University of Minnesota
School of Physics and Astronomy

Spring 1998 GRADUATE WRITTEN EXAMINATION

Friday, March 27, 1998 Part I 9:00 A.M. - 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed book examination. You may use calculators.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit. Also write the relevant problem number of each such piece of paper. **BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER**, so that no sheet contains work for more than one problem, and use only one side of the paper. Your completed problems should be put into the manila envelope provided.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Permittivity constant	ε_o	8.85×10^{-12} F/m
Permeability constant	μ_o	1.26×10^{-6} H/m
Electron charge to mass ratio	e/m_e	1.76×10^{11} C/kg
Proton rest mass	m_p	1.67×10^{-27} kg
Ratio of proton mass to electron mass	m_p/m_e	1840
Neutron rest mass	m_n	1.68×10^{-27} kg
Muon rest mass	m_μ	1.88×10^{-28} kg
Planck constant	h	6.63×10^{-34} J·s
Electron Compton wavelength	λ_c	2.43×10^{-12} m
Molar gas constant	R	8.31 J/mol·K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2·K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Gravitational constant	G	6.67×10^{-11} m^3/s^2·kg
Bohr radius	a_o	5.29×10^{-11} m
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg

Short questions

1. An elevator ascends with a constant upward acceleration $a = 1.2$ m/s^2. At the instant t_o, when its upward speed is 2.4 m/s, a loose bolt drops from the ceiling of the elevator which is at a height $H = 2.8$ m above the floor. The bolt hits the floor of the elevator at time t_1. Find

   (a) the time of flight $\Delta t = t_1 - t_o$ of the bolt from ceiling to floor, and
   (b) the distance the bolt has fallen relative to the elevator shaft.

2. A time-independent magnetic field is given by $\mathbf{B} = 2xyb\,\mathbf{i} + y^2a\,\mathbf{j}$.

   (a) What is the relationship between the constants a and b?
   (b) Determine the steady current density $\mathbf{J}$ that gives rise to this field.

3. Compare the resolution of the Hubble telescope (diameter ≈ 2m) observing in visible light ($\lambda \approx 500$ nm) with a radio telescope/interferometer observing the hydrogen line ($\lambda = 21$ cm) with two dishes (antennae) on opposite sides of the earth (diameter ≈ 8,000 miles ≈ 13,000 km). Estimate the resolution of each in seconds of arc and give the ratio.

4. The ground state energy of a single, isolated particle in a cubical box (with impenetrable walls) is E_o. What will be the ground state energy of a system of 10 such particles if they are non-interacting and if

   (a) they have spin 0?
   (b) they have spin 1/2?

5. A fish tank containing water (mass M) is sliding down a frictionless inclined plane. What is the angle of the water relative to the horizontal plane? Assume a steady state with no sloshing.

6. A tennis ball of mass $m = 60$ g and radius $R = 3.1$ cm is immersed in water at a depth $L =$ 10 m. When released, it eventually surfaces and jumps out of the water, to a height $h = L/10 =$ 1.0 m. Find the amount of heat in Joules that is released in the process just described above. The density of water is 1 g/cm^3.

7. Find the heat capacity of a particle with three energy levels as a function of temperature T. Assume that the two highest energy levels are degenerate and are Δ higher than the ground level.

8. A particle of rest mass m is moving in a straight line when it disintegrates into a pair of photons which emerge moving in directions making equal angles of 60° on opposite sides of this line.

   (a) What was the initial particle's speed, in units of c?
   (b) Calculate the energies ε_1 and ε_2 of the final state photons, in units of mc^2.

9. Two arbitrary shaped objects 1 and 2 with net charges $+Q_o$ and $-Q_o$, respectively, are surrounded by a uniform ohmic medium with constant conductivity σ.

(a) Derive the expression for the total curent to object 1 at time $t = 0$.
(b) Find an expression for the charge on the object 1 as a function of time.

10. A 100-W light bulb is placed in a refrigerator whose compressor consumes 10 W. If the refrigerator is operating at 100% of theoretical efficiency and the insulation is perfect, what is the temperature inside the refrigerator? The ambient temperature is 300 K.

11. A spacecraft is travelling at a speed of $0.9c$ from the earth towards the star Vega (in the $+x$ direction) as seen by an observer at rest relative to the earth. A small payload is ejected from the rocket towards Vega with velocity of $0.8c$ (as seen by observers in the spacecraft).

(a) Derive from the Lorentz transformations, the x component of the velocity of the payload as seen by observers on the earth.

(b) Suppose an identical payload is ejected transverse to the direction of travel of the rocket (in the y direction) with the same $0.8c$ velocity relative to the spacecraft. What is the y component of the velocity as observed in the earth's fixed frame?

12. A particle of mass m bound by a one-dimensional harmonic oscillator potential, $V = (1/2)m\omega^2x^2$, is in thermal contact with a heat bath at temperature T. What is the thermally averaged mean square displacement $\langle x^2 \rangle_T$ in the two extreme cases:

(a) $\hbar\omega \gg kT$,

(b) $\hbar\omega \ll kT$.

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Spring 1998 GRADUATE WRITTEN EXAMINATION

Saturday, March 28, 1998 Part II 9:00 A.M. - 1:00 P.M.

Part II of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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Long problems

1. How long does it take for a bungee jumper to momentarily come to rest for the first time at the lowest point of his ride given that the unstretched length of the **massless** bungee cord is 10 m, its spring constant is 60 N/m and the mass of the jumper is 60 kg. One end of the cord is attached to the jumper and the other, to the platform from which the jumper leaps. Assume that the jumper steps off the platform with negligible initial velocity.

2. A particle is in a 1-dimensional infinite square well of width L and its wave function is given by $\psi(x) = c(x^2 - Lx)$

   (a) Are the boundary conditions satisfied by this function? What is (are) the appropriate condition(s)?

   (b) Find c.

   (c) Find $\langle x \rangle$.

   (d) Find $\langle E \rangle$.

3. One mole of a monoatomic ideal gas undergoes a process for which the relation between pressure and volume is $p = p_o + \alpha/V$, where both p_o and α are positive. The gas expands from an initial volume V_1 to a final volume V_2. Find

   (a) the change in the internal energy of the gas,
   (b) the work done by the gas on its surroundings, and
   (c) the amount of heat transferred to the gas by its surroundings.

4. (a) Show that the density of states dN/dE at the Fermi surface of a degenerate gas of free electrons as a function of N (number of electrons) and the Fermi energy, E_F, is given by $dN/dE = 3N/\,2E_F$.
   (b) Use this result to estimate the spacing of unoccupied energy levels above the Fermi surface, in electron volts, for a 1 cm^3 sample of lithium (density = 0.53 g/cm^3).

5. Show that the magnetic field near the center of a set of Helmholtz coils, *i.e.* two coils of radius R, separated by a distance of R, is uniform to second order in the displacements from the midpoint along the line connecting the centers of the coils. The currents in the two coils flow in the same direction.

6. Consider the transmission of a beam of particles of mass m and momentum $p = \hbar k$, in one dimension, incident on a rectangular potential barrier of height V_o and extending from $x = 0$ to $x = L$, in the special case that the energy E of the incident particles <u>is exactly equal</u> to the barrier height V_o.

   (a) Calculate the transmission and reflection coefficients T and R.
   (b) Check some properties of your answers in part (a): is probability conserved ? Do T and R have the expected limiting values for L very large or very small ?
   (c) For what value of the de Broglie wavelength of the particles is the transmitted fraction equal to 1/2?