University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

FALL 1999 - PART I

Friday, September 3, 1999 - 9:00 AM – 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You are required to do Question 1 and eleven other questions that you may select from Questions 2 through 18. Note that on average you can afford to spend about 15 minutes on each question. If you do more than eleven questions from the group of Questions 2 through 18, all your answers will be graded, but you will only receive credit for your eleven best efforts.

This is a closed book examination. You may use calculators. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant **PROBLEM NUMBER. BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER,** so that no sheet contains work for more than one problem. Use only one side of the paper: If some problems require more than one sheet for their solution, then be sure to indicate "page 1", "page 2",... etc. under the problem number entered on the sheets. All your completed problems should be put in the manila envelope provided.

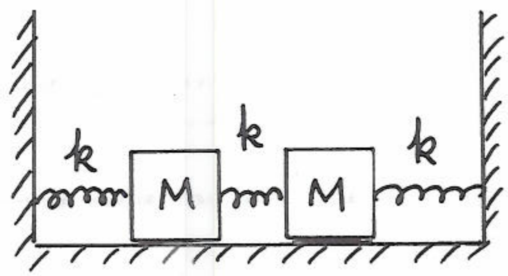
Graduate Written Exam Fall 1999 - Part I

1. Give numerical estimates for the following quantities, in the units indicated:.

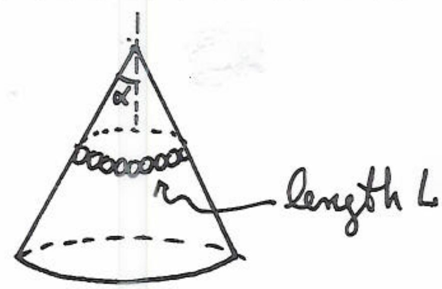
   (a) Rest mass of the electron (in eV/c^2)
   (b) Ionization energy of the ground state of the hydrogen atom (in eV)
   (c) Average binding energy of a neutron in a medium-weight nucleus (in eV)
   (d) Energy of an electron at the Fermi surface of copper (in eV)
   (e) Energy of a photon of visible light (in eV)
   (f) Kinetic energy of a thermal neutron (in eV)
   (g) Kinetic energy of a small car going 60 miles per hour (in eV)
   (h) Lifetime of a neutron (in seconds)
   (i) Temperature corresponding to the λ-point of helium (in K)
   (j) Surface temperature of the Sun (in K)
   (k) Intrinsic spin of the photon (in units of $h/2\pi$)

2. A particle of mass M is initially at rest. It is struck by a moving particle of mass m. The collision is perfectly elastic, and the particles are constrained to move along the same straight line. What fraction of the kinetic energy of the incoming particle is transferred to M by the collision ? Use non-relativistic mechanics in deriving your answer.

3. Two bodies of mass M are connected by three massless springs, each of spring constant k, as shown in the figure. The masses are constrained to move in the x-direction on a frictionless horizontal surface. Find, <u>preferably by inspection</u>, the frequencies of the normal modes of oscillation of this system.



4. A single closed loop of chain of mass m and length L rests on the surface of smooth frictionless cone (see the figure). The chain lies in a horizontal plane, while the axis of the cone is vertical. The half-angle of the cone is α. Determine the tension in the chain.



5. The objective lens of a camera has a focal length of 5.0 cm. The distance between the lens and the film may be adjusted so that an object anywhere between infinity and one meter from the lens may be focused on the film. By placing a supplementary lens (a close-up lens) in front of and close to the regular lens of the camera, objects as close as 25 cm may be photographed.

   (a) What is the nature of this lens ? That is, give its focal length and state whether it is convergent or divergent.
   (b) With this lens in place, how far away is the most distant object that can be brought into focus ?

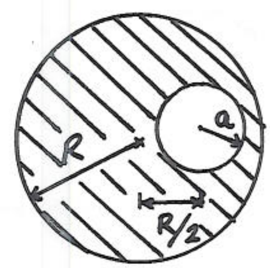
6. Compare the approximate angular resolution of the following two optical systems:

   (a) The 200-inch diameter telescope on Mt. Palomar, for light of wavelength 5000 Angstroms, assuming that this telescope is diffraction limited.
   (b) Two radio-telescopes, operating as an interferometer, separated by a distance of 2000 miles, respectively 140 feet and 85 feet in diameter, for 18 cm radio waves.

7. A solid aluminium sphere in a uniform magnetic field rotates relative to the field. For each of the following two situations, discuss qualitatively the current distribution:

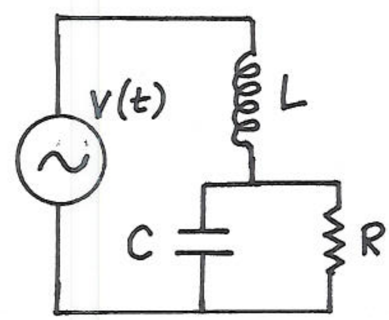
   (a) the axis of rotation is perpendicular to the magnetic field
   (b) the axis of rotation is parallel to the magnetic field

8. An otherwise solid conducting sphere of radius R contains a single spherical cavity of radius a. The center of the cavity is at a distance R/2 from the center of the conducting sphere. The cavity is entirely within the sphere, so a < R/2. This conducting sphere carries a net charge Q. Find the electrostatic potential everywhere outside the conducting sphere if



   (a) the cavity is empty
   (b) the cavity contains a point charge q at its center, with the net charge of the conducting sphere still Q.

9. The AC circuit illustrated in the diagram was patented by Charles Proteus Steinmetz. The applied voltage is given by $V(t) = V_o \sin \omega t$, with a frequency fixed at $\omega = 1/(LC)^{1/2}$. Determine the amplitude and phase of the current through the resistor R. Express your answer in terms of the amplitude V_o of the applied voltage and the other circuit parameters.



10. One mole of an ideal monoatomic gas is contained in a thermally insulated cylinder of total volume V. Initially, the gas is confined to half of the cylinder by a thin partition. This partition is suddenly removed (without doing work on the gas), allowing the gas to fill the entire volume of the cylinder. Determine,

   (a) The resulting change ΔU (in Joules) of the internal energy of this system.
   (b) What is wrong with the following statement: No heat flowed into the system during the expansion. Therefore, since $\Delta S = \int dQ/T$, it follows that $\Delta S = 0$ for this process.
   (c) The actual change ΔS (in J/K) in the entropy of this system.

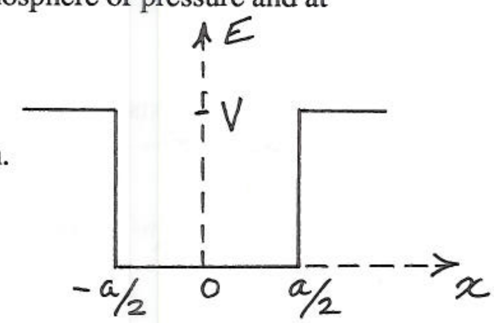
11. Because of increasing levels of greenhouse gases in the atmosphere, the mean air temperature on Earth will probably rise, making for longer growing seasons and resulting in higher shrubs and trees, with a consequent increase in the moment of inertia of the Earth . Supposing this happens (!),

(a) Assume this growth process is equivalent to raising a thin ring of mass m to a height h above the ground: Calculate the fractional change $\Delta T/T$ in the length of the day, assuming that the Earth is a sphere of moment of inertia I_o about its axis, and that the vegetation is concentrated in a ring around the equator.
(b) Does the total kinetic energy of rotation increase, decrease, or stay the same ?

12. Invoke the energy band theory of crystalline solids to explain the differences between the electrical properties of a conductor, a semi-conductor and an insulator.

13. The cross-section for collision between helium atoms is about 10^{-16} cm^2 . Estimate the mean free path of helium atoms in helium gas at one atmosphere of pressure and at room temperature.

14. Suppose a particle of mass m moves in one dimension in a square potential well of width a and depth V, as shown. Find the minimum depth for which there will exist at least two discrete levels, the ground state of even parity and the first excited state of negative parity.



15. The rotational level spacings of the HD (hydrogen-deuterium) molecule are of the order of 5×10^{-3} eV, the vibrational level spacings are of the order of 0.5 eV, and the electronic level spacings are of the order of 5 eV. Sketch a curve showing the molar specific heat at constant volume as a function of temperature, from T = 30 K to T = 20,000 K. Note that the boiling point for HD is 22 K, and neglect the fact that HD in fact dissociates at 6,000 K.

16. The normalized wave function of the electron in the ground state of the hydrogen atom may be written as $\psi(r) = (1/\pi a_o^3)^{1/2} \exp[-r/a_o]$. Here, r is the magnitude of **r**, the radius vector between the proton and the electron, and a_o is the Bohr radius. Give an expression for the expectation value of r in the ground state. You should express your answer as a constant times a_o. This constant should be specified as completely as possible !

17. Consider two identical particles moving in a one-dimensional harmonic oscillator potential. The Hamiltonian operator is

$$H = -(\hbar^2/2m)\,[\partial^2/\partial x_1^2 + \partial^2/\partial x_2^2] \;+\; (m\omega^2/2)\,[\,x_1^2 + x_2^2\,]$$

while the two lowest single-particle spatial wave functions for the harmonic oscillator are

$$\psi_0(x) = c_0 \exp[\ -m\omega^2 x^2/2\hbar\ ]\ \ , \text{with } E_0 = \hbar\omega/2$$

$$\psi_1(x) = c_1\, x \exp[\ -m\omega^2 x^2/2\hbar\ ]\ \ , \text{with } E_1 = 3\hbar\omega/2$$

and where c_0 and c_1 are normalization constants that can be left unspecified. Construct the full ground state wavefunction of the two-particle system in the two cases

(a) the particles are spin-zero Bosons
(b) the particles are spin-1/2 Fermions.

18. The Λ^0 is a particle that decays spontaneously into a proton and a pi-meson, according to $\Lambda^0 \to p + \pi^-$. Determine the numerical value of the momentum of both the proton p and the pi-meson π^-, in the center-of-mass system (i.e. the reference frame in which the decaying Λ^0 particle is at rest) and in units of MeV/c. The masses of the particles of interest here are:

$$m_\Lambda = 1114 \text{ MeV}/c^2$$

$$m_p = 938 \text{ MeV}/c^2$$

$$m_\pi = 139 \text{ MeV}/c^2$$

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GRADUATE WRITTEN EXAMINATION

FALL 1999 - PART II

Saturday, September 4, 1999 - 9:00 AM – 1:00 PM

Part II of this exam consists of six problems of equal weight: You may select any six of the nine questions in this part of the exam. If you do more than six problems, all of your answers will be graded, but you will receive credit only for your six best efforts.

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Graduate Written Exam Fall 1999 - Part II

1. The purpose of this problem is to test your understanding of the definitions of various dynamical quantities.

   A particle of mass m is projected vertically from the surface of the Earth, with an initial kinetic energy E. Taking the acceleration due to gravity to be a constant of magnitude g during the motion, and neglecting the Earth's rotation, write down the following for an inertial frame of reference:

   (a) Newton's equation of motion
   (b) The kinetic and potential energies
   (c) The lagrangian, and the Euler-Lagrange equations
   (d) The hamiltonian
   (e) Hamilton's equations

2. (a) Write down Maxwell's equations for the electromagnetic field in the presence of a charge density $\rho(\mathbf{r},t)$ and a current density $\mathbf{j}(\mathbf{r},t)$.
   (b) Show that Maxwell's equations lead to the equation of continuity for the charge and current densities.
   (c) Show how the equation of continuity implies the law of conservation of electric charge.

3. Three coplanar slits each of width a are arranged parallel to each other with a separation d between the centers of adjacent slits. They are illuminated at normal incidence by monochromatic plane waves of light of wavelength λ. The light emerging from the slits passes through a lens of focal length $F = 1000$ d. An interference pattern is formed in the focal plane of this lens.

   (a) Assuming that $a \ll d$, determine the form of this pattern. Make a sketch of this pattern, with appropriate labeling of the scales of the abcissa and ordinate.
   (b) Now consider the slit width to be given by $a = d/2$. What new element enters the problem ? As in part (a), determine the form of the pattern, and make a sketch of it.

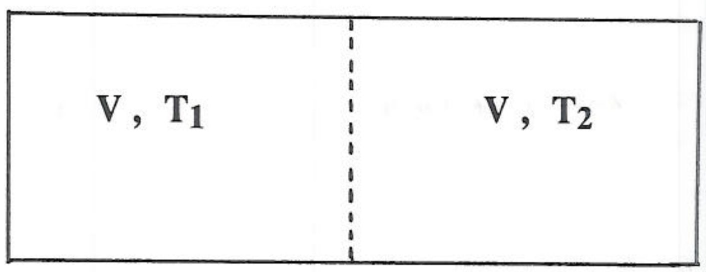
4. An atmosphere whose pressure and density as a function of height satisfy the relation $p = A\,\rho^{\gamma}$ with A = constant and γ = adiabatic index, is called an adiabatic atmosphere.

   (a) Write down the equation (relating the change in pressure with height) that expresses the condition of hydrostatic equilibrium, assuming the acceleration due to gravity, g, to be constant over the entire height of the atmosphere.
   (b) Show that the temperature of an adiabatic atmosphere in hydrostatic equilibrium decreases linearly with height, and find a formula for the constant of proportionality Use this to estimate the temperature gradient for the Earth's lower atmosphere, in ^{o}C/meter.

5. A beam of charged particles consists of a mixture of protons and K^+ mesons, all with equal charge +e and equal momentum 1.0 GeV/c. Outline the design of a Cerenkov counter which, when placed in the beam, will be sensitive only to the K^+ mesons. The rest energies of protons and K^+ mesons are respectively $m_p c^2 = 938$ MeV and $m_K c^2 = 494$ MeV. To construct your counter, you have available the following materials: Lucite (n = 1.450), water (n = 1.331) and Freon gas (n = 1.009).

6. The container shown in the figure below has a volume 2V divided into two equal parts by a thin, porous, insulating wall with holes whose diameters are small compared to the mean free paths of gas molecules. N moles of gas of molar mass M are put in the container, the left side being maintained at a temperature T_1 and the right side at a temperature $T_2 > T_1$. Let A be the total area of the holes in the wall. When equilibrium is established,

   (a) Compute the particle density and pressure on the left side of the porous wall.
   (b) Same question for the right side of the porous wall.



7. A particle of mass m moves in one dimension in a square well potential with walls of infinite height at a distance L apart (to be concrete, you may take V = 0 for $0 < x < L$ and V = infinity elesewhere). The particle is known at time t = 0 to be in a state consisting in an equal admixture (with zero relative phase!) of the two lowest energy eigenstates of the system.

   (a) Write down the wavefunction for this particle at t = 0 and at an arbitrary later time t.
   (b) Calculate the probability as a function of time that the particle will be found in the right-hand side of the potential well, i.e. at some $x > L/2$.

8. A hemisphere of uniform density, radius R and mass m, rests on a plane surface on its spherical side. The hemisphere rocks back and forth with a small amplitude, and no slipping. Calculate the frequency of these small oscillations.

9. Discuss briefly some of the experimental information supporting the use of Fermi-Dirac statistics to describe many-electron, many-nucleon, or any other many-Fermion systems that you may know of. Give at least three examples, and for each case you discuss, show qualitatively how the Pauli exclusion principle explains the observed facts.