University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Fall 2015 – PART I

Tuesday, August 25th, 2015 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

1. A positive point charge q is fixed at a distance $a = 10$ cm above a grounded and conducting plane. An equal negative charge $-q$ is at a distance b above the plane, on the segment perpendicular to the plane that goes from the plane to the positive charge. Compute the value of b for which no force is acting on the negative charge, neglecting gravity.

2. Consider a free Fermi gas consisting of N spin ½ particles of mass m in 2 dimensions, confined to a square with area $A = L^2$. (a) Find the Fermi energy ε_F (in terms of N, A, and m), (b) Derive a formula for the density of states. (Hint: You should find that it is a constant, independent of ε), (c) Find the average energy per particle in terms of ε_F.

3. An electron in a hydrogen atom does not fall to the proton because of quantum motion (which may be accounted for by the Heisenberg uncertainty relation for an electron localized in the volume with size r). This is true because the absolute value of the Coulomb potential energy goes to minus infinity with decreasing distance to the center r relatively slowly, like $-1/r$. Is such an "atom" stable for any potential behaving as $-1/r^s$? If not, find the range of values of s at which the "atom" is stable, so that "the electron" does not fall to the center.

4. Estimate the average velocity (in m/s) and the mean free path (in m) of nitrogen molecules in this room. Hint: recall that atmospheric pressure is about 10^5 N/m^2.

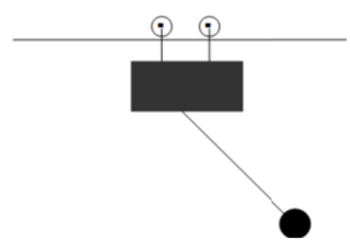
5. A coaxial cable of length $l = 1$ m is made of two thin coaxial copper cylinders with diameters $a = 1$ cm and $b = 2$ cm separated by air. At one end of the cable the internal and external cylinders are connected by a short wire. At the other end internal and external cylinders are connected to opposite poles of a battery. The current runs on the external cylinder to the opposite end of the cable and then returns back to the battery via the internal cylinder. Calculate the cable inductance L in H (Henry).

6. A particle of mass m can move in 1 spatial dimension x. Its wave function is

$$\psi(x,t) = Ne^{-a|x|-ibt}$$

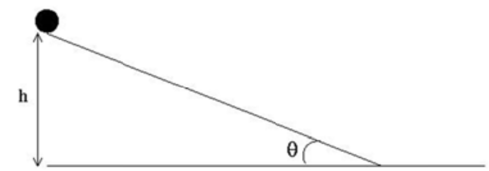
where t is time, and N, a, and b are positive constants. Find the potential $V(x)$ which governs the motion of this particle. (Hint: take into account the discontinuity in the slope of the wave function at $x = 0$.)

7. Consider the arrangement shown at right with a rectangular cart with mass M that can move horizontally along a rail on massless pulleys. A sphere with mass m is attached to the cart by a massless string of length L. If the string is always taut and there are no dissipative forces, what is the period of small oscillations for the sphere?



8. Consider a particle which has only two energy states, $E_1 = 0$, $E_2 = \varepsilon$. (a) Compute the average energy $\langle E \rangle$ of such particle in a reservoir with temperature T. (b) Calculate the heat capacity C_V of a system of N such non-interacting particles.

9. A solid cylinder with a uniform density has a mass M and radius R is released with zero initial speed from the top of an inclined surface, as shown in the figure. (a) Calculate explicitly the moment of inertia of the cylinder; (b) How long does it take for it to reach the bottom of the surface, assuming that it rolls without sliding? (c) Suppose a hollow cylindrical shell with the same mass M and radius R is released at the same time as the cylinder. Does it reach the bottom of the inclined surface sooner or later than the cylinder? Explain your reasoning.



10. A particular Cherenkov particle detector consists of a tank full of water. A tau-antineutrino passes through the detector and collides with a nucleus of a hydrogen atom in the detector. The tau-antineutrino and proton annihilate in the collision to produce an anti-tau and a neutron. (a) What is the minimum energy the tau-antineutrino needs to have in the rest frame of the detector in order for this interaction to happen? (b) Using the minimum energy from part (a), what is the magnitude and direction (with respect to the incoming anti-neutrino) of the momentum of the produced anti-tau? The anti-tau has a mass of 1.8 GeV/c^2. The proton and neutron each have a mass of 0.9 GeV/c^2. Assume the antineutrino is massless.

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GRADUATE WRITTEN EXAMINATION

Fall 2015 – PART II

Wednesday, August 26th, 2015 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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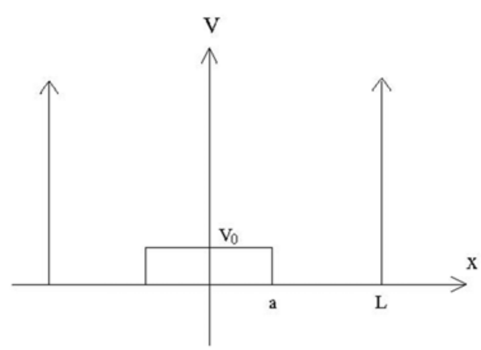
1. A long linear solenoid has length l, radius r, and n turns per unit length. The solenoid is part of a circuit having a resistor of resistance R and a generator of emf $\mathcal{E}$. The emf of the generator is very slowly increased in such a way that the current I in the windings of the solenoid grows as $I = a\,t$, where a is a small positive constant and t is time. (a) Compute the total magnetic energy inside the solenoid. (b) Compute the induced electric field inside and outside the solenoid. (c) Compute the flux of the Poynting vector through the solenoid (evaluate **B** just inside the solenoid), and show that is is equal to the rate of increase with time of the magnetic energy inside the solenoid. (d) When the current reaches the value I_0, the generator is switched off. Compute how the current evolves from this time on, and show that energy is conserved.

2. You decide to do a Rutherford-type scattering experiment to find the composition of an unknown material. You shoot a beam of oxygen nuclei ($m=16m_p$, where m_p is the proton mass) at the material, and find that the oxygen nuclei that are scattered by 60° have 54% of their initial kinetic energy. What is the mass of the nucleus that scattered the oxygen, in units of the proton mass? You can assume that the velocity of the oxygen beam is much less than the speed of light.

3. Consider a particle of mass m in an infinite one dimensional potential well of width $2L$ (i.e., $V = 0$ for $-L < x < L$ and is infinite otherwise). (a) Compute the (properly normalized) eigenfunctions and the corresponding eigenenergies. (b) Assume now that the potential is perturbed by a small potential of height V_0 and width $2a$, where $a \leq L$ (see Figure). Compute the corrections to the eigenenergies to leading order in V_0. (c) Compute the perturbed energies in the limit of $a = L$, and compare them with the exact solutions in this limit.



4. A metallic ball with radius R is immersed and suspended in a weakly conducting medium with a uniform conductivity σ in the middle of a large metallic vessel (e.g., salty water in a metallic bathtub). (a) One wire from a battery is attached to the ball and the second wire is attached to the vessel. Calculate the resistance of the media. You can assume that the current is spherically symmetric around the ball. (This is how a standard plasma probe tests the ionization degree of a plasma.) (b) After the ball is charged to a charge Q_0, the battery is disconnected at $t = 0$ and the ball discharges with time. Find how the ball charge Q depends on time and the characteristic time of this discharge by writing a simple differential equation for $Q(t)$ (this characteristic time is called the Maxwell time).

5. A hemoglobin molecule can bind four O_2 molecules. Assume that ε is the energy of each bound O_2, relative to O_2 at rest at an infinite distance. Let $\lambda = \exp(\mu/kT)$ denote the absolute activity of the free O_2, where μ is the chemical potential. (a) Find the appropriate partition function. (b) What is the probability that one and only one O_2 is adsorbed on a hemoglobin molecule? Carefully sketch the result qualitatively as a function of λ. (c) What is the probability that all four O_2 are adsorbed? Sketch this result also as a function of λ.