University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Spring 2018 – PART I

Thursday, January 11th, 2018 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Sun's Mass	M_S	1.99×10^{30} kg
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

The electron in a hydrogen atom is in a state described by the following superposition of normalized energy eignestates u, with real $A > 0$,

$$\psi(r,\theta,\phi) = \frac{1}{5}(3u_{100} + Au_{211} - 2u_{21-1} + 3u_{321})$$

where the subscripts represent the quantum numbers $\{n, l, m_l\}$.

a) Calculate A such that this wavefunction is normalized.

b) Find the expectation value of the energy in this state, in terms of the ground state energy of hydrogen E_1.

c) Find the expectation values of L^2 and L_z in this state.

Problem 2

An electron is confined to the interior of a hollow spherical cavity of radius R with impenetrable walls. Find an expression for the pressure exerted on the walls of the cavity by the electron in its ground state, recalling that the Laplacian in spherical polar coordinates is given by

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + (\textit{angular part})$$

Problem 3

Consider a "ballistic pendulum" made up of a heavy uniform rod 1 meter long suspended vertically from one end. A bullet traveling horizontally hits this rod at a distance of 75cm from the pivot point and becomes embedded in the rod, causing the rod to deflect by an angle of 30°. Assuming that the mass of the bullet is 1% of the mass of the rod, find the speed of the bullet. (Hint: you can neglect the mass of the bullet compared to the rod when determining the moment of inertia).

Problem 4

Find the value of c for which the force given below is conservative. Determine the potential energy function for this force.

$$\mathbf{F} = \frac{z}{y}\mathbf{i} + \frac{cxz}{2y^2}\mathbf{j} + \frac{x}{y}\mathbf{k}$$

Problem 5

Consider a spherical shell of radius R that has an imposed potential given by

$$V(\theta) = V_0 cos^2\theta$$

a) Determine the potential inside and outside of the sphere in terms of a Legendre polynomial expansion.

b) Calculate the radial electric field inside and outside, and find the surface charge density.

c) What is the total charge on the spherical shell?

Problem 6

Consider a co-axial cable made up of an inner conductor of radius a and an outer conductor of radius b. The inner conductor is solid and contains a constant current density j. The outer conductor is a thin shell and contains a surface current that exactly balances the current of the inner conductor.

a) Determine the surface current density on the outer conductor.

b) Determine the magnetic field everywhere in space and calculate the total magnetic energy per unit length.

c) Find the inductance per unit length of this cable.

Problem 7

A system with an equidistant energy spectrum ($\epsilon_n = n \cdot \Delta, n = 0, 1, 2, \ldots$) is populated with identical bosons of spin $s = 0$. If $N \gg 1$ is the number of bosons occupying the lowest two orbitals and the occupation of the ground state is twice the occupation of the lowest excited state, find the:

a) Temperature

b) Chemical potential

c) Occupancy of the second excited orbital ($n = 2$)

Problem 8

Two identical classical monatonic ideal gases with the same temperature τ and the same number of atoms N are contained in two vessels of volumes V_1 and V_2 which are then connected. The combined system is isolated from the environment. After the system has reached equilibrium, what is the:

a) Total energy

b) Pressure

c) Change of entropy

d) Change of temperature

Problem 9

A particle of mass m is given a kinetic energy equal to n times its rest energy. What is its speed and momentum?

Problem 10

A typical neutron star has approximately the same mass as the sun but is as dense as a proton. Estimate the radius of a neutron star and the gravitational binding energy released in its formation.

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GRADUATE WRITTEN EXAMINATION

Spring 2018 – PART II

Friday, January 12th, 2018 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

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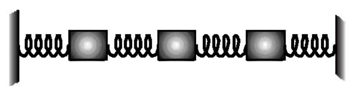
Problem 1

Consider the infinite square well in one dimension, extending from $-a/2$ to $+a/2$. A particle of mass m is sitting in its ground state. At time $t = 0$ the size of the well doubles, so that instead of going from $-a/2$ to $a/2$, it goes from $-a$ to a. If one then measures the energy, what is the most probable result, and what is the probability of getting it?

Problem 2

Consider a system as shown in the figure with three equal masses m connected with identical springs of spring constant k, with the whole system attached to two immovable walls. The masses are constrained to move only in the horizontal direction.

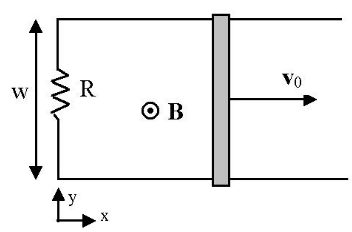


a) Write down the equations of motion for each mass and find the normal mode frequencies (Hint: this gives a cubic equation, but you should be able to find an obvious common factor).

b) Determine the eigenvectors (i.e., the ratio between the amplitudes of the three mases in each mode). Assume that the first mass has an amplitude of 1 for each mode. Sketch the motion in each mode.

Problem 3

Consider a "sliding bar" generator made up of a U-shaped wire with width w and having a resistance R with the current closed by a movable bar with initial velocity $\mathbf{v}_0$ as in the figure. A magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ comes out of the page.



a) Find the emf generated and the current that flows through the circuit. Draw a diagram showing the direction of the current flow.

b) Determine the magnetic force on the bar, and solve for its motion, assuming the bar has mass M.

c) Show that the energy dissipated by the resistor is equal to the kinetic energy lost by the bar.

Problem 4

The energy of a photon gas contained to a volume V and in thermal equilibrium with a reservoir at temperature τ is given by a Stefan-Boltzmann law, $U = \alpha V \tau^4$.

a) Find the heat capacity C_V and the entropy σ.

b) Find the free energy.

c) Derive the "photon gas law", i.e., find $p(V, \tau)$.

d) Calculate the work performed by the photon gas in isothermal expansion from $V_1 = V$ to $V_2 = 2V$.

e) Calculate the heat transferred to the photon gas when it expands at constant pressure from $V_1 = V$ to $V_2 = 2V$.

Problem 5

A radioactive isotope of bismuth, ${}^{210}_{83}Bi$ undergoes beta-decay into polonium with a mean lifetime τ_1 of 7.2 days. In turn, the polonium alpha-decays into lead with a mean lifetime τ_2 of 200 days. Denote the number of bismuth and polonium nuclei at time t respectively by $N_1(t)$ and $N_2(t)$.

a) Write out the nuclear reactions corresponding to both decays, carefully accounting for the atomic and mass numbers Z and A of the nuclei involved.

b) The number of (parent) bismuth nuclei evolves with time according to the differential equation

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

where $\lambda_1 = 1/\tau_1$ is the decay constant of bismuth. Write down the corresponding equation for the number of polonium nuclei $N_2(t)$ produced by bismuth decay, with λ_2 as the polonium decay constant.

c) Solve these equations for $N_2(t)$, given the initial conditions $N_1(0) =$ N and $N_2(0)$=0. Hint: First solve for $N_1(t)$ and use this result together with a simple integrating factor in the equation for $\frac{dN_2}{dt}$.

d) Since we start with no polonium at $t = 0$, and that after a long enough time all of the polonium produced will have decayed, there will be a time t^* at which the number of polonium nuclei and correspondingly, the rate of α-particle emission will reach a maximum. What is that time, in days?