terms of the angular momentum.

   (c) Show that the period of these radial oscillations equals the period of the circular orbit.

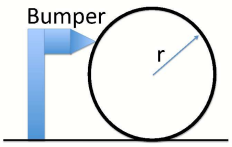
GRADUATE WRITTEN EXAMINATION, August 2009

Short Problems *Best 10 of 12*

1. What is the interaction energy between two parallel electrostatic dipoles,with dipole moments $\vec{p}_1$ and $\vec{p}_2$ respectively, separated by a distance d? Assume the line joining the dipoles is perpendicular to the direction of the dipoles.

2. A radioactivity counter has a precisely known mean expected background rate of 63 counts/min. In the presence of a test sample a technician measures 94 counts in one minute. Roughly, how long does the technician need to take data in order to quote the sample rate with an uncertainty of 10%?

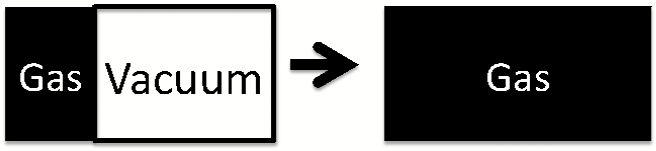
3. The bumpers of a billiard table are designed so that there is no horizontal reaction force that the table surface exerts on the ball during a rolling collision. What will be the height of the bumpers with respect to the radius of the billiard ball? The moment of inertia for a billiard ball is $I = \frac{2mr^2}{5}$.



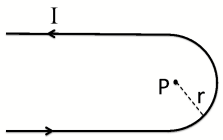
4. What is the value of the matrix element $< j\prime, m\prime|[J^+, J^-]|j, m >$?

5. Assume a football player (American football) can throw horizontally more easily than vertically. The initial speed of the football varies as $v_0 cos(\theta)$ where θ is the initial elevation angle. What elevation angle achieves the maximum horizontal distance? Neglect air resistance.

6. One mole of an ideal gas is confined to one third of a thermally insulated container (see figure). The membrane ruptures and the gas freely expands. What is the change in entropy?



7. A long wire with current I is bent so that it forms a long U shape. Find the expression for the magnetic field ($\vec{B}$) at the point P, which is at the center of the semi-circle. Clearly indicate the units and your system of units.

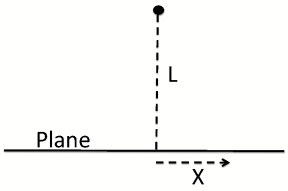


8. A number, N, of Maxwell-Boltzmann particles are distributed among 4 energy states: The energy of the nth state is nkT where n=0,1,2 and 3. What is the energy of the system as a function of temperature?

9. Matrix A is Hermitian and unitary. Derive the possible eigenvalues.

10. Suppose the electron is 100 times more massive than it's known mass. What would be the energy of a radiated photon in a hydrogen atom when the electron transitions from the first excited state to the ground state? What would be the radius of the first Bohr orbit?

11. A charge q is a distance L from a conducting grounded plane (infinite in extent). Find an expression for the charge density of the plane as a function of the distance x in the figure. The variable x represents the radial distance from the point on the plane that is directly below the charge.



12. Derive the Ideal Gas Law for a 2-dimensional gas. It may be helpful to consider the average force exerted by a molecule on the wall of a container.