University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Fall 2018 – PART I

Monday, August 20th, 2018 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth’s mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling’s Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

A function $\Psi(x,t)$ is a solution to the Schroedinger equation for a potential $V(x)$:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = [\,-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)]\ \Psi(x,t) \tag{1}$$

Consider now the potential $V'(x) = V(x) + V_0$ where V_0 is a constant.

a) Is $\Psi(x,t)$ a solution of (1) with the potential $V(x)$ replaced by $V'(x)$? If not, can it be modified so that it is a solution?

b) Compare with the situation in classical mechanics.

c) Discuss whether the above results would have experimental consequences.

Problem 2

Consider the Earth-Sun system as a quantum gravitational analog of a hydrogen atom.

a) What would be, in actual units, the Bohr radius of the system?

b) By equating the actual energy of the system to the Bohr formula, estimate the quantum number n of the Earth.

Problem 3

A particle moves in one dimension under the action of a quadratic velocity dependent retarding force. (Assume the initial velocity is v_0, and the motion is horizontal.)

a) Derive the expression for the position as a function of time.

b) Does this particle ever stop (i.e., at $t = \infty$ is the distance traveled finite)?

Problem 4

A bullet of mass m and velocity v_0 strikes a target object of mass M, which is resting on a frictionless support. If the bullet emerges with velocity $v_0/2$, find the fraction of the initial kinetic energy which is deposited into the target as frictional heat in terms of the ratio of bullet mass to target mass $\gamma = m/M$.

Problem 5

Two conducting metal objects are embedded in a weakly conducting material of conductivity σ.

a) Show that the resistance between them is related to the capacitance by $R = \epsilon_0/\sigma C$.

b) Suppose that you connect the two objects with a battery so that the potential difference is V_0. Show that when you disconnect the battery, the potential decreases exponentially and determine the time constant.

Problem 6

A satellite in geostationary orbit is used to transmit data via electromagnetic radiation. The satellite is at a height of 35,000 km above the surface of the earth, and we assume it has an isotropic power output of 1 kW.

a) What is the amplitude E_0 of the electric field vector of the satellite broadcast as measured at the surface of the earth?

b) A receiving dish of (projected) radius R focuses the electromagnetic energy incident from the satellite onto a receiver which has a surface area of 5cm^2. How large does the radius R need to be to achieve an electric field vector amplitude of 0.1mV/m at the receiver?

Problem 7

Consider a one-particle system capable of three states ($\epsilon_n = n \cdot \Delta, n = 0, 1, 2$) in thermal contact with a reservoir at temperature τ. In the limits $\tau/\Delta \to \infty$ and $\tau/\Delta \to 0$, find the

a) energy.

b) free energy.

c) heat capacity.

Problem 8

Consider a non-interacting gas of $N \gg 1$ ^4He atoms at temperature τ in a volume V at pressure p. If half of the atoms are in the ground state ($\epsilon = 0$), find the chemical potential μ.

Problem 9

A radioactive nucleus in an excited state decays (at rest) with a lifetime of $\tau = 10^{-7}s$ to its ground state through the emission of a gamma ray of energy $E = 15$ keV.

a) What is the wavelength of the photon emitted in this decay (in nm)?

b) What is the natural line width of the excited level (in eV)?

c) What is the length of the photon wave train (in meters)?

Problem 10

The laboratory differential cross section for proton scattering in a certain process is $d\sigma/d\Omega = a + bcos^2\theta$ with a and b constants with the numerical values $a = 420\mu$b/sterad and $b = 240\mu$b/sterad. Note that 1 barn (b) $= 10^{-24}$cm^2.

a) What is the total cross section (numerical value)?

b) A counter mounted at 10 cm from the target at an angle of 60° has an effective area of 0.1 cm^2. The target thickness is 10^{-4}cm and the number of atoms per cm^3 in the target is 10^{22}. What is the counting rate, when the beam current is 1 μA?

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GRADUATE WRITTEN EXAMINATION

Fall 2018 – PART II

Tuesday, August 21st, 2018 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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Problem 1

Recall that for a charged particle in a magnetic field the quantum Hamiltonian is:

$$H = (1/2m)(\vec{p} - q\vec{A})^2$$

where $\vec{B} = \nabla \times \vec{A}$, and $\vec{p} \to -i\hbar\nabla$.
Show that:

$$\frac{d\langle \vec{r} \rangle}{dt} = \langle (\vec{p} - q\vec{A})/m \rangle$$

Is this result consistent with classical mechanics?

Problem 2

A simple pendulum (mass m) is suspended from a cart (mass M) which moves without friction along a horizontal track.

a) Taking the position of the cart on the track as X and the angle of the pendulum as θ, find the equations of motion.

b) Show how one of these reduces to the expected special case solutions if we i) fix the position of the cart or ii) imagine the pendulum mass is stationary with respect to the cart.

Problem 3

Tritium ions are confined inside a thin wall spherical vessel with a radius of R. (Tritium ions are positively charged isotopes of Hydrogen with one proton and two neutrons, and you can assume that they have the mass of three protons). When the hot gas of tritium ions is stored inside the vessel, it forms layers with different concentrations such that the charge density is $\rho = a_0 r$ where a_0 is a positive constant and r is the radial position from the center of the vessel. The goal is to inject additional Tritium ions into the container so that they can reach the center of the vessel. To know the requirements for your injection system, you need to map out the potential created by the ionic gas.

a) What is the potential as a function of radial distance r from the center of the vessel? Make sure you determine the function for all regions of space and assume $V = 0$ at infinity. Give your answer in terms of R and a_0.

b) Sketch a graph of the potential from part a).

c) Assume the vessel has a radius of 1.00m and $a_0 = 1.88 \times 10^{-6}\mathrm{C/m}^4$. If you are a long distance away from the vessel (you can use the approximation that $r = \infty$), at what minimum speed would you need to shoot a Tritium ion towards the vessel if you want it to reach the center of the vessel?

Problem 4

The energy spectrum of a system consists of two orbitals with one-particle energies $\epsilon_1 = -\epsilon$ and $\epsilon_2 = \epsilon$. The system is in thermal and diffusive contact with a reservoir at temperature τ and chemical potential μ. Assuming that each orbital can be occupied by no more than one particle and that $\mu = 0$, find:

a) All possible states of a system, (ϵ, N), and the grand partition function $\bar{Z}$.

b) The probability that the system is in a state with zero energy.

c) The Probability for the system to be occupied by one particle.

d) The average number of particles in the system, $\langle N \rangle$, and average energy, $\langle U \rangle$.

e) $\langle N \rangle$ and $\langle U \rangle$ in the limits $\tau \to 0$ and $\tau \to \infty$.

Problem 5

A spaceship travels at a constant velocity $v = 0.8c$ with respect to the Earth. Denote the spaceship frame coordinates by a prime ($'$). At time $t = t' = 0$ by Earth and spaceship clocks, respectively, a light signal is sent from the tail (back end) of the spaceship towards the nose (front end) of the spaceship, just as the tail of the spaceship (at $x' = 0$ in the spaceship frame) passes the Earth (at $x = 0$ in the Earth frame). The length of the spaceship, measured in a frame in which it is at rest, is L.

a) At what time, by spaceship clocks, does the light signal reach the nose of the spaceship?

b) At what time, by Earth clocks, does the light signal reach the nose of the spaceship?

Now suppose there is a mirror at the nose of the spaceship which immediately reflects the light signal back to the tail of the spaceship.

c) At what time, by spaceship clocks, does the light signal finally return to the tail of the spaceship?

d) At what time, by earth clocks, does the light signal finally return to the tail of the spaceship?

All answers should be expressed in terms of L and c.