Problem 1

A circle of rope of density ρ, cross-sectional area a and radius $R \gg \sqrt{a}$ is spinning at an angular velocity ω about an axis through the center of the circle. What is the tension in the rope? You may ignore gravity.

Problem 2

Consider a quantum system consisting of two non-interacting spin-1/2 particles. A measurement has been performed with the following result (in units of $\hbar$):

$$(S_1)_z = 1/2 \qquad\qquad (S_2)_x = 1/2$$

where S_i is the spin operator of the i-th particle, and the other subscripts denote the projection onto the z- and x-axis respectively. What is the probability that the total spin of the system is 1?

Recall that the spin-1/2 operators are given in terms of the Pauli matrices,

$$\boldsymbol{S} = \frac{\boldsymbol{\sigma}}{2}, \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Problem 3

In the CO molecule, the carbon (A=12) and oxygen atoms (A=16) are separated by 1.13×10^{-10} m.

(a) Give the energy of the first rotational excited state in eV.

(b) Give the frequency of the lowest energy transition in Hz.

Problem 4

The Poisson probability distribution is often written as:

$$P(n,\mu) = \frac{\mu^n}{n!}\exp(-\mu)$$

(a) If μ is the average number of events seen in a particular experiment in a given time interval, what is the significance of n and of $P(n,\mu)$?

In the very first instance of a multi-messenger observation in astrophysics, the underground detector of the IMB experiment observed a burst of 8 neutrino interactions in the space of 10 minutes, in coincidence with the visual observation of the supernova SN1987A. The corresponding normal background rate had been established previously to two events per day.

(b) What then is the probability of eight background events occurring in the detector in one day?

(c) What is the probability that the eight neutrino events seen in the ten minutes in coincidence with the supernova were actually due to a fluctuation in the background?

Problem 5

A solid metal sphere two meters in diameter is isolated in the center of a large room and is charged to a potential of 100 kV. How much heat in J will be released if the sphere is suddenly grounded?

Problem 6

The radius of the planet Mars is 3.4×10^6 m, its surface gravity is 3.7 m/s^2 and it is about 1.52 times farther from the Sun than the Earth is. From this information, find

(a) the mass of the planet Mars, in kg.

(b) the length of the Martian "year", in Earth years.

Problem 7

Suppose the electron had spin-3/2 instead of spin-1/2. What would then be the atomic numbers Z of the three lowest-mass noble gases, i.e. the equivalents of helium, neon and argon?

Problem 8

The magnetic moment per atom in a solid is $\mu = 10^{-23}$ J/T. What magnetic field B (in Tesla) must be applied at T = 77 K if twice as many atoms are to have their magnetic moments aligned parallel to the field as there are antiparallel?

Problem 9

$^{62}_{29}Cu$, an unstable nucleus with a half-life of approximately 10 minutes, is produced in an accelerator at a constant rate of R nuclei per second. Starting with no $^{62}_{29}Cu$ at all, how much time does it take, in minutes, for the number of $^{62}_{29}Cu$ nuclei to reach 80% of its equilibrium value?

Problem 10

A steady current density **J** gives rise to a time-independent magnetic field $\boldsymbol{B} = 2xy\,a\,\hat{\boldsymbol{x}} + \,y^2\,b\,\hat{\boldsymbol{y}}$, where a and b are constants in appropriate SI units (T/m^2).

(a) How are a and b related?

(b) What is the current density **J** ?

Problem 1

Consider the quantum mechanical problem of two identical non-relativistic particles, each of mass m and spin $1/2$, confined to a one-dimensional infinite square well of width L (that is, $V = 0$ for $0 < x < L$ and $V \to \infty$ everywhere else). In the first part of the problem the particles are not mutually interacting.

Note: For a single particle of mass m, the normalized eigenfunctions and corresponding energies are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}}\, sin\left(\frac{n\pi x}{L}\right), \quad E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

(a) Write down the three lowest energy eigenstates of the system which are also eigenstates of total spin, including both the spatial and spin parts of the wave functions. Give the energy of each state and make an energy diagram indicating the energies and degeneracies.

(b) Now introduce a delta function interaction $V(x_1 - x_2) = \lambda\, \delta(x_1 - x_2)$. Find the resulting shifts in the energies of the states found in part (a) in first order perturbation theory in λ. Indicate how the energy levels are shifted in a diagram as in (a).

Problem 2

A point particle of mass m moving along the x-axis $(-\infty < x < \infty)$ is subject to a force (here, a and b are positive constants of suitable dimensionality)

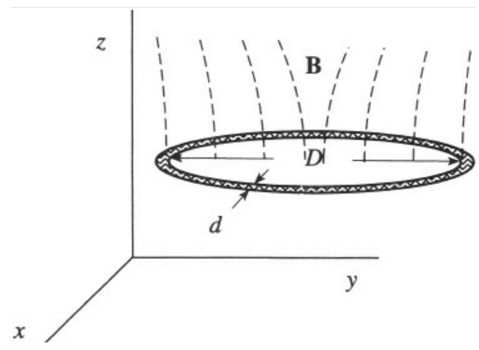
$$F(x) = -a\, e^{-bx}\,(1 - e^{-bx})$$

(a) Obtain and sketch the corresponding potential $V(x)$, adjusted so that $V(x) \to 0$ as $x \to +\infty$. For what values of the total energy is the resulting motion periodic?

(b) What is the period of small oscillations about equilibrium?

Problem 3

A conducting circular loop made of wire of diameter d, resistivity ρ, and mass density ρ_m is falling from a great height h in a magnetic field pointing in the z-direction, $\boldsymbol{B} = B_0\,(1 + kz)\hat{\boldsymbol{z}}$, where k is some constant. The loop, of diameter D, is always parallel to the xy plane (see illustration).



Disregarding air resistance, find the terminal velocity of the falling loop.

Hint: The force exerted by a magnetic field on a magnetic dipole is $\boldsymbol{F} = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{B})$.

Problem 4

A starship leaves the Earth at a speed v = 0.6 c. When a clock on the starship says that one hour has elapsed since departure, the starship sends a light signal back to Earth.

(a) Suppose that Earth and starship clocks were synchronized to zero at the time of departure. According to *Earth* clocks, when was the signal sent?

(b) Again according to *Earth* clocks, how long after the starship's departure did the signal arrive at Earth?

(c) Finally, now according to *starship* clocks, how long after the starship's departure did the signal arrive at Earth?

Problem 5

Two monoatomic ideal gases, each occupying a volume $V = 1.0 \text{ m}^3$, are separated by a removable insulating partition. They have different temperatures and pressures: $T_1 = 350$ K, $p_1 = 10^3 \text{ N/m}^2$, and $T_2 = 450$ K, $p_2 = 5 \times 10^3 \text{ N/m}^2$, respectively. The partition is then removed, and the gases are allowed to mix while remaining thermally isolated from the outside.

(a) What are the final temperature T_f (in K) and final pressure p_f (in N/m^2) ?

(b) What is the net change in entropy due to the mixing of the gases ?

Physical constants

$c = 2.998 \times 10^{8}\ \text{m/s}$ (speed of light in vacuum)

$e = 1.601 \times 10^{-19}\ \text{C}$ (electron charge)

$h = 6.626 \times 10^{-34}\ \text{J s}$ (Planck's constant)

$\hbar = h/2\pi = 1.054 \times 10^{-34}\ \text{J s} = 6.582 \times 10^{-22}\ \text{MeV s}$

$1/4\pi\varepsilon_0 = 8.9875 \times 10^{9}\ \text{N m}^2/\text{C}^2$ (Coulomb's constant)

$\mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2$ (permeability of free space)

$m_e = 9.11 \times 10^{-31}\ \text{kg} = 0.511\ \text{MeV}/c^2$ (electron mass)

$m_p = 1.67 \times 10^{-27}\ \text{kg} = 938.3\ \text{MeV}/c^2$ (proton mass)

$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} = 931.5\ \text{MeV}/c^2$ (atomic mass unit = mass of ^{12}C atom/12)

$\text{G}_N = 6.674 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$ (gravitational constant)

$k = 1.381 \times 10^{-23}\ \text{J/K}$ (Boltzmann's constant)

$N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$ (Avogadro's number)

$R = 8.31\ \text{J/mol} \cdot \text{K}$ (molar gas constant)

Standard atmosphere $= 1.013 \times 10^{5}\ \text{N/m}^2$

Molar volume of ideal gas at STP $= 22.4 \times 10^{-3}\ \text{m}^3/\text{mol}$

Astronomical data

Earth mass $= 5.974 \times 10^{24}$ kg
Earth radius $= 6.378 \times 10^{6}$ m
Solar mass $= 1.990 \times 10^{30}$ kg
Solar radius $= 6.960 \times 10^{10}$ m
1 Astronomical Unit (AU) $= 1.496 \times 10^{11}$ m (Earth-Sun distance)