University of Minnesota
School of Physics and Astronomy

Fall 1998 GRADUATE WRITTEN EXAMINATION

Friday, September 18, 1998 Part I 9:00 A.M. - 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed book examination. You may use calculators.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit. Also write the relevant problem number of each such piece of paper. **BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER**, so that no sheet contains work for more than one problem, and use only one side of the paper. Your completed problems should be put into the manila envelope provided.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Permittivity constant	ε_o	8.85×10^{-12} F/m
Permeability constant	μ_o	1.26×10^{-6} H/m
Electron charge to mass ratio	e/m_e	1.76×10^{11} C/kg
Proton rest mass	m_p	1.67×10^{-27} kg
Ratio of proton mass to electron mass	m_p/m_e	1840
Neutron rest mass	m_n	1.68×10^{-27} kg
Muon rest mass	m_μ	1.88×10^{-28} kg
Planck constant	h	6.63×10^{-34} J·s
Electron Compton wavelength	λ_c	2.43×10^{-12} m
Molar gas constant	R	8.31 J/mol·K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2·K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Gravitational constant	G	6.67×10^{-11} m^3/s^2·kg
Bohr radius	a_o	5.29×10^{-11} m
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg

1998 fall GWE Questions

Short problems

1. A rock is found to contain 4.20 mg of ^{238}U and 2.00 mg of ^{206}Pb. Assume that the rock contained no lead at the time of its formation, so that all the lead now present is due to the decay of the uranium originally present in the rock. Find the age of the rock given that the half-life of ^{238}U is 4.47×10^9 yr. The decay times of all intermediate elements are negligibly short.

2. A particle of mass m is located in a three-dimensional cubic region with absolutely impenetrable walls. The side of the cube has length a.

   (a) What are the allowed values of the energy for this particle ?
   (b) With the ground state identified as the first level, what is the energy difference between the third and fourth energy levels of the particle ?
   (c) What is the energy and degeneracy of the sixth level ?

3. A planet (of mass m) moves in an elliptic orbit around the Sun (of mass M) so that its minimum and maximum distances from the Sun are equal to r_1 and r_2 respectively. In terms of the given quantities, find the angular momentum L of the planet relative to the center of the Sun.

4. An electric charge q is located at a distance d away from an infinitely large conducting plane. What force would the charge exert on the plane?

5. The refractive index of glass can be represented approximately by the empirical relation $n = A + B\lambda^{-2}$, where λ is the wavelength of light in vacuum. What are the corresponding phase and group velocities of light in glass ? Do your formulae reduce to what you expect if there is no dispersion ?

6. With what speed would you have to go through a red light ($\lambda_R = 620$ nm) in order to have it appear green ($\lambda_G = 540$ nm) ?

7. Two identical bodies, each characterized by a heat capacity at constant pressure C which is independent of temperature, are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially, their temperatures are T_1 and T_2, respectively; finally, as a result of the operation of the heat engine, the bodies will attain a common final temperature T_f.

   (a) What is the total amount of work W done by the engine? Express the answer in terms of C, T_1, T_2, and T_f.

   (b) Use arguments based upon entropy considerations to derive an inequality relating T_f to the initial temperatures T_1 and T_2.

   (c) For given initial temperatures T_1 and T_2, what is the maximum amount of work obtainable from the engine?

8. A block of mass M moves without friction on a horizontal surface at a speed v_0. A small cube of mass m is then dropped onto the front edge of the block, with zero initial horizontal speed. With μ the coefficient of friction between the block and the cube, and neglecting the cube's dimensions as compared to those of the block, what is the minimum length L of the block so that the cube will not slip off it ?

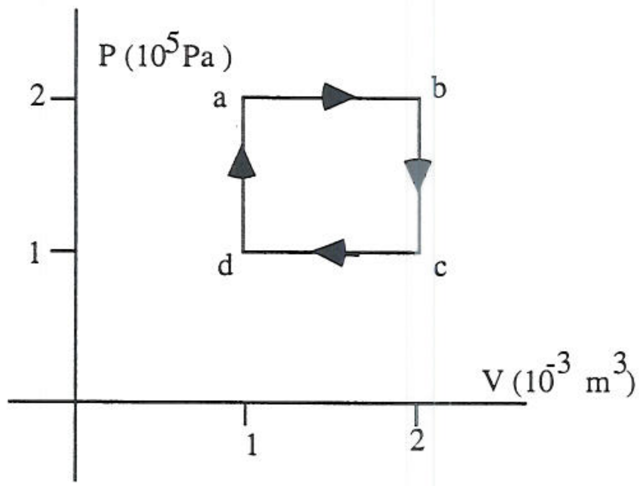
9. An electron, of speed v directed at an angle θ with respect to the x-axis, is at the origin of coordinates when it enters a region of constant magnetic field $\mathbf{B} = B\hat{x}$.

   (a) Qualitatively describe the subsequent motion of the electron.

   (b) At what distance from the origin will the electron's trajectory encounter the x-axis ?

10. A length L of tape (flexible and not stretchable) is tightly wound. It is then allowed to unwind as it rolls down a long and steep incline that makes an angle θ with the horizontal, the upper end of the tape being tacked down. How long does the tape take to unwind completely? Note that the moment of inertia of a disk is half as much as that of a hoop of the same radius and the same mass.

11. A gas in run through the following process quasistatically.
   (a) How much work is done by or to the system in the cycle a-b-c-d-a?
   (b) How much heat is absorbed or released by the gas during the cycle?



12. An electron in the Coulomb field of a proton is in a state described by

$$|\psi\rangle = \frac{1}{6}[4|1,0,0\rangle + 3|2,1,1\rangle - |2,1,0\rangle + \sqrt{10}|2,1,-1\rangle],$$

where n, l, and m in the $|n,l,m\rangle$'s are the principal, angular momentum and magnetic quantum numbers.

   (a) What is the $\langle E \rangle$?

   (b) What is the energy uncertainty ΔE in this state?

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Fall 1998 GRADUATE WRITTEN EXAMINATION

Saturday, September 19, 1998 Part II 9:00 A.M. - 1:00 P.M.

Part II of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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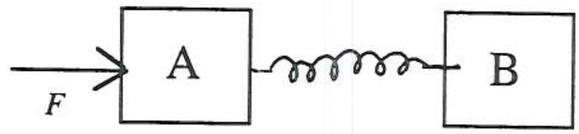
Long problems

1. Two identical objects of mass m are connected by a spring of spring constant k. At $t = 0$, the masses are at rest at their equilibrium positions. Object A is then subjected to an external force $F = F_0 \cos\omega t$ ($t > 0$). Calculate the motion of object B.



2. The electric field in an empty, square metal box of sides of length $L = \pi/k$ in the x and y directions is given by

$$\mathbf{E} = E_0\hat{z} \cos kx \cos ky \cos\omega t.$$

Find an expression for the corresponding magnetic field $\mathbf{B}$ in the box, spelling out how ω and k, and the electric and magnetic field amplitudes are related, respectively.

3. In a model in which the charges in a conductive material obey the equation of motion

$$m\frac{dv}{dt} = qE(t) - \frac{mv}{\tau}.$$

Find the velocity as a function of time in the presence of a harmonic electric field, $E(t) = E_0 \cos\omega t$, which was adiabatically switched on from the distant past (much longer ago than the period). Thus determine the two conductivity functions associated with the parts of the velocity which are in and out of phase with the electric field, respectively. Assume that the volume density of charge carriers is n. Find the power dissipated per charge per unit volume in such a system, averaged over the period $T = 2\pi/\omega$.

4. A (horizontal) long cylindrical vessel with a frictionless piston of mass $m = 200$ g contains 0.1 mol of helium. The gas is heated in such a way that the piston, initially at rest, is given a constant acceleration. Neglect the pressure outside the piston.

   (a) How much heat Q (in J) should be supplied to the gas to allow the piston to attain a speed of $v = 1$ m/s ?

   (b) By how much does the temperature of the gas change in this process in degrees C ?

5. Consider a space ship of mass m designed to carry people between two identical stars a distance R apart. The ship is supposed to work by use of a perfectly relecting light sail of area A. You are to neglect any light from sources other than the two stars. Express the light flux in terms of the solar flux S per unit area from the nearest star at the starting distance a from that star. Assume that the masses of the stars are each M and that the force on the sail is large enough to overcome the gravitational attraction so that the light pushes the ship away from the starting point. Assume that the sail is a perfect reflector.

(a) Calculate the velocity of the ship at the half way point of the trip.

(b) Write an expression for the time required for the trip in terms of the parameters given and a single integral. Neglect any relativistic effects.

6. A particle, of mass m, is confined in a one-dimensional infinite square well: The potential inside the well $(-L < x < L)$ is zero and the potential outside the well $(|x| > L)$ is infinite.

(a) List the normalized wavefunctions and corresponding energies of the ground state and the first two excited states of the particle.

Now suppose that a weak time independent perturbation in the form of a potential $V(x) = U_0\, x/L$ depending linearly on the position is imposed inside the well (U_0 is a constant).

(b) Calculate the correction to the energies of the ground state and first two excited states (considered in part (a)), to *first order* in perturbation theory.

(c) Calculate the correction to the energy of the ground state only, to *second order* in perturbation theory, keeping only the first contributing term in the series. The following trigonometric identities may prove useful:

$$\sin A \cos B = [\sin(A+B) + \sin(A-B)]/2$$
$$\cos A \cos B = [\cos(A+B) + \cos(A-B)]/2$$
$$\sin A \sin B = [\cos(A-B) - \cos(A+B)]/2$$

Even if you don't arrive at a precise result, you should at least write down the functional dependence on the given parameters that you expect as the answer to part (c).