University of Minnesota
School of Physics and Astronomy

Spring 1999 GRADUATE WRITTEN EXAMINATION

Friday, March 26, 1999 Part I 9:00 A.M. - 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed book examination. You may use calculators. A list of some of the physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit. Also write the relevant problem number of each such piece of paper. **BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER**, so that no sheet contains work for more than one problem, and use only one side of the paper. Your completed problems should be put into the manila envelope provided.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Permittivity constant	ε_o	8.85×10^{-12} F/m
Permeability constant	μ_o	1.26×10^{-6} H/m
Electron charge to mass ratio	e/m_e	1.76×10^{11} C/kg
Proton rest mass	m_p	1.67×10^{-27} kg
Ratio of proton mass to electron mass	m_p/m_e	1840
Neutron rest mass	m_n	1.68×10^{-27} kg
Muon rest mass	m_μ	1.88×10^{-28} kg
Planck constant	h	6.63×10^{-34} J•s
Electron Compton wavelength	λ_c	2.43×10^{-12} m
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2•K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Bohr radius	a_o	5.29×10^{-11} m
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg

Short Questions

1. A radioactive nucleus in an excited state decays (at rest) with a lifetime $\tau = 10^{-7}$ s to its ground state through the emission of a gamma ray of energy E = 15 keV.

   (a) What is the wavelength of the photon emitted in this decay (in nm) ?
   (b) What is the natural line width of the excited level (in eV) ?
   (c) What is the length of the photon wave train (in meters) ?

2. An iron sphere two meters in diameter is isolated in the center of a large room and is charged to a potential V of 100,000 Volts. How much heat (in Joules) will be liberated if the sphere is suddenly grounded ?

3. You are walking on the ice, in the middle of a large Minnesota lake in winter, when you come across a hole cut straight through the ice, probably by an ice fisherman. Looking down the hole, you notice that the water level is 6 cm below the surface of the ice. What is the thickness of the ice covering the lake ? Density of ice = 0.9 g/cm^3.

4. A racing car with rear-wheel drive has its center-of-mass at a height H above the ground and at a distance D in front of its rear axle. Its wheel base (distance between front and rear axles) is L. What minimum forward acceleration is required for our racing car's front wheels to lift off the ground ? What minimum coefficient of friction between the rear tires and the ground is required to achieve this ?

5. The magnetic moment per atom in a solid has magnitude $\mu = 10^{-23}$ J/T . What magnetic field, in tesla, must be applied at T = 77 K if twice as many atoms are to have their magnetic moments aligned parallel to the field as there are antiparallel ?

6. The electron in a hydrogen atom is in a state described by the following superposition of normalized energy eigenstates u,

   $$\psi(r,\theta,\phi) = [\, 3\, u_{100} + A\, u_{211} - 2\, u_{21\text{-}1} + 3\, u_{321}\,]/5$$

   where the subscripts represent the quantum numbers $\{n, \ell, m_\ell\}$.

   (a) Calculate A > 0 such that the wave-function is normalized.
   (b) Find the expectation value of the energy in this state, in terms of the ground state energy of hydrogen E_1.
   (c) Find the expectation values of L^2 and of L_z in this state.

7. The radius of the planet Mars is 3.4×10^6 m, its surface gravity is 3.7 m/s^2, and it is about 1.52 times farther from the Sun than the Earth is. Find

   (a) the mass of the planet Mars, in kg.
   (b) the length of the Martian "year", in Earth years.

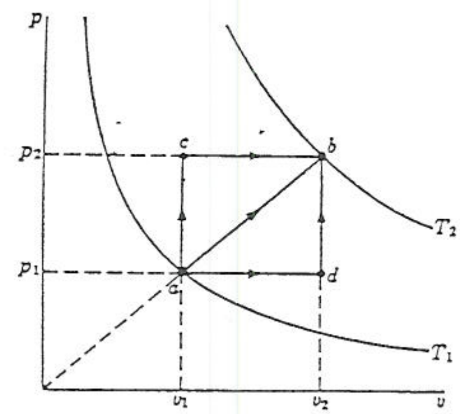
8. The electric field of a plane electromagnetic wave propagating in free space is given by $\mathbf{E} = (\mathbf{i}+\mathbf{j})E_o \sin(kz+\omega t)$ with $E_o = 10^5$ V/m. What then is the associated magnetic field of this wave (in Tesla), written in a similar form ?

9. A particle of mass m is given a kinetic energy equal to n times its rest energy. What are its speed and momentum ?

10. A small ball of mass m hangs from a string of length L. The ball and string are brought to a horizontal position and then released. At what point will the net acceleration of the ball be directed horizontally ? State your answer in terms of the angle the string then makes with the vertical.

11. A tunnel leading straight through a hill is found to greatly amplify pure tones at 135 Hz and 138 Hz. Find the minimum length of the tunnel, in meters. The speed of sound is $c_s = 343$ m/s.

12. An ideal gas for which $c_v = 5\,R/2$ is taken from point "a" to point "b" in the p-V plane along the two paths acb and ab shown in the Figure, in which $p_2 = 2\,p_1$ and $V_2 = 2\,V_1$ (ignore path adb). Find the heat Q supplied to the gas, per mole, in each process: Express your answer in terms of R and T_1 only.



University of Minnesota
School of Physics and Astronomy

Spring 1999 GRADUATE WRITTEN EXAMINATION

Saturday, March 27, 1999 Part II 9:00 A.M. - 1:00 P.M.

Part II of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

This is a closed book examination. You may use calculators. A list of some of the physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

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Long Problems

1. A charge $+Q$ is distributed uniformly on a circle of radius R, in the x-y plane, and with its center at the origin. You need to calculate the frequency ω of small oscillations for a particle of mass m and charge $+q$, constrained to the x-y plane, about the origin.

   (a) First, find the electric field vector (due to the ring of charge $+Q$) on the z-axis as a function of z.
   (b) Using your result from part (a), along with Gauss's law in differential form, determine the electric field vector in the x-y plane near the origin.
   (c) Now calculate the oscillation frequency ω as specified in the question.

2. Two monatomic ideal gases, each occupying a volume $V = 1.0\ m^3$, are separated by a removable insulating partition. They have different temperatures $T_1 = 350$ K and $T_2 = 450$ K, and different pressures $p_1 = 10^3\ N/m^2$ and $p_2 = 5 \times 10^3\ N/m^2$. The partition is then removed, and the gases are allowed to mix while remaining thermally isolated from the outside.

   (a) What are the final temperature T_f (in K) and pressure p_f (in N/m^2) ?
   (b) What is the net change in entropy due to mixing (in J/K) ?

3. A particle of mass m is confined by infinitely high potential walls to a rectangular region of the x-y plane bounded by $0 < x < L$ and $0 < y < 2L$.

   (a) Solve this quantum mechanical problem, providing a formula for the allowed energy levels and the corresponding normalized wave-functions.
   (b) List the five lowest levels, with their energies (in units of the ground state energy) and corresponding degeneracies.

4. A spaceship travels at a constant velocity $v = 0.8\ c$ with respect to Earth. Denote spaceship-frame coordinates by a prime ($'$). At $t = t' = 0$ by Earth and spaceship clocks, respectively, a light signal is sent from the tail (back end) of the spaceship towards the nose (front end) of the spaceship. The length of the spaceship, as measured in a frame in which it is at rest, is L.

   (a) At what time, by spaceship clocks, does the light signal reach the nose of the spaceship ?
   (b) At what time, by Earth clocks, does the light signal reach the nose of the spaceship ?

   Now suppose that there is a mirror at the nose of the spaceship which immediately reflects the light signal back to the tail of the spaceship.

   (c) At what time, by spaceship clocks, does the light signal finally return to the tail of the spaceship ?
   (d) At what time, by Earth clocks, does the light signal finally return to the tail of the spaceship ?

   All answers should be expressed in terms of L and c.

5. A proton of kinetic energy 1.0 keV is scattered elastically through a 90^o angle by a helium nucleus initially at rest. What are:

(a) the de Broglie wavelength of the scattered proton, in nm ?
(b) the kinetic energy (in eV) and scattering angle (measured from the direction of motion of the incoming proton) of the recoiling helium nucleus ?

6. Two monochromatic, coherent point light sources of the same wavelength λ, are located on a line perpendicular to a distant screen. The closest source, S_1, is at a distance D from the screen, while the more distant one, S_2, is a distance d farther away as shown in the diagram: the screen is far away in the sense that both D and d are much larger than the wavelength of the light sources. *Assuming that there is an interference maximum at the point* A where the line drawn through the sources intersects the screen, find a formula for the radius r of the first bright ring around A on the screen, and get a numerical value assuming $\lambda = 600$ nm, D = 3.0 m and d = 1.0 m. Recall that the expansion $(1+x)^{1/2} = 1 + x/2 + O(x^2)$ can be used when $x \ll 1$.