Searching for Stochastic Gravitational Wave Background with LIGO

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LIGO-G060024-00-D

Outline

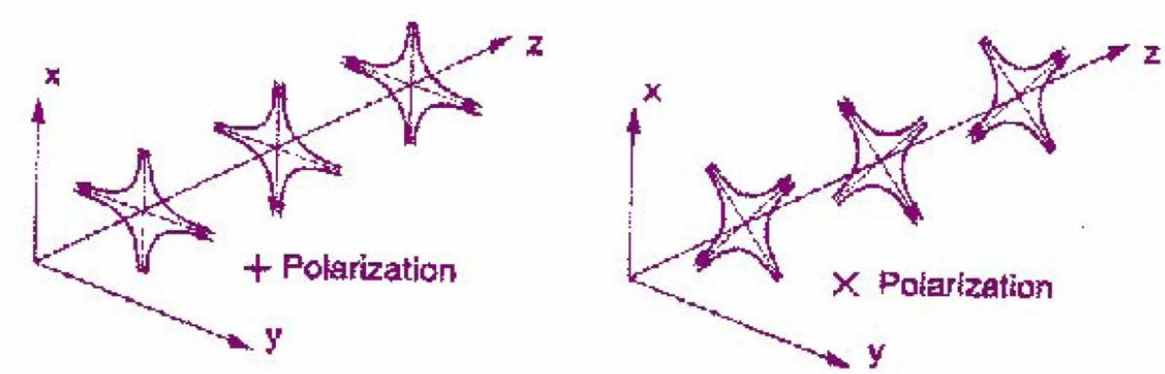
- LIGO Experiment:
  - » Overview
  - » Status
  - » Future upgrades
- Stochastic background of gravitational waves:
  - » Search method
  - » Recent results
  - » Outlook for the future
- Implications of the stochastic search:
  - » Beginning to constrain theoretical models

Gravitational Waves

- Newtonian gravity: instantaneous action at a distance.
- General Relativity: the "signal" travels at the speed of light.
- Weak field limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Einstein's field equations reduce to the wave equation:

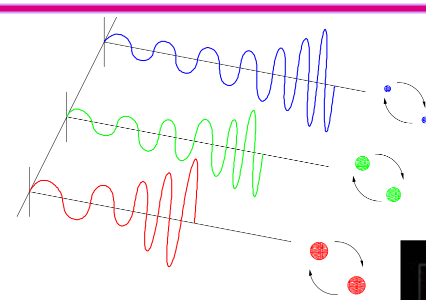
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{\mu\nu} = 0$$

- Two polarizations: $h = ah_+ + bh_\times$ $a,b \sim f(\omega t - \mathbf{k} \cdot \mathbf{x})$

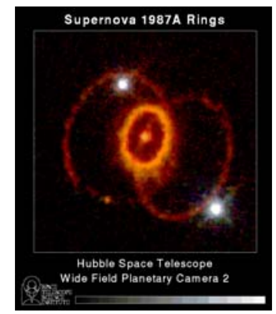


Sources of Gravitational Waves

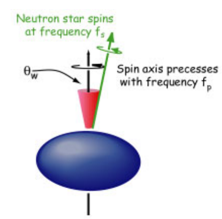
- Compact binary inspiral: *"chirps"*
  - » NS-NS waveforms are well described
  - » BH-BH need better waveforms
  - » search technique: matched templates



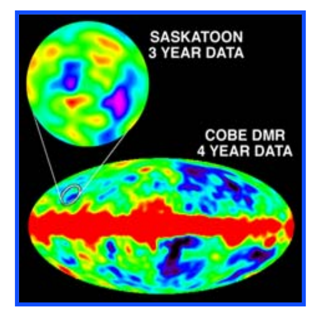
- Supernovae / GRBs: *"bursts"*
  - » burst signals in coincidence with signals in electromagnetic radiation



- Pulsars in our galaxy: *"periodic"*
  - » search for observed neutron stars
  - » all sky search (computing challenge)
  - » r-modes

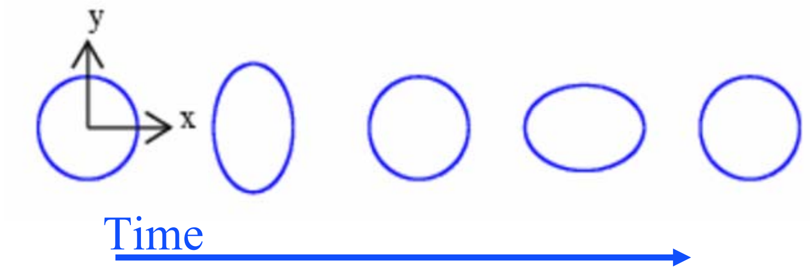


- Incoherent superposition of many sources
  - » Cosmological or astrophysical *"stochastic background"*
- Unexpected?



Interferometers as Gravitational Wave Detectors

- Gravitational wave effectively stretches one arm while compressing the other.



Interferometers as Gravitational Wave Detectors

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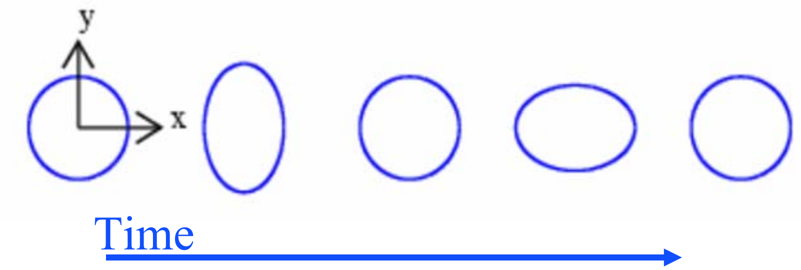
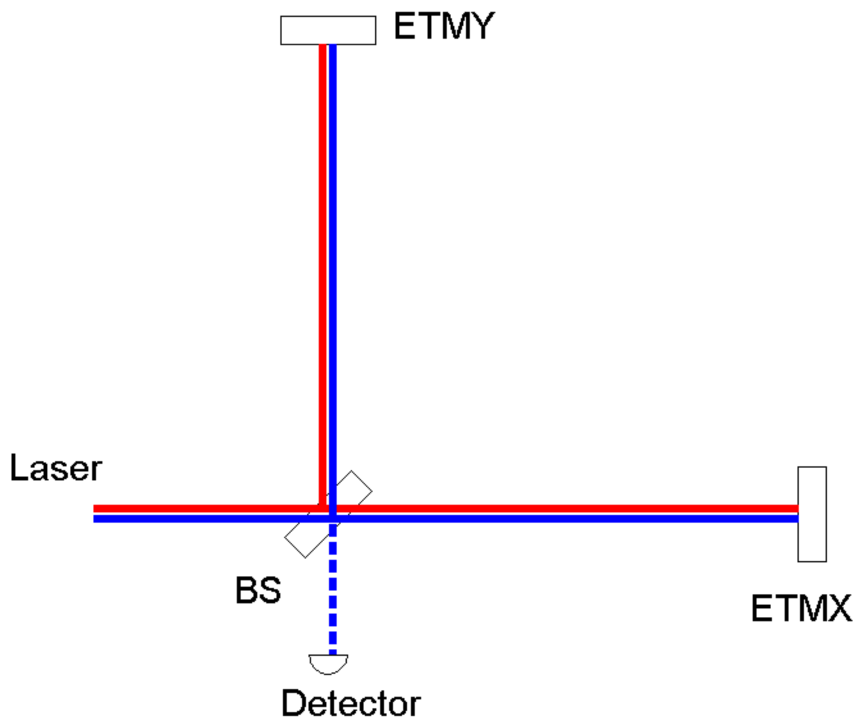
- Interferometer measures the arm-length difference.
  - » Suspended mirrors act as "freely-falling".
  - » Dark fringe at the detector.





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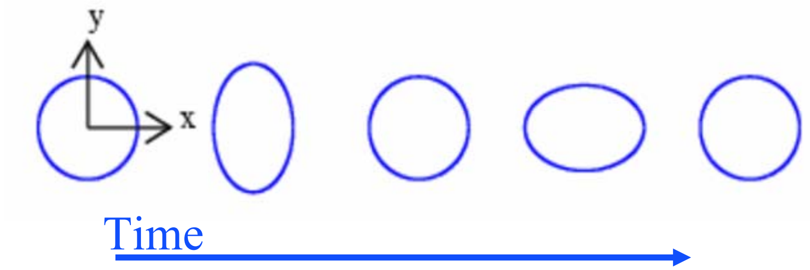
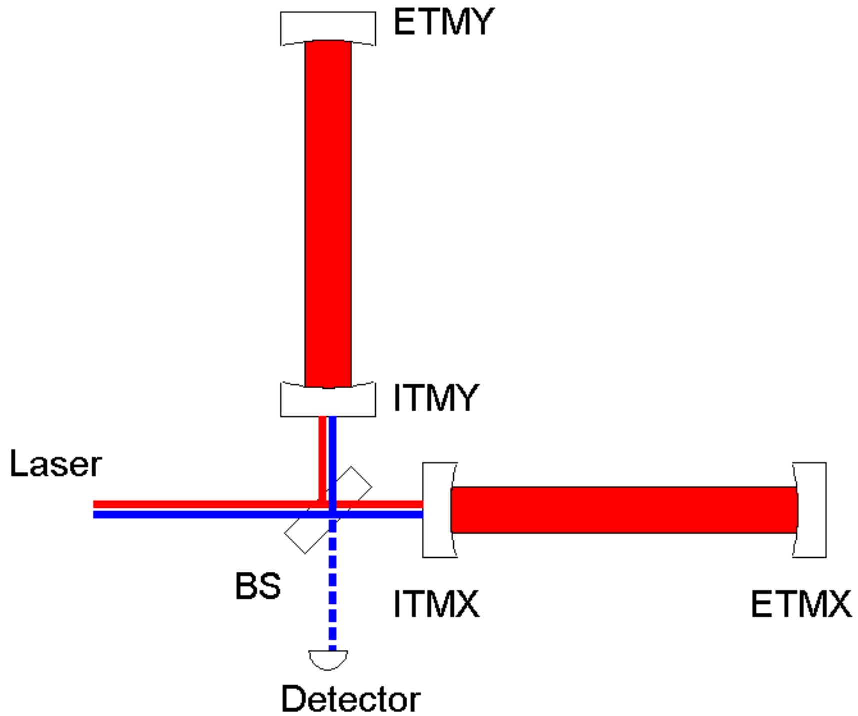
  - » Effectively increase arm length ~100 times.





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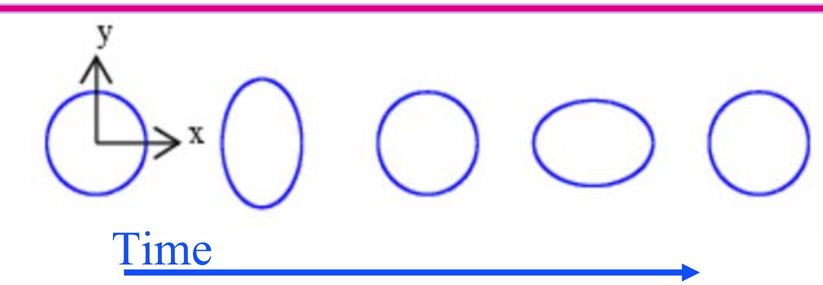
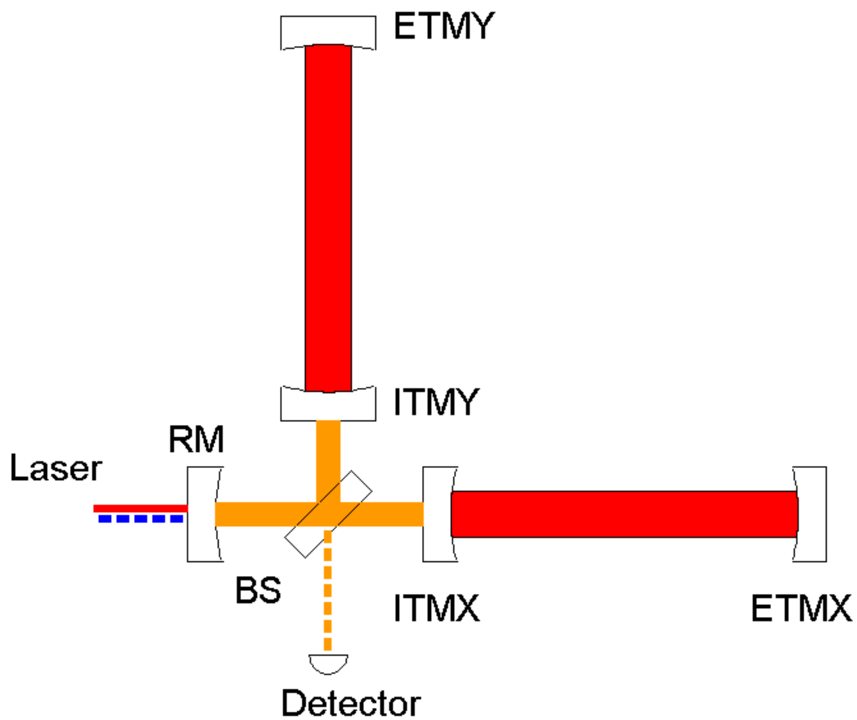
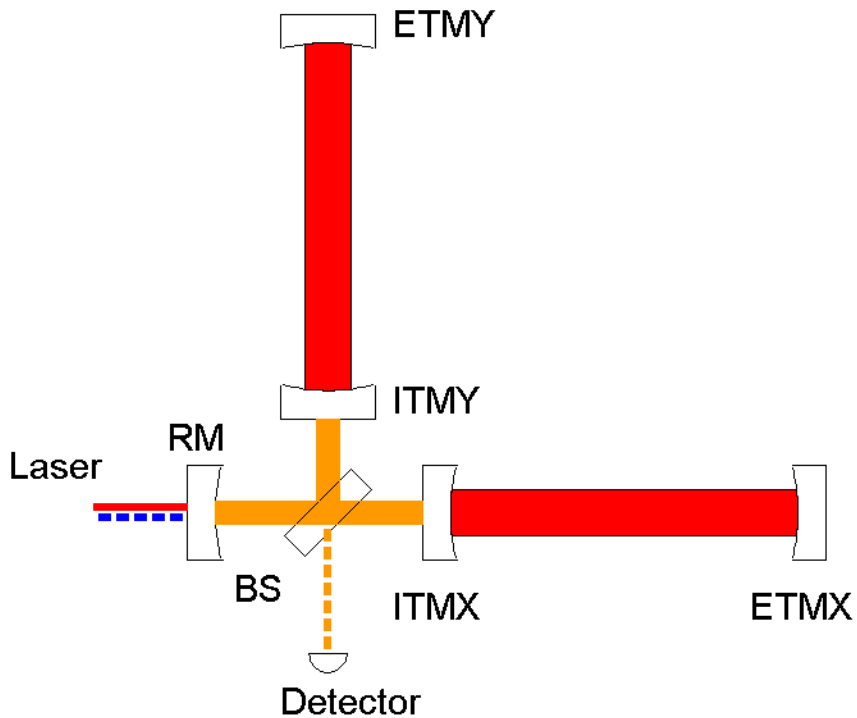
- Fabry-Perot cavities in the arms
  - » Effectively increase arm length ~100 times.
- Power-recycling mirror
  - » Another factor of ~40 in power.





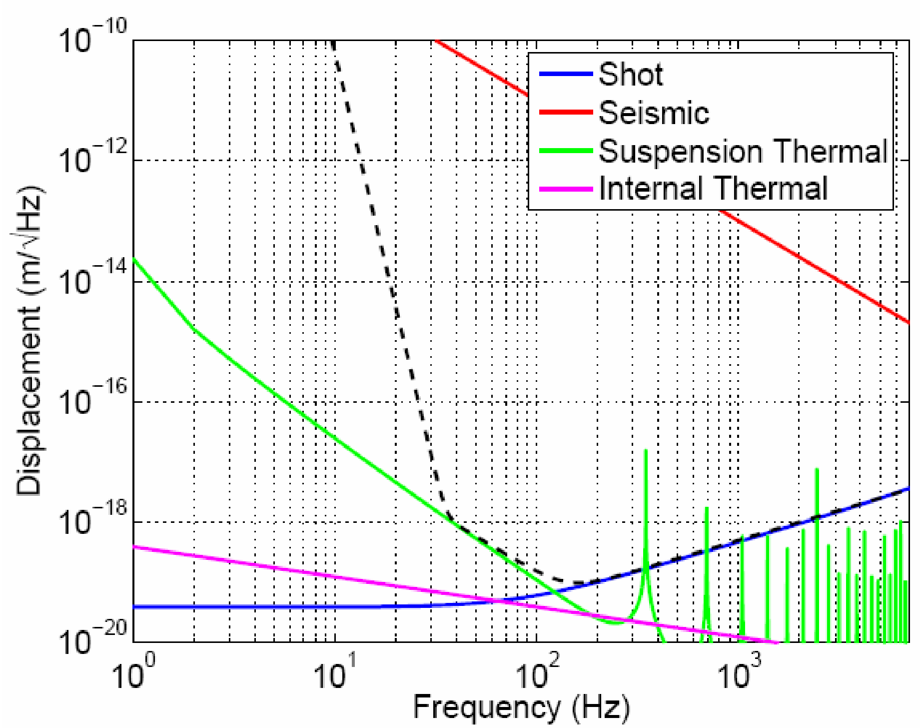
LIGO Sensitivity

- Rough sensitivity estimate
  - » Input laser power: ~5 Watt
- Sensitivity (ΔL) ~ λ (~ 10^{-6} m)
  / Number of Bounces in Arm (~100)
  / Sqrt(Number of Photons (~10^{21}))
  ~ 3 × 10^{-19} m
- Strain Sensitivity:
  - » h = $\Delta L / L \sim 10^{-22}$
  - » L = 4 km



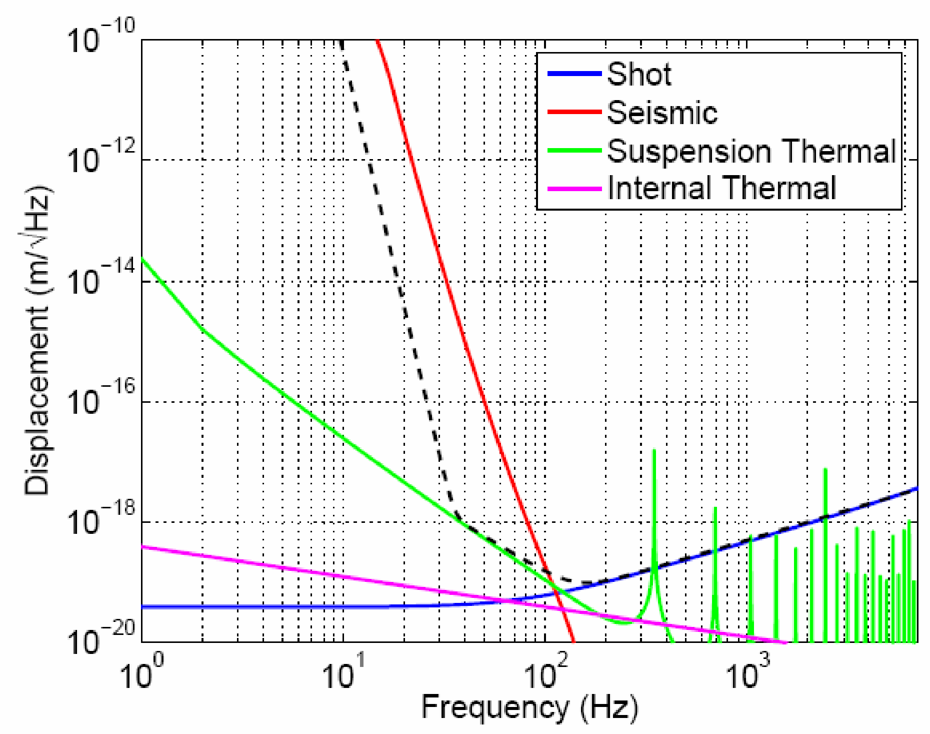
LIGO Sensitivity

- Seismic Noise



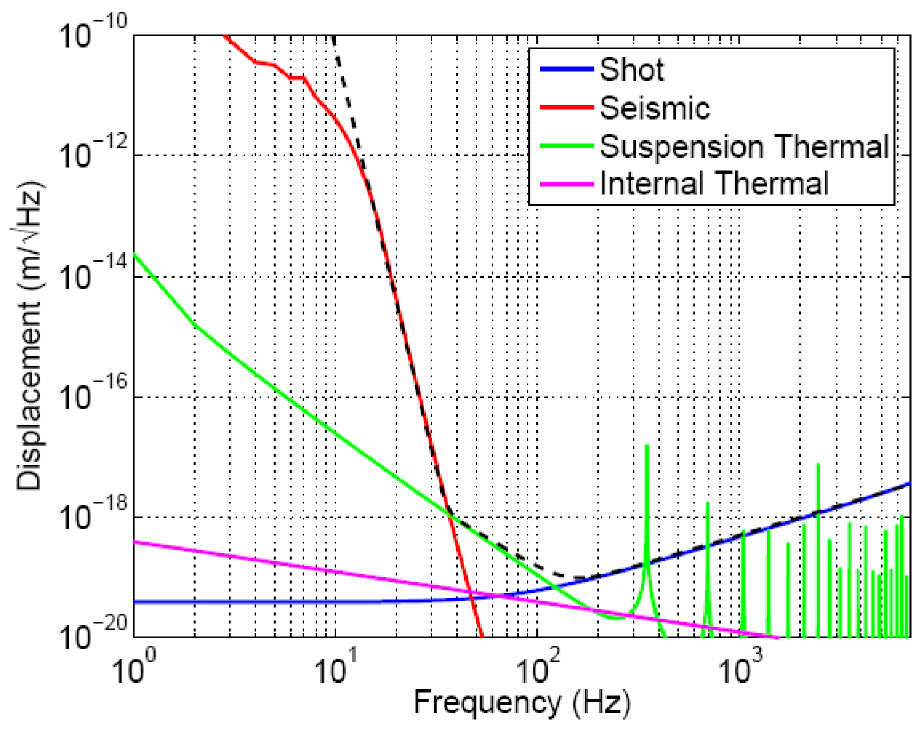
LIGO Sensitivity

- Seismic Noise
  - » Active and passive isolation



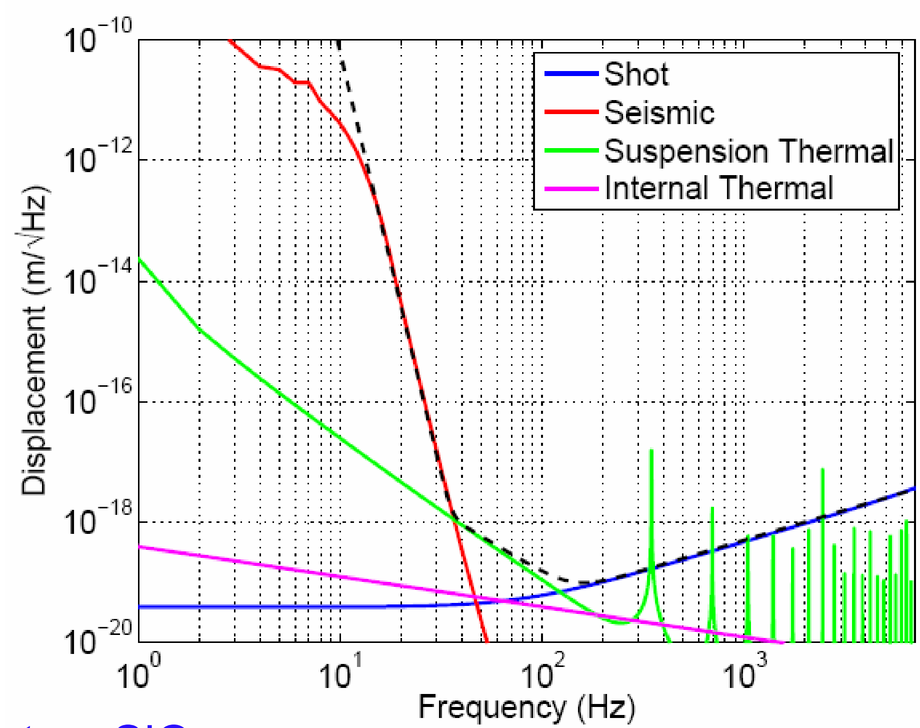
LIGO Sensitivity

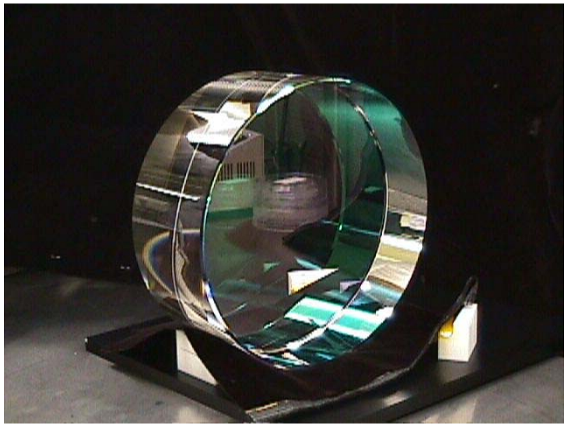
- Seismic Noise
  - » Active and passive isolation
  - » Suspensions
  - » Effective "Seismic Wall" at 40 Hz



LIGO Sensitivity

- Seismic Noise (<40 Hz)
  - » Active and passive isolation
  - » Suspensions
  - » Effective “Seismic Wall” at 40 Hz
- Thermal Noise (40-150 Hz)
  - » Suspension wires
  - » Internal mirror modes
- Shot noise (>150 Hz)

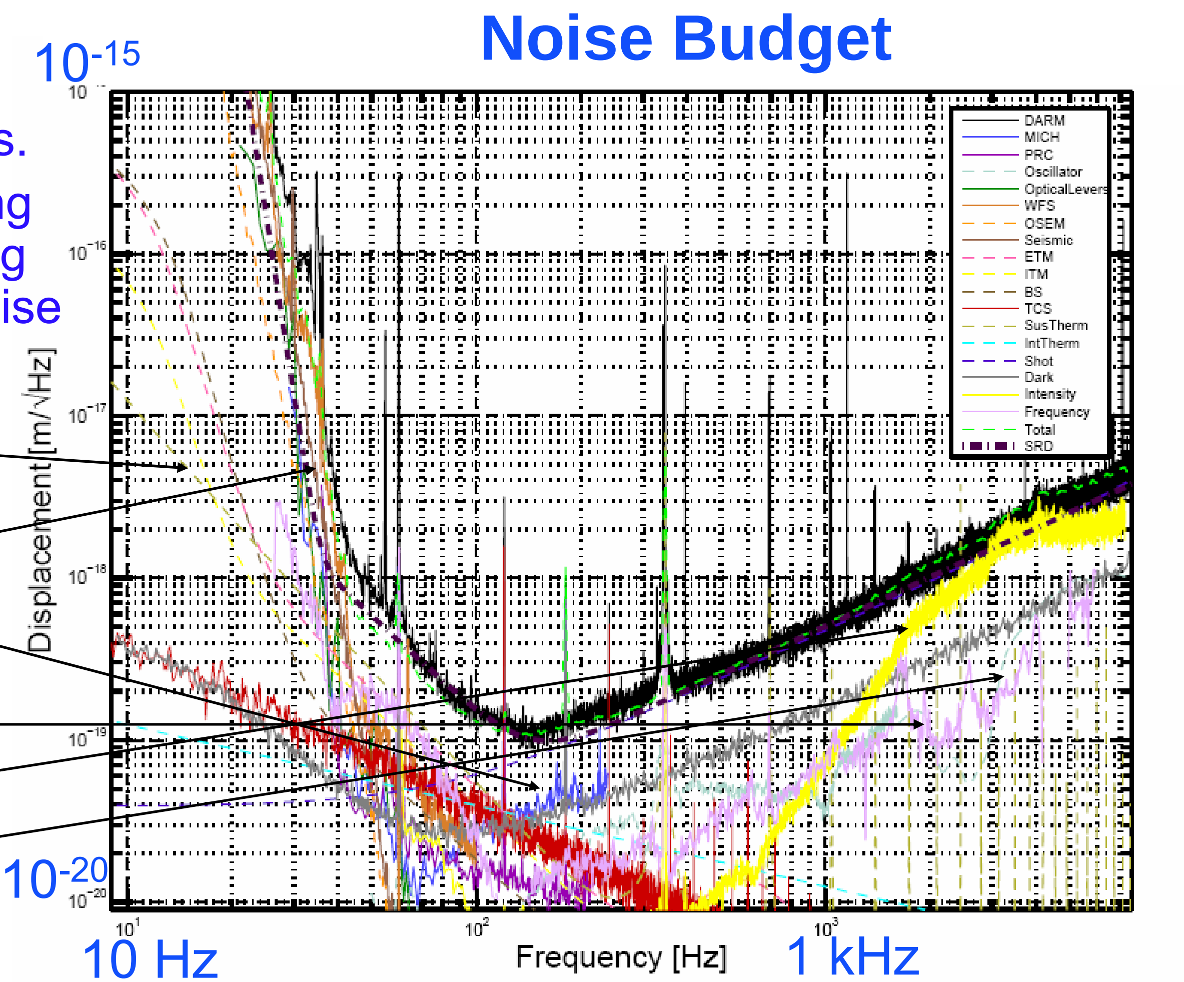




- Substrates: SiO_2
  - » 25 cm Diameter, 10 cm thick
  - » Internal mode Q’s > 2×10^6
- Polishing
  - » Surface uniformity < 1 nm rms (λ / 1000)

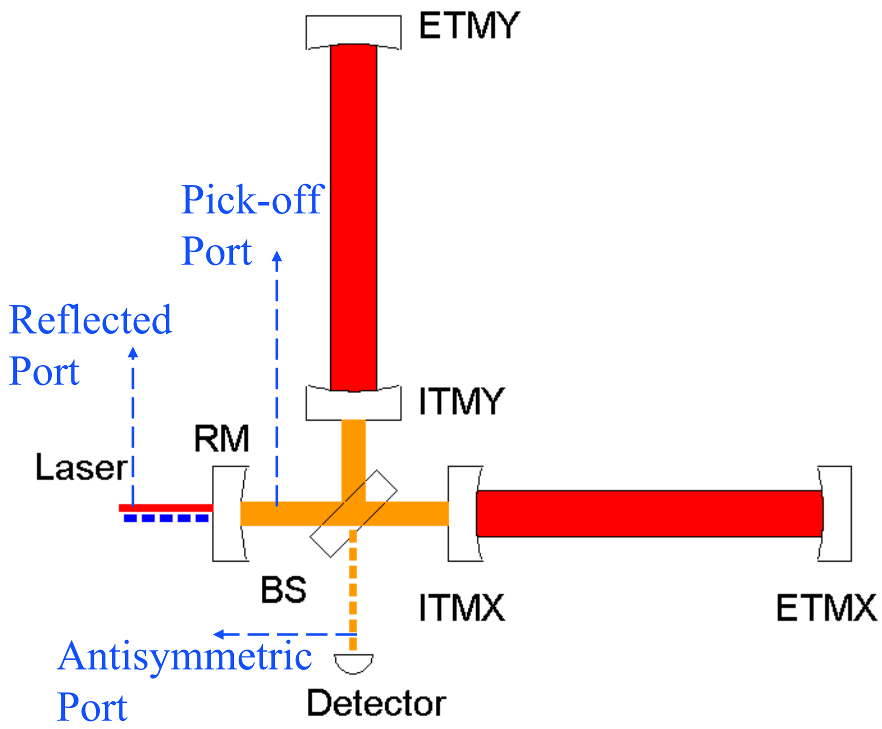
Technical Noise Sources

- In practice, there are many other, technical, noise sources.
  - » Much of the commissioning effort devoted to identifying and suppressing these noise sources.
- Electronics, 60 Hz harmonics etc.
- Angle-to-length couplings
- Auxiliary length degrees of freedom
- Laser frequency noise
- Laser intensity noise
- Oscillator phase noise

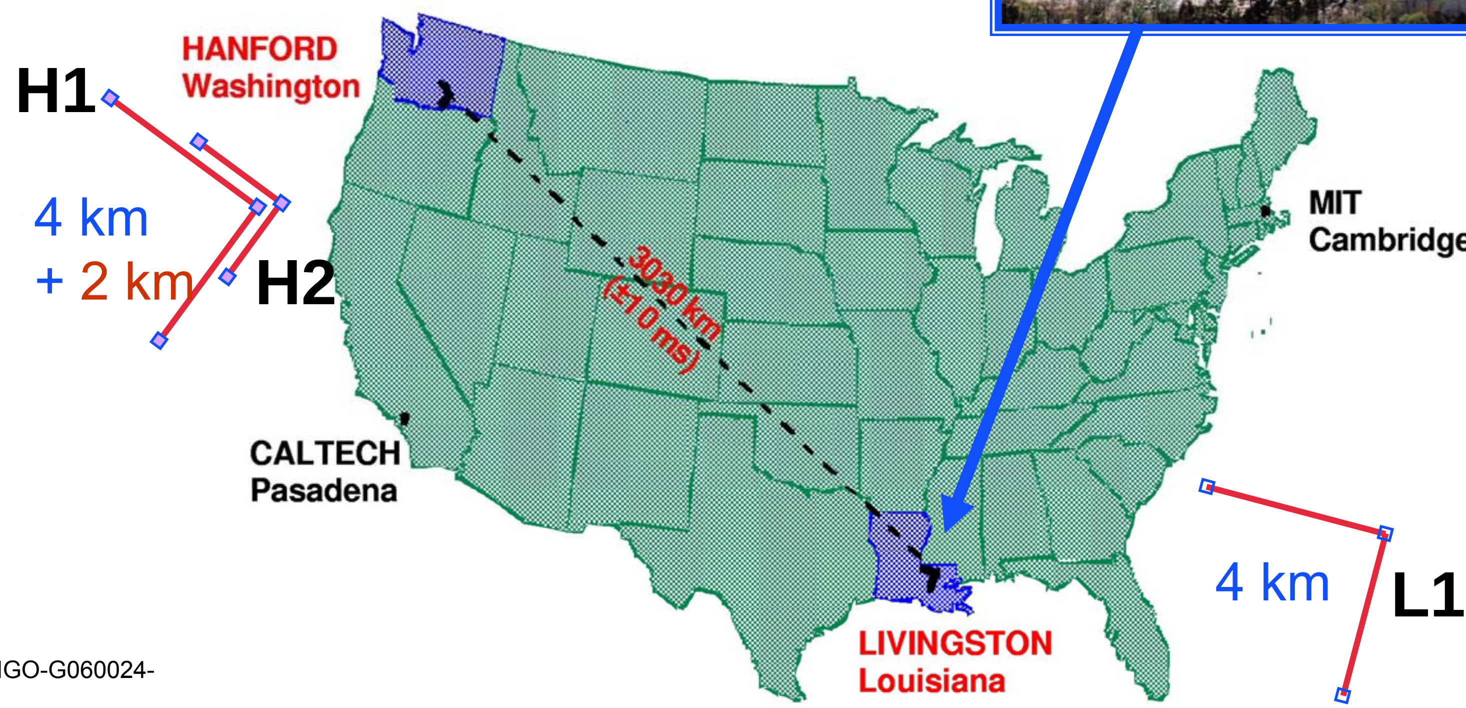


Interferometer Controls

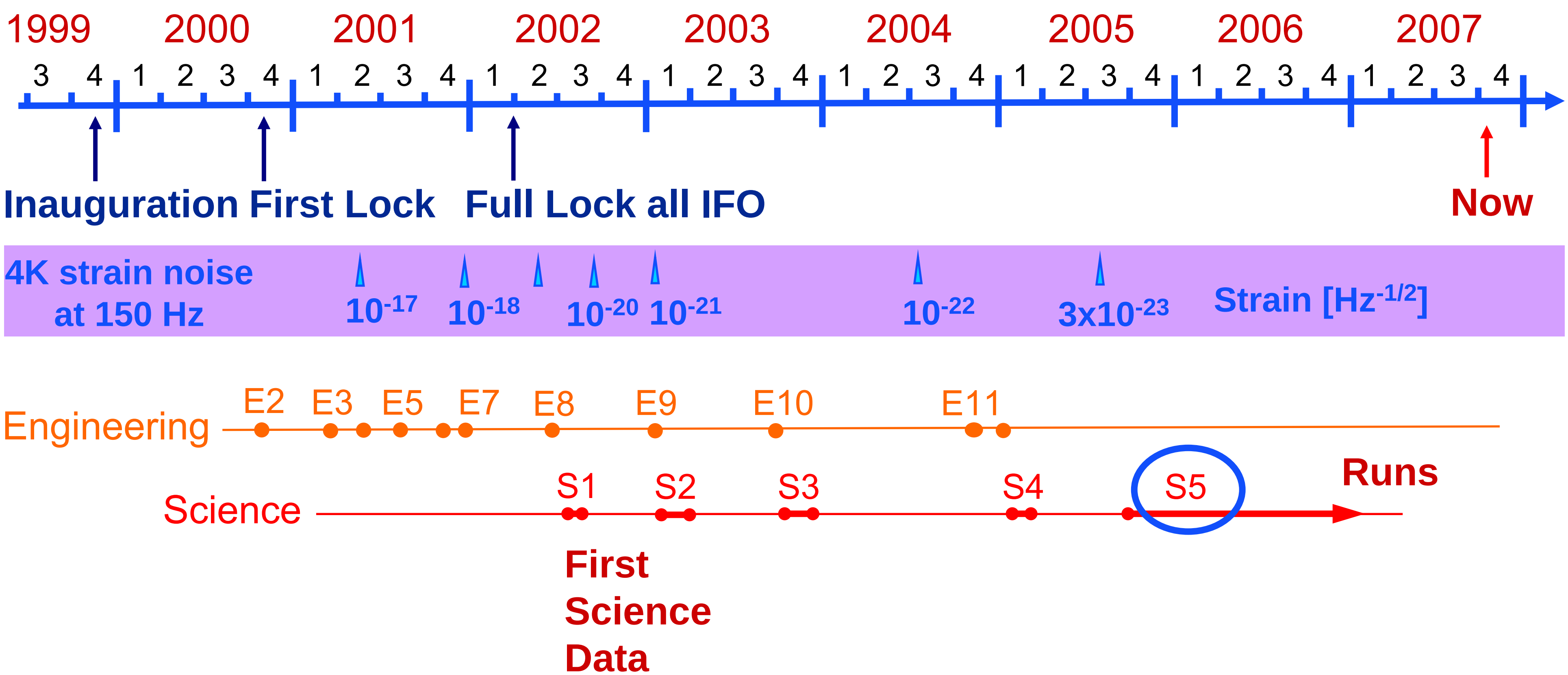
- 6 mirrors: many degrees of freedom
  - » Length
  - » Angular
- Input field is phase modulated:
  - » $E_{in} = E_0 \, e^{i*\Gamma*\cos(\omega t)}$
  - » Carrier + sidebands at ±25 MHz.
  - » Carrier resonates in arms, sidebands do not!
- Sample the beam at several locations, measure with photodiodes.
  - » Output voltage is demodulated.
  - » Pound-Drever-Hall locking.



LIGO Observatories

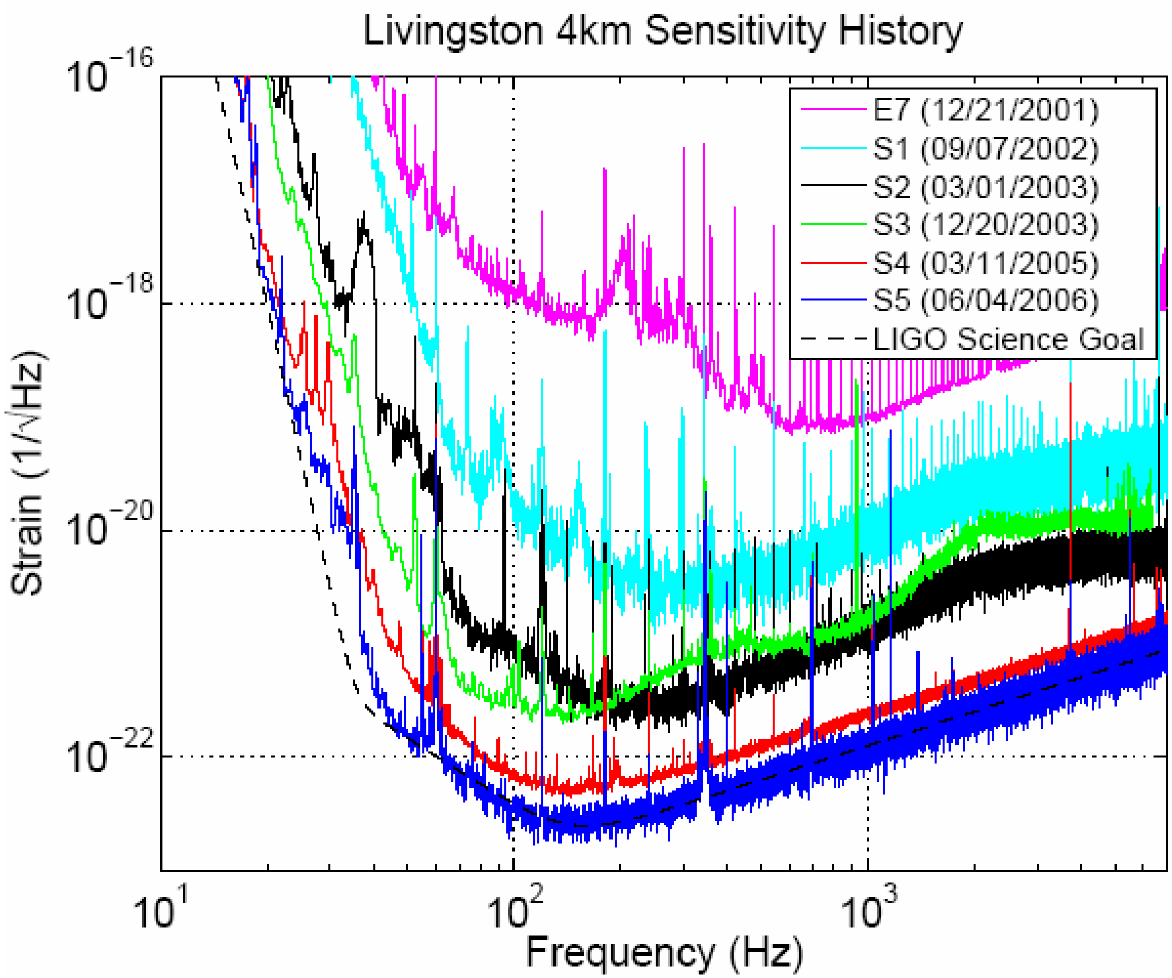


Brief History of LIGO

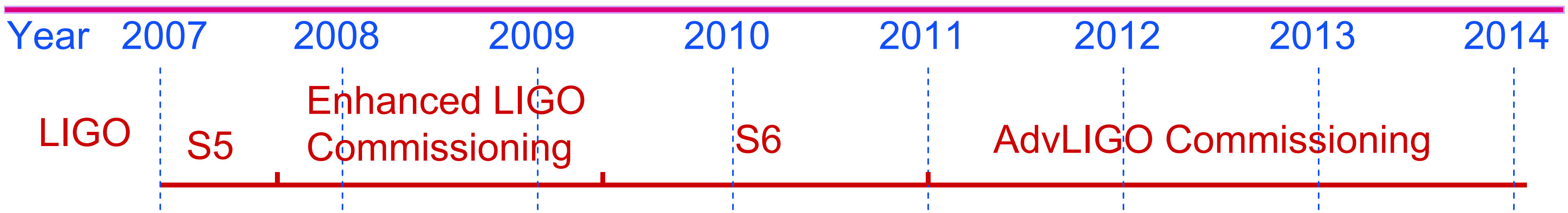


Sensitivity History

- ~4 orders of magnitude over ~4 years.
- S5 started: 1-year long run at design sensitivity.



Expected Future Timeline

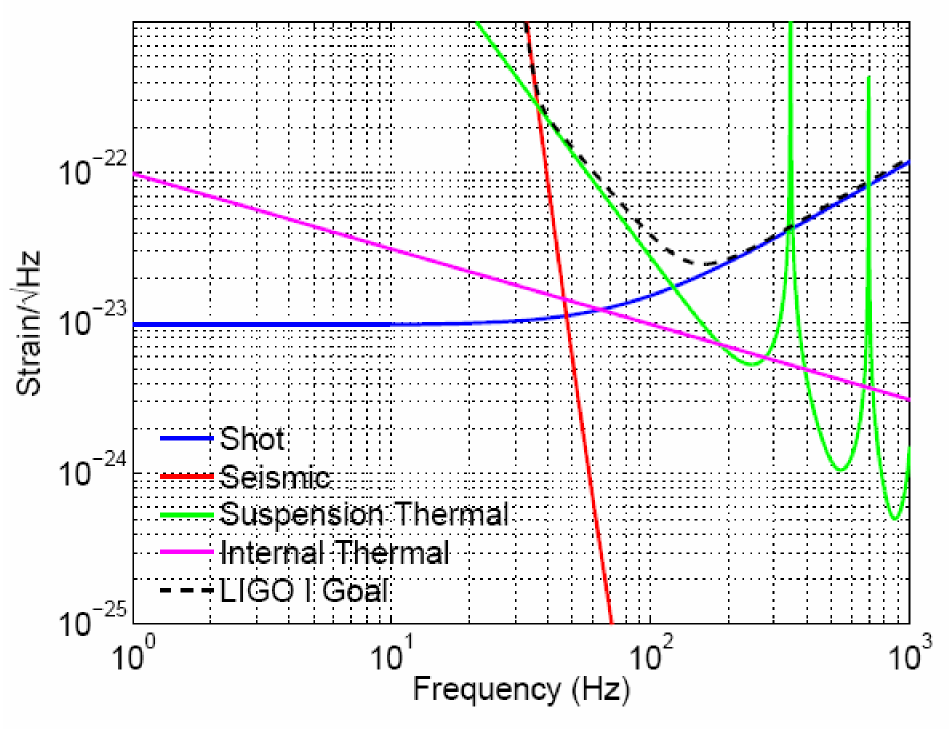


Enhanced LIGO

- Includes several relatively minor upgrades
  - Slightly more powerful laser
  - Output port sensing chain to be moved inside vacuum, and onto a seismically isolated platform.
  - Mode cleaner in the output port to remove undesired spatial modes.
  - New readout scheme (DC)
- Improve interferometer sensitivity 2-3 times above ~100 Hz.
- Test some of the Advanced LIGO concepts.

Advanced LIGO

- Keep the same facilities, but redesign all subsystems.
  - » Improve sensitivity over the whole frequency range.



Advanced LIGO

- Keep the same facilities, but redesign all subsystems.
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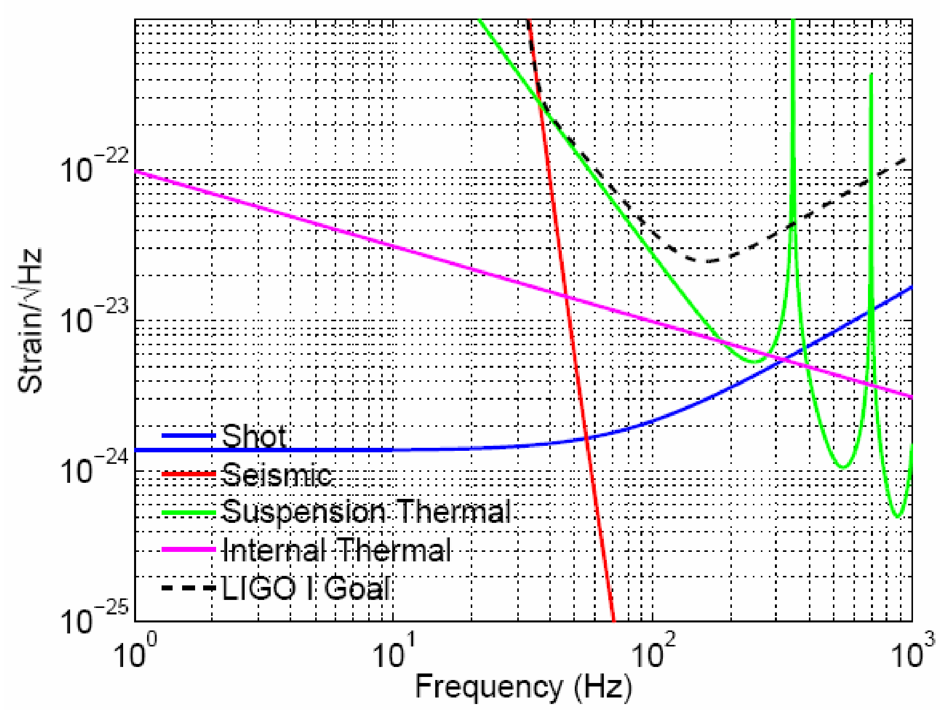
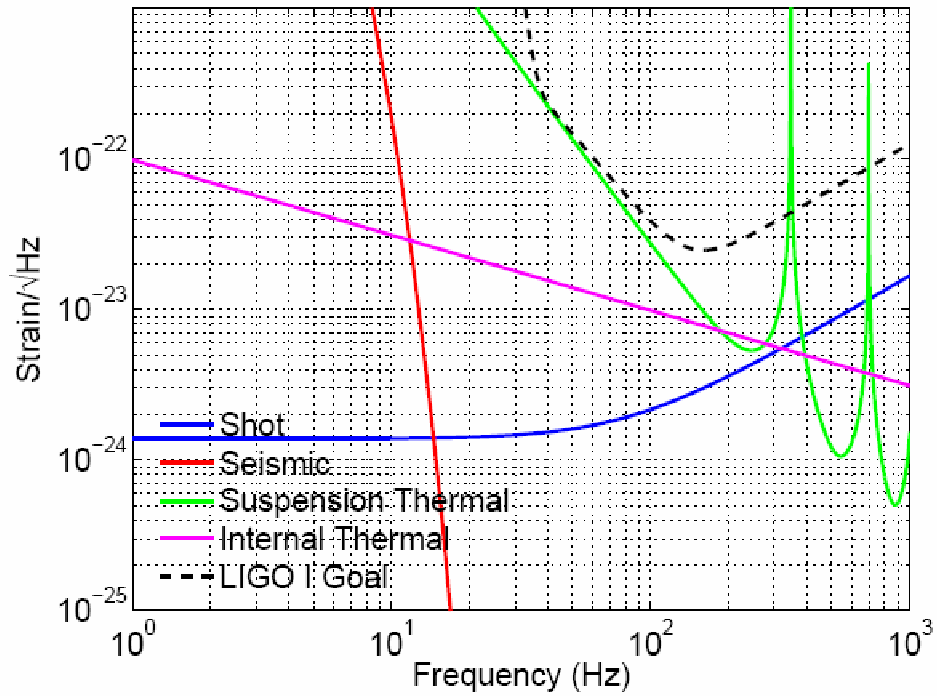
- Increase laser power in arms.



Advanced LIGO

- Keep the same facilities, but redesign all subsystems.
  - » Improve sensitivity over the whole frequency range.
- Increase laser power in arms.
- Better seismic isolation.
  - » Quadruple pendula for each mass



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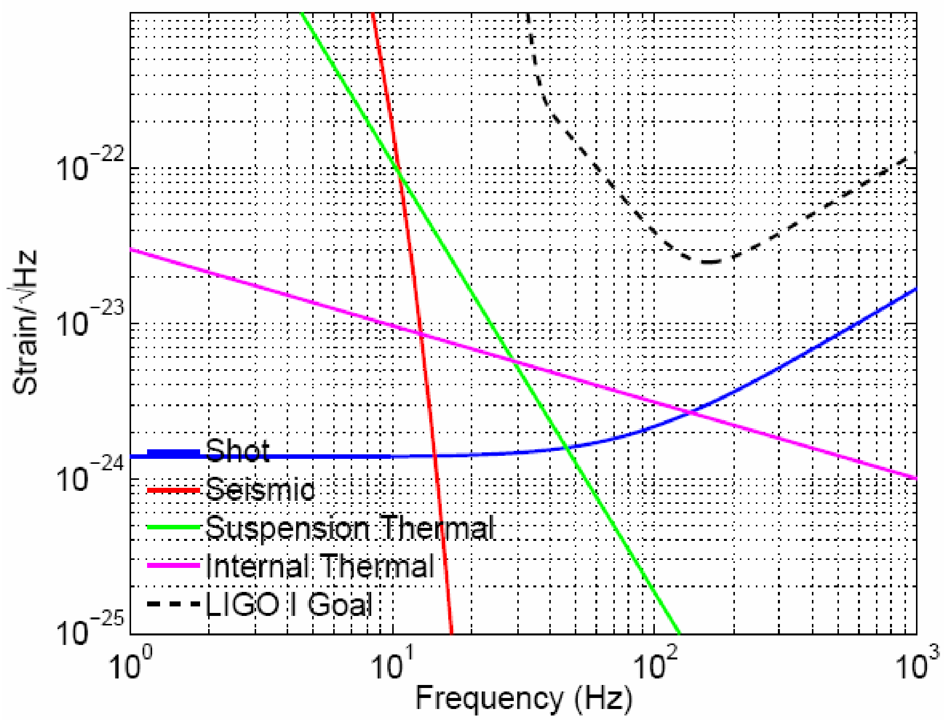
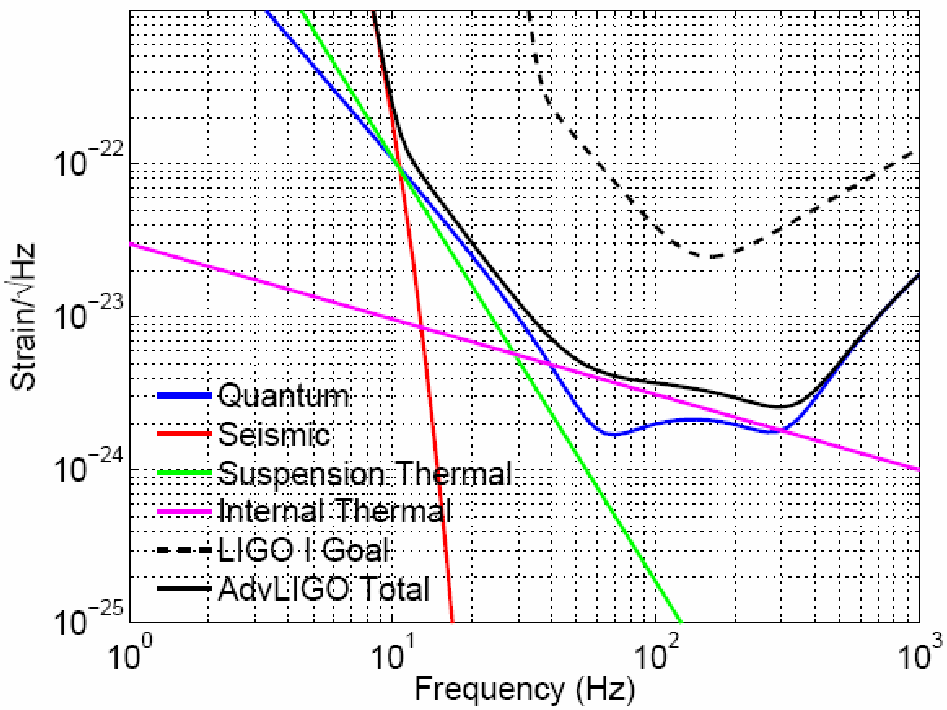
- Larger mirrors to suppress thermal noise.
- Silica wires to suppress suspension thermal noise.



Advanced LIGO

- Keep the same facilities, but redesign all subsystems.
  - » Improve sensitivity over the whole frequency range.
- Increase laser power in arms.
- Better seismic isolation.
  - » Quadruple pendula for each mass
- Larger mirrors to suppress thermal noise.
- Silica wires to suppress suspension thermal noise.
- “New” noise source due to increased laser power: radiation pressure noise.
- Signal recycling mirror
  - » Allows tuning sensitivity for a particular frequency range.
- Installation expected to start in 2011.
  - » First results: ~2014.



Beyond Advanced LIGO

- Third generation GW interferometers will have to confront (and beat) the Standard Quantum Limit
  - » Shot noise ~ $P^{-1/2}$
  - » Radiation pressure noise ~ $P^{1/2}$
  - » Together define an optimal power and a maximum sensitivity for a “conventional” interferometer
  - » Require non-classical states of light, special interferometer configurations, …
- Underground interferometers?
  - » Significant reduction in seismic noise and gravity gradient could allow exploring low frequencies (~1 Hz)
  - » Further suppression of thermal noise
    - – Cryogenic?
    - – Larger mirror masses?
  - » Lower power?

Stochastic Background of Gravitational Waves

- Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab}\,\dot{h}^{ab} >$$

- Characterized by log-frequency spectrum:

$$\Omega_{GW}(f) = \frac{1}{\rho_c}\,\frac{d\rho_{GW}(f)}{d\ln f}$$

- Related to the strain power spectrum:

$$S(f) = \frac{3H_0^2}{10\pi^2}\,\frac{\Omega_{GW}(f)}{f^3}$$

- Strain scale:

$$h(f) = 6.3\times 10^{-22}\,\sqrt{\Omega_{GW}(f)}\left(\frac{100\ \mathrm{Hz}}{f}\right)^{3/2}\ \mathrm{Hz}^{-1/2}$$

Detection Strategy

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 \; s_1(t_1)\; s_2(t_2)\; Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \; \tilde{s}_1^*(f)\; \tilde{s}_2(f)\; \tilde{Q}(f)$$

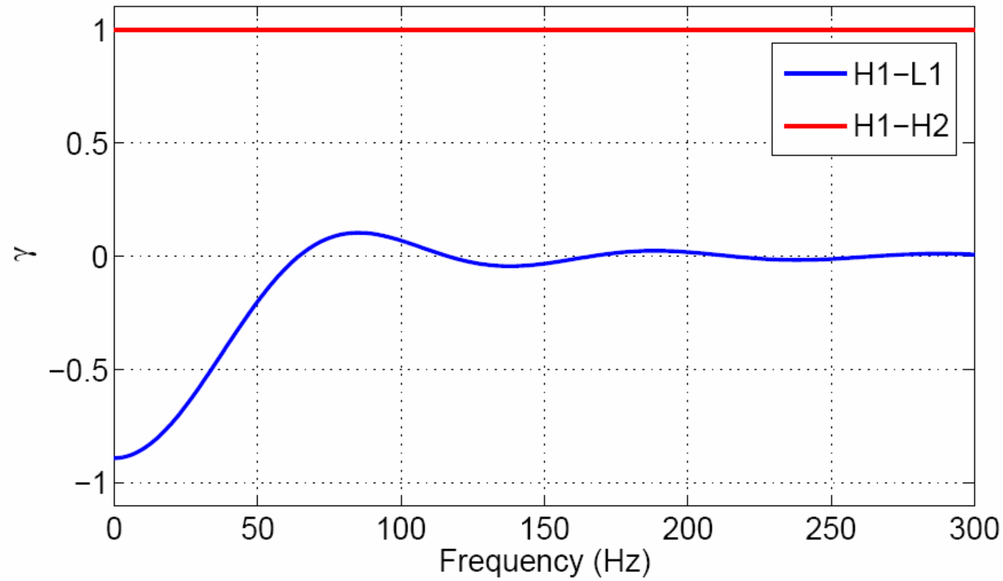
- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df \; P_1(f)\; P_2(f)\; |\, \tilde{Q}(f)\, |^2$$

- Optimal Filter

$$\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(f)\; \Omega_t(f)}{f^3\; P_1(f)\; P_2(f)}$$

Overlap Reduction Function



For template: $\Omega_t(f) = \Omega_\alpha (f/100\ \mathrm{Hz})^\alpha$

Choose *N* such that: $< Y >= \Omega_\alpha T$

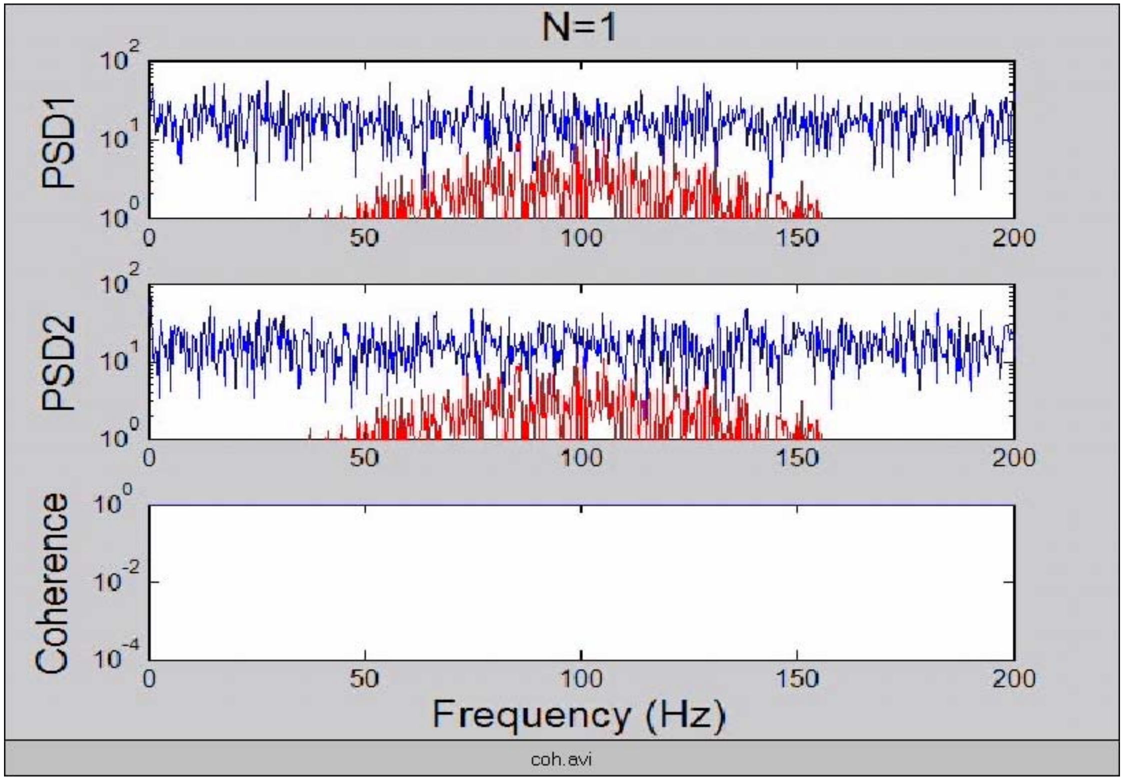
Example: White Noise

Define:

- $s_i(f) = n_i(f) + h(f)$
- $n_i(f)$ – white gaussian noise
- $h(f)$ – gaussian, colored, signal

- $Q(f) = 1$
- Coherence – "normalized" cross spectrum, averaged over N samples:

  $Coh = CSD^2 / PSD_1 / PSD_2$

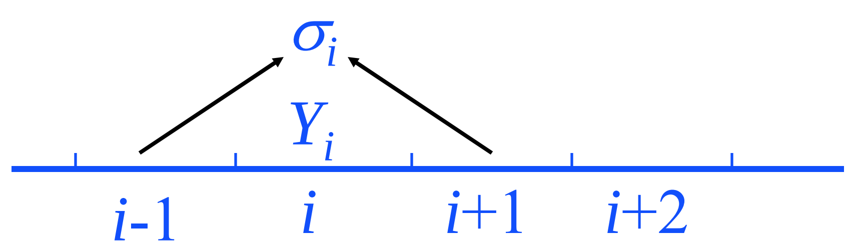


Analysis Details

- Data divided into segments:
  - » Y_i and σ_i calculated for each interval *i.*
  - » Weighed average performed.
- Sliding Point Estimate:
  - » Avoid bias in point estimate
  - » Allows stationarity ($\Delta\sigma$) cut
- Data manipulation:
  - » Down-sample to 1024 Hz
  - » High-pass filter (40 Hz cutoff)
- 50% overlapping Hann windows:
  - » Overlap in order to recover the SNR loss due to windowing.

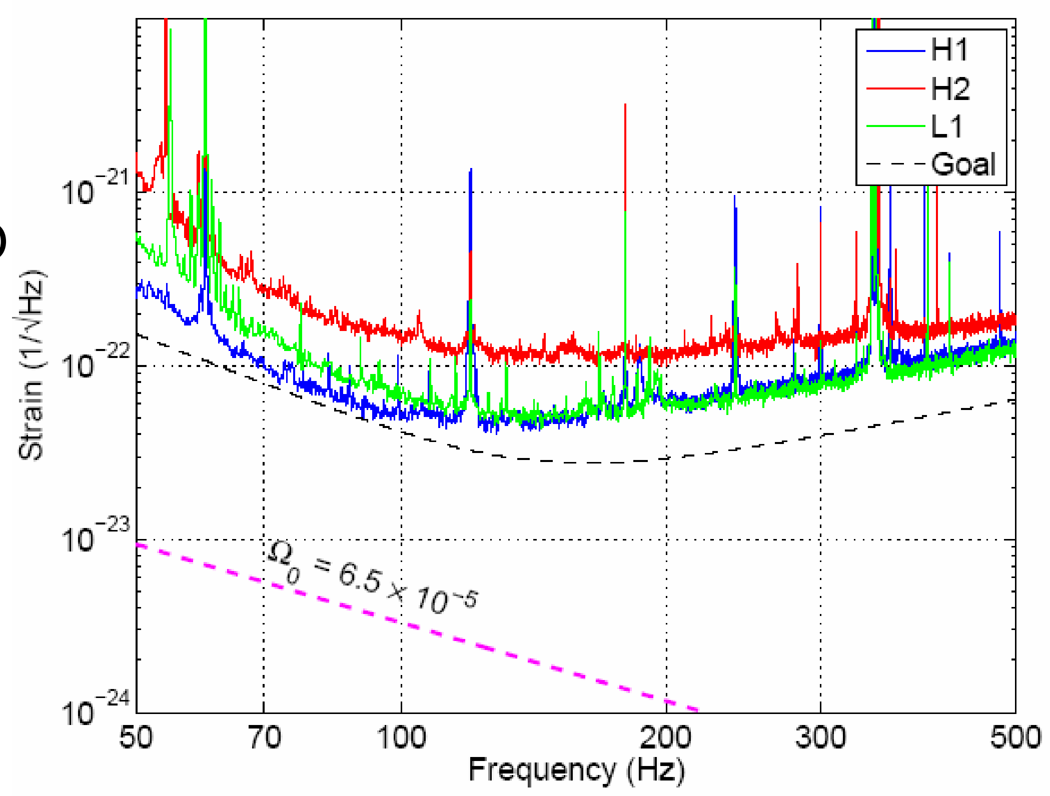
$$Y_{\text{opt}} = \frac{\sum_i \sigma_i^{-2} Y_i}{\sum_i \sigma_i^{-2}}$$

$$\sigma_{\text{opt}}^{-2} = \sum_i \sigma_i^{-2}$$



Science Run S4

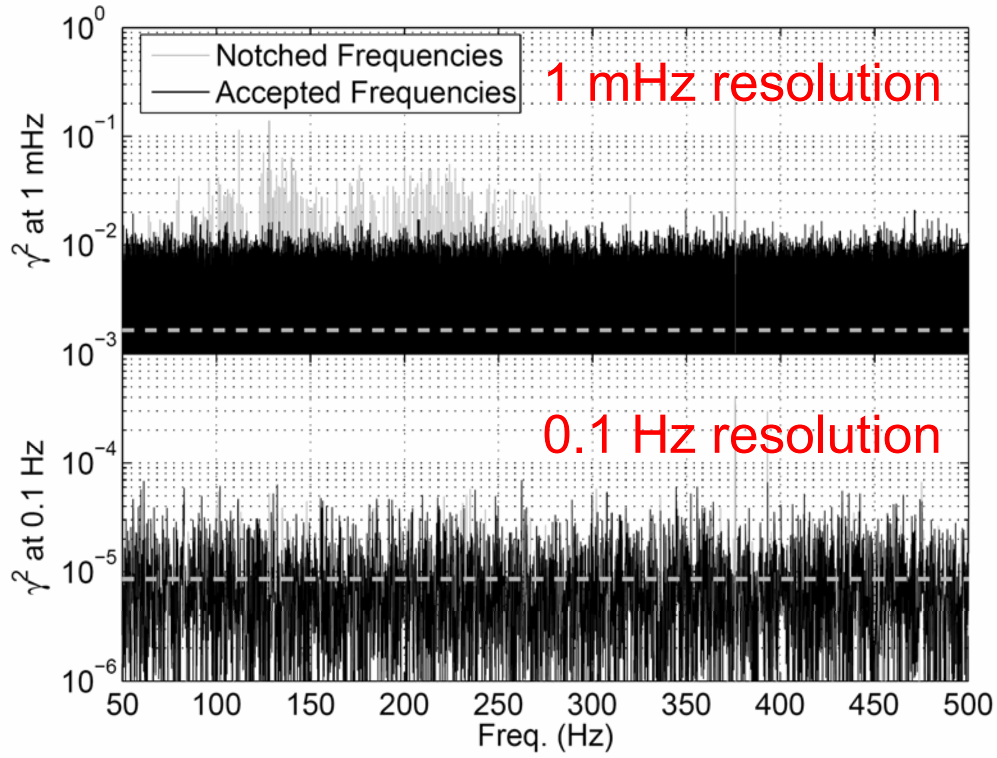
- LIGO S4 science run took place between 22 February 2005 and 23 March 2005.
- Within a factor of ~2-3 from LIGO design sensitivity.
- Factor of ~10 improvements at some frequencies, compared to science run S3.



S4: H1L1 Coherence

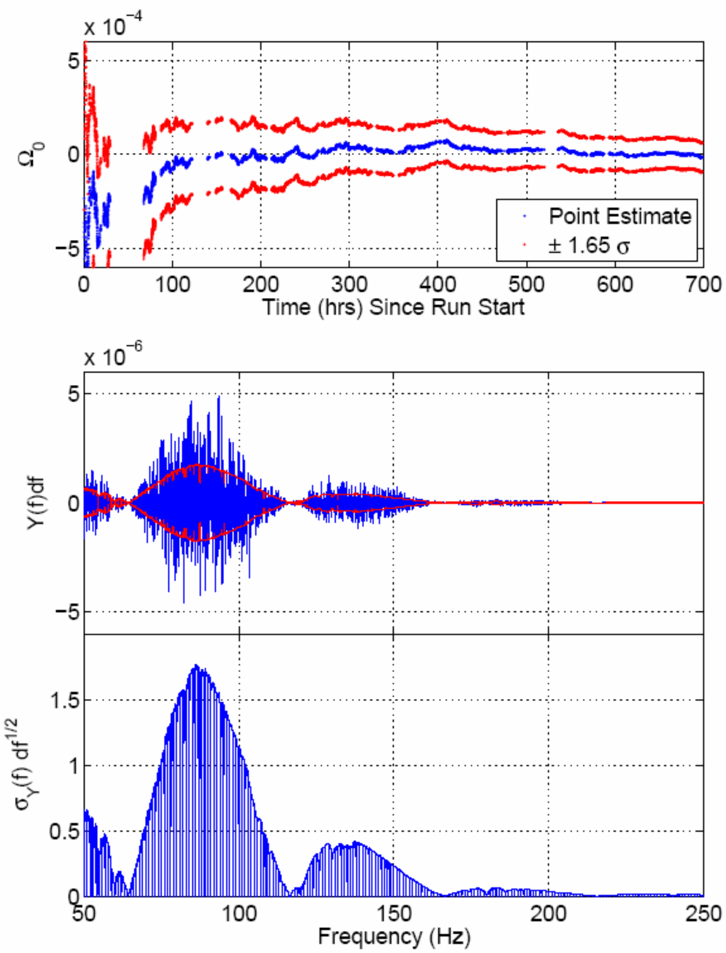
- Calculated over all of S4.
  - » Using the same data as in stochastic analysis.
- At 1 mHz resolution, many 1 Hz harmonics are observed.
  - » Sharp features, not visible at 0.1 Hz resolution.
  - » One source was the GPS synchronization signal.
  - » Expect improvement for S5.
- Also see simulated pulsar lines.

$$\text{Coh} = \text{CSD}^2 / \text{PSD}_1 / \text{PSD}_2$$

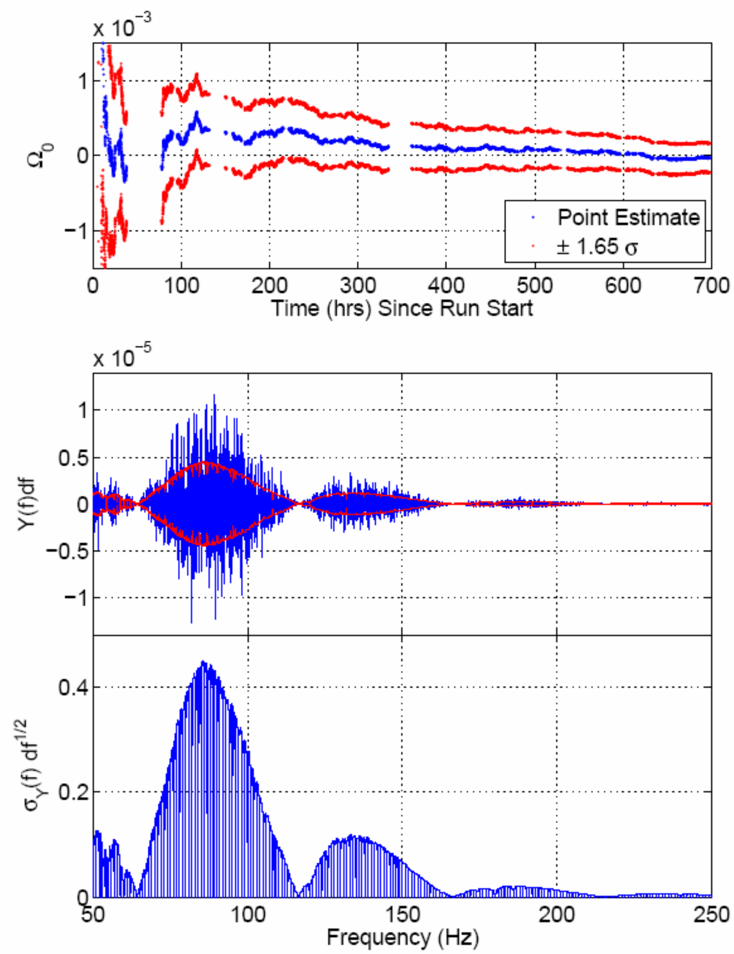


Results (1)

H1L1

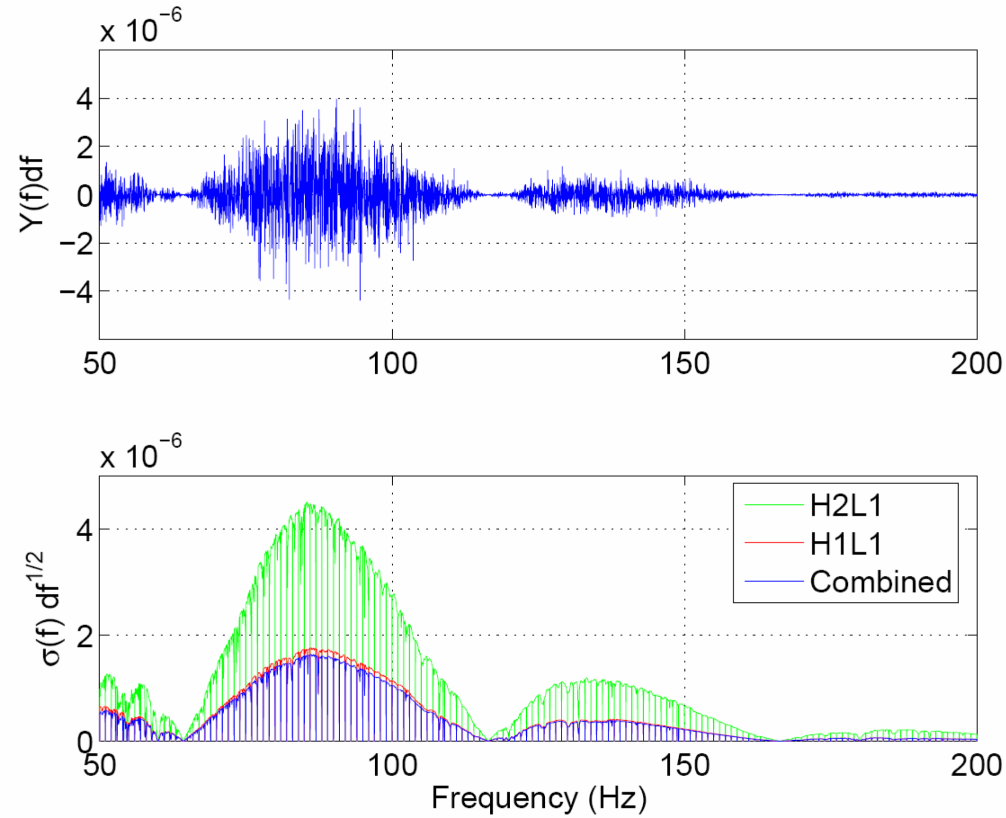


H2L1



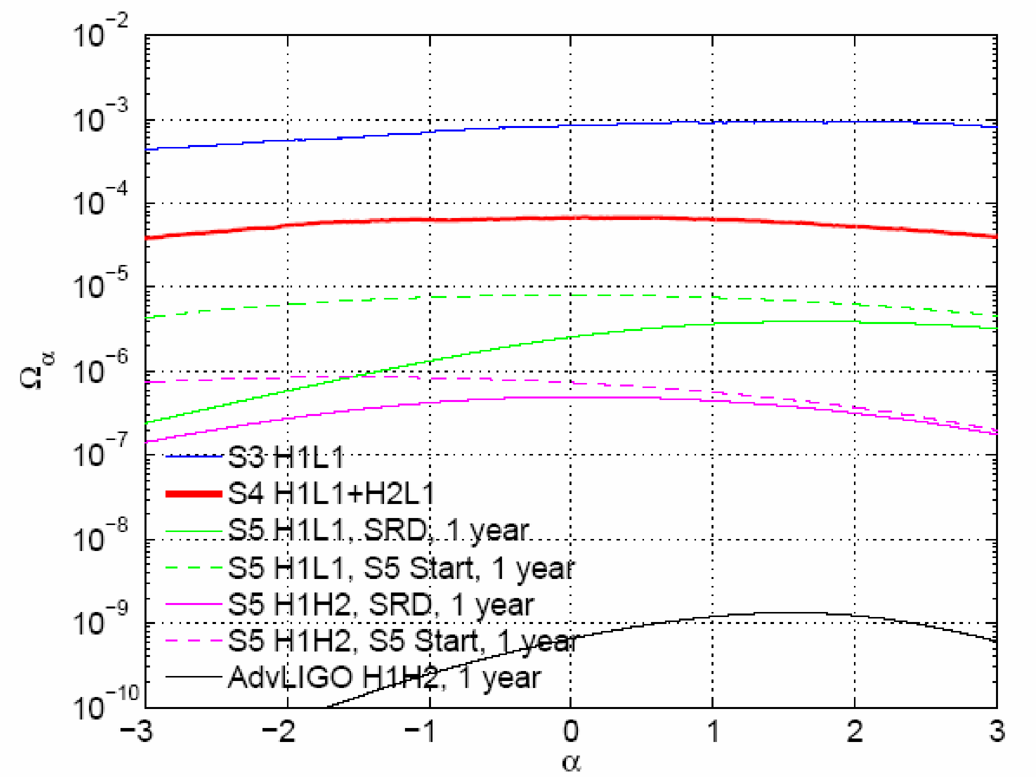
Results (2)

- Combined H1L1 + H2L1:
  - **$\Omega \pm \sigma_\Omega = (-0.8 \pm 4.3) \times 10^{-5}$**
  - H = 72 km/s/Mpc
  - 51-150 Hz (includes 99% of inverse variance)
- Bayesian 90% UL:
  - Prior on Ω: S3 Posterior
  - Marginalize over calibration uncertainties
    - Gaussian priors with standard deviation 5% for L1, 8% for H1 and H2.
  - 90% UL: **6.5×10^{-5}**



Reach as a Function of Spectral Slope

$$\Omega_t(f) = \Omega_\alpha (f/100\ \text{Hz})^\alpha$$

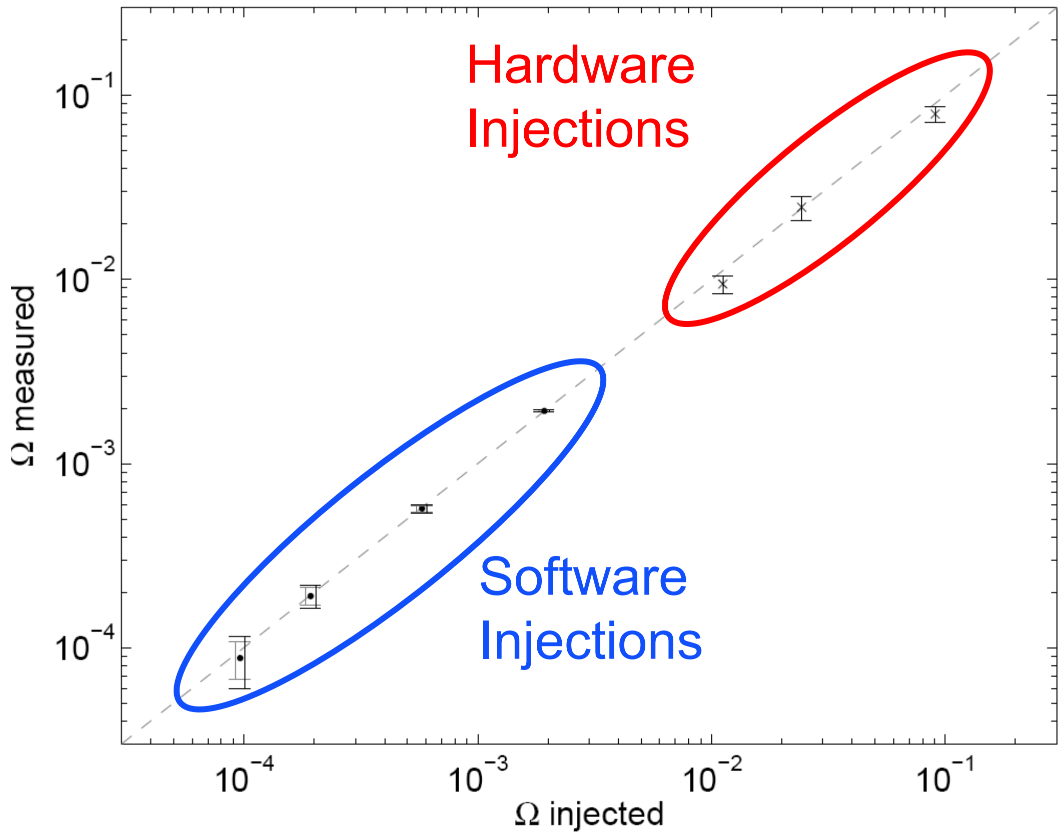


- S3 H1L1: Bayesian 90% UL.
- S4 H1L1+H2L1: Bayesian 90% UL.
- Expected S5: design strain sensitivity and 1 year exposure.
  - For H1L1, sensitivity depends on frequency band.

- AdvLIGO: sensitivity optimized for binary neutron star search, and 1 year exposure.

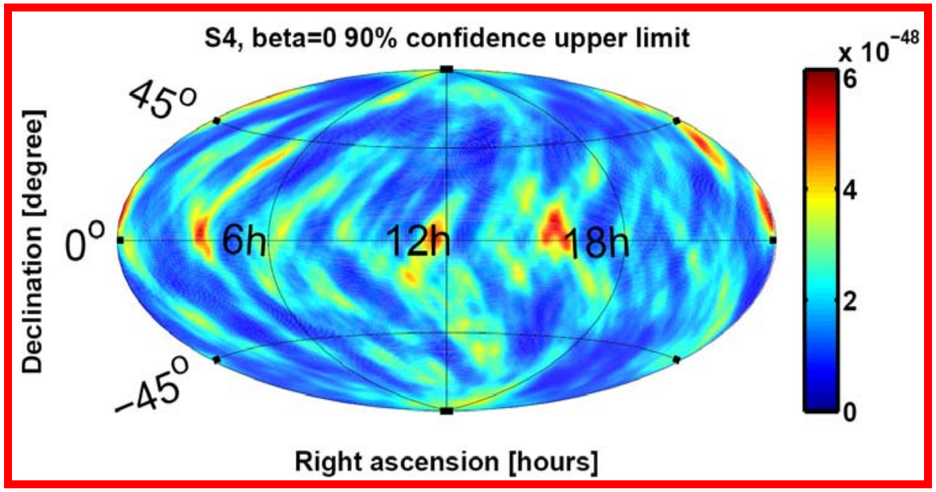
Signal Injections

- Software injections:
  - » Signal added to data in software.
  - » Successfully recovered down to $\Omega \sim 10^{-4}$.
  - » Theoretical error agrees with the standard error over 10 trials.
- Hardware injections:
  - » Physically moving the mirrors.
  - » Successfully recovered (within errors).

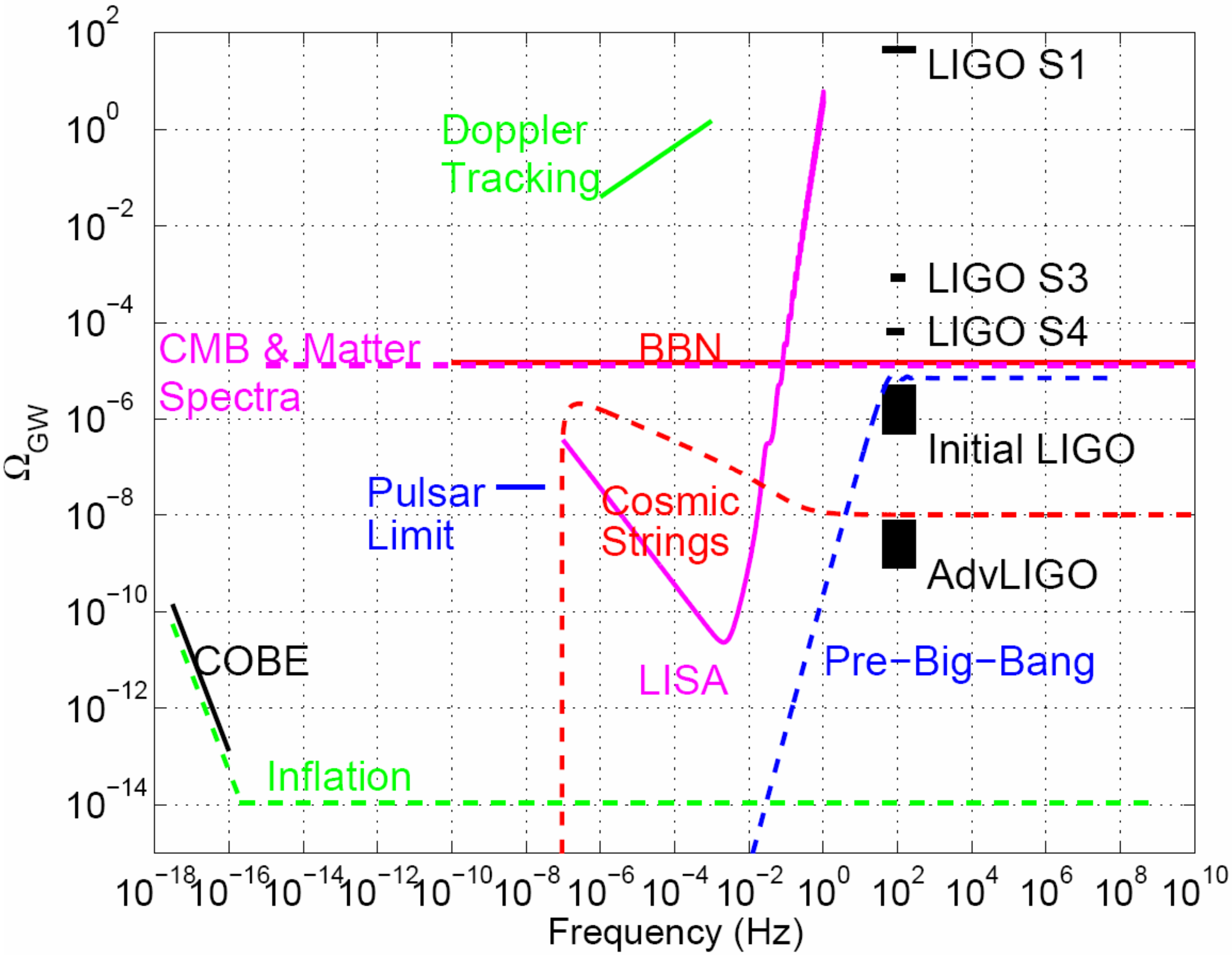


Future Stochastic Searches

- S5 data analysis in progress.
- H1H2 pair offers potentially 10x better sensitivity, but susceptible to instrumental correlations
  - » Study coherence with auxiliary (environmental etc.) channels to identify “bad” frequencies.
  - » Time-shift one detector with respect to the other by ~1-sec. GW correlations should disappear, but instrumental correlations may survive.
- Search at the higher resonance frequencies of the arm cavities (37.5, 75 kHz)
- Include non-LIGO interferometers in the search (VIRGO, GEO)
- Directional searches
  - » Search for point-sources (S. Ballmer, S4 run)
  - » Generate stochastic GW map (analogous to CMB maps)
  - » Spherical-harmonic decomposition



Landscape



Pre-Big-Bang models and Cosmic Strings models:

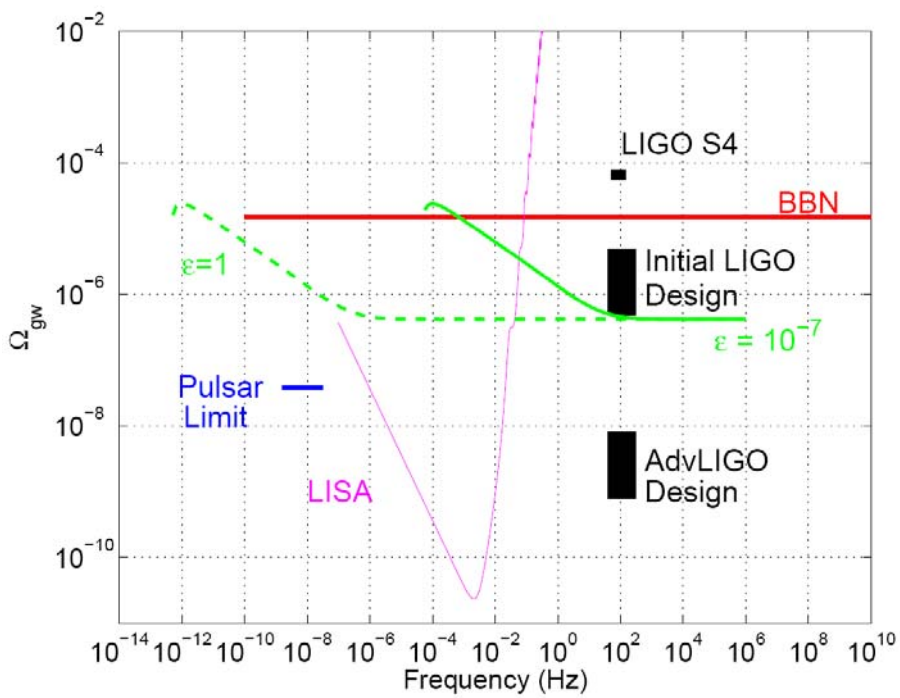
- can easily escape other experimental bounds

- accessible to LIGO.

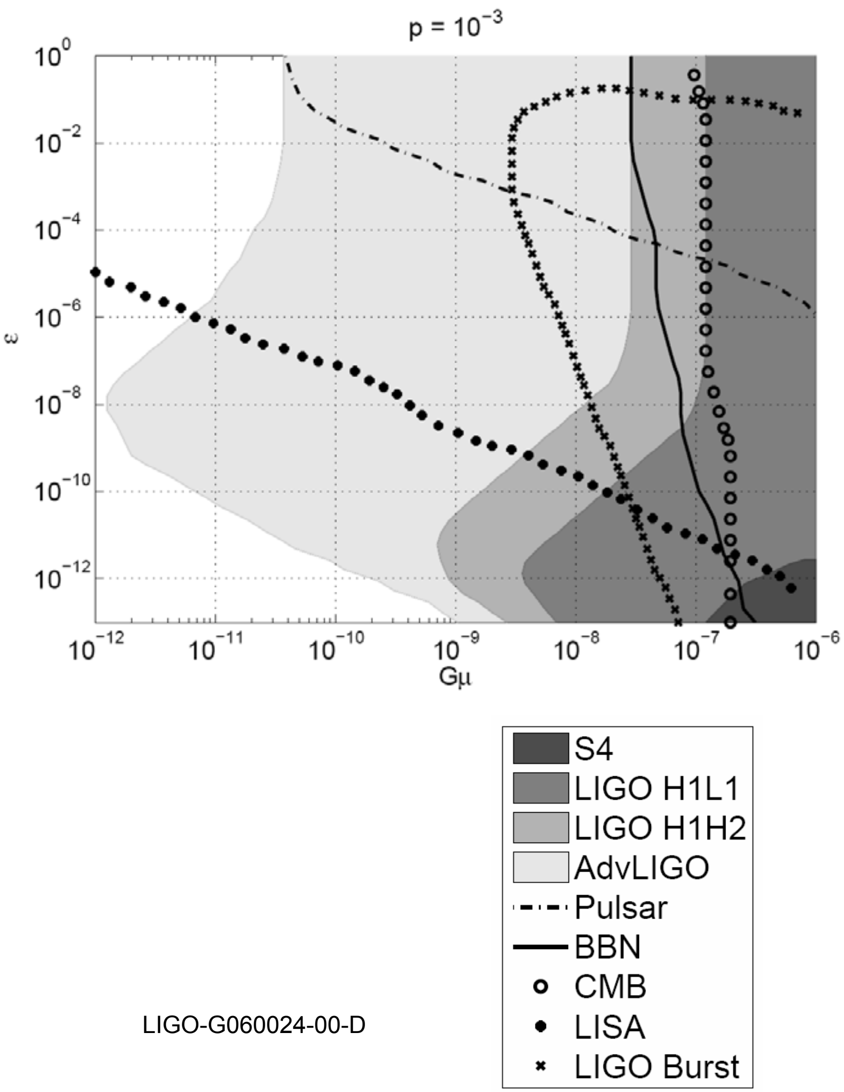
Cosmic Strings: Model

- Topological defects formed during phase transitions in the early Universe.
- Or, fundamental or Dirichlet strings (in string theory).
- Cosmic string cusps, with large Lorentz boosts, can create large GW signals.
- Look for the stochastic background created by superposing cusp signals throughout the Universe.
- Calculation done by Siemens, Mandic & Creighton, PRL98, 111101 (2007).
  - » Update on Damour & Vilenkin, PRD71, 063510 (2005).
  - » There are uncertainties in the calculation.
  - » Some of them can be resolved by improving simulations of cosmic strings networks.

Small-loop Case
$p = 5\times 10^{-3}$
$G\mu = 10^{-7}$



Small Loop Case



- If loop-size at formation is determined by gravitational back-reaction, the loops are small and of the same size.
- Parameters:
  - » loop-size parametrized by: $10^{-13} < \varepsilon < 1$
  - » String tension: $10^{-12} < G\mu < 10^{-6}$
    - – Upper bound from CMB observations.
  - » Reconnection probability: $10^{-3} < p < 1$
    - – Determines the density of strings.
- Spectrum has a low-frequency cutoff.
  - » Determined by the string length and the angle at which we observe the cusp.
- Small ε or $G\mu$ push the cutoff to higher frequencies.
- Spectrum amplitude increases with $G\mu$ and with $1/p$.

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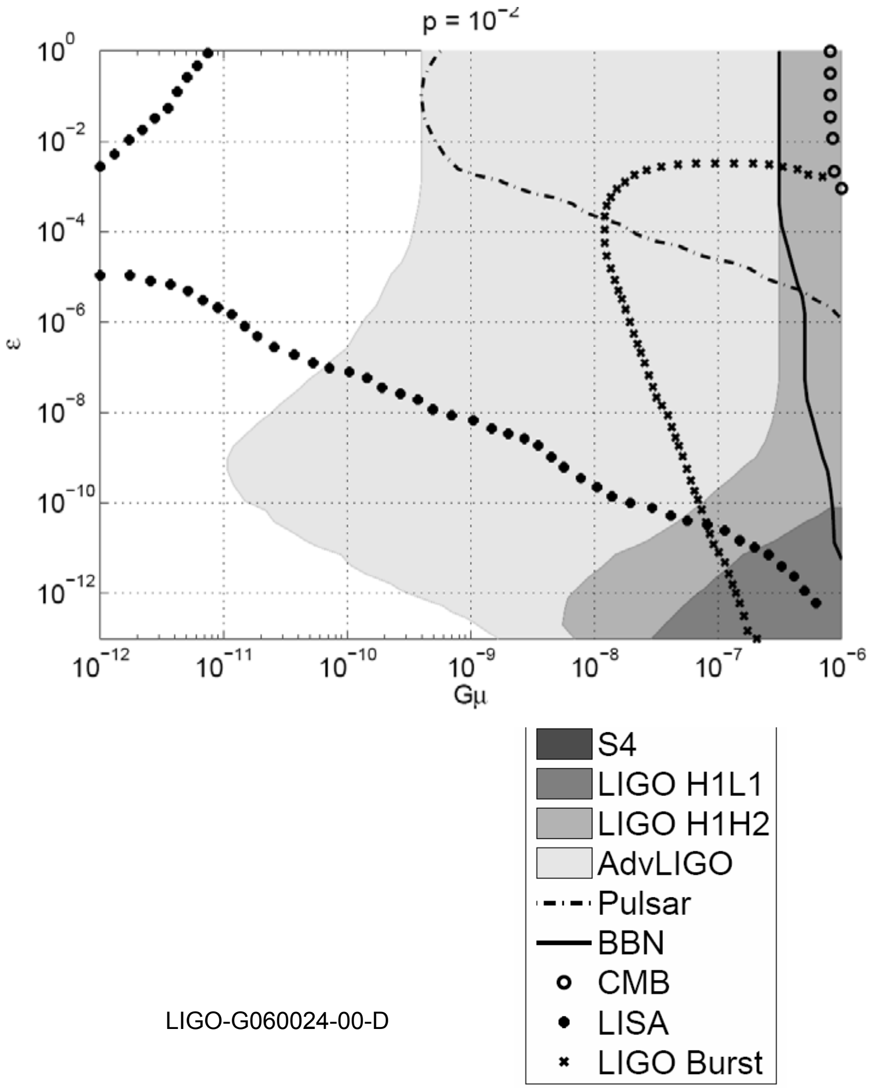

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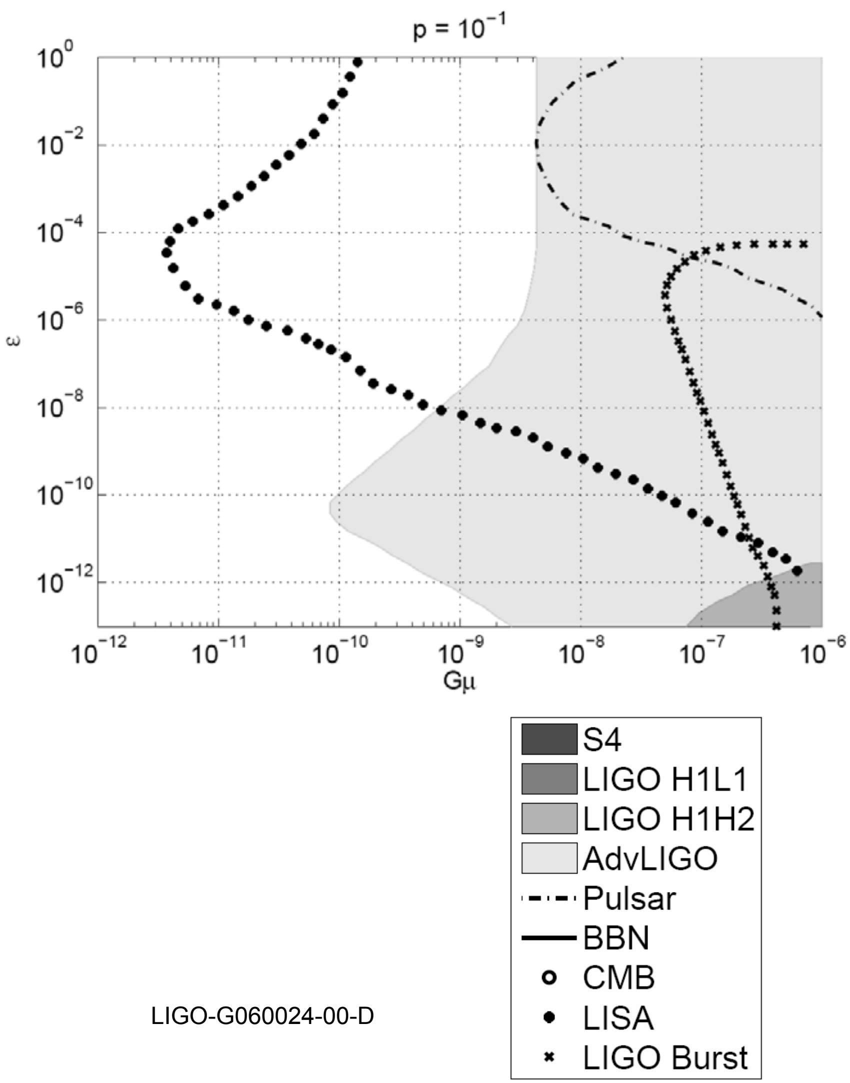

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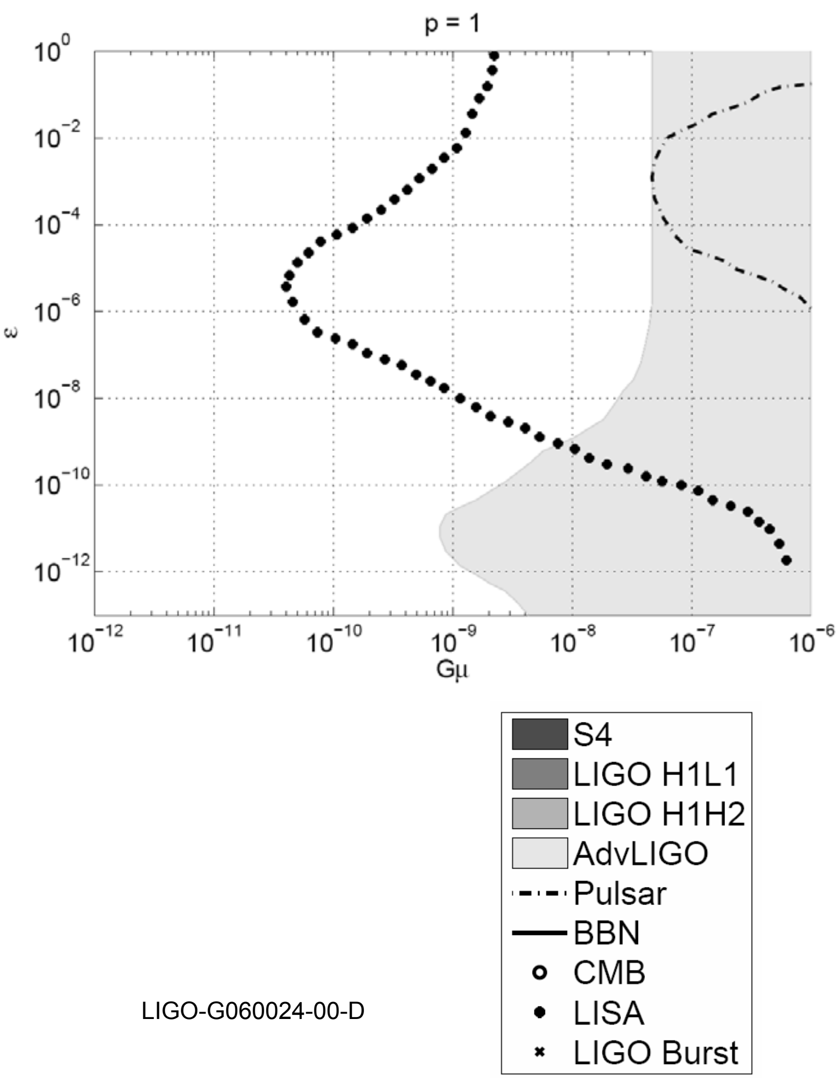

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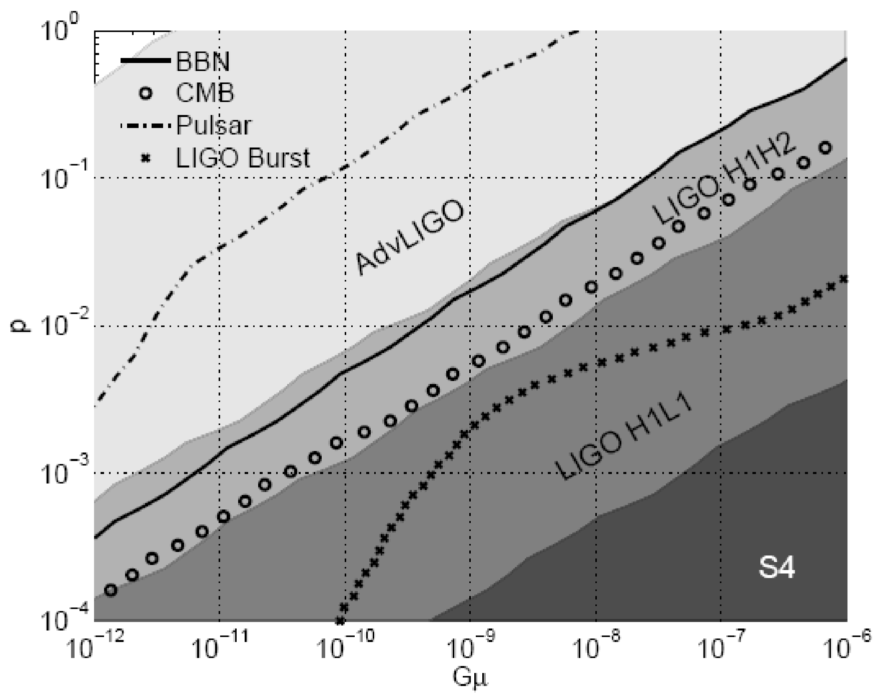
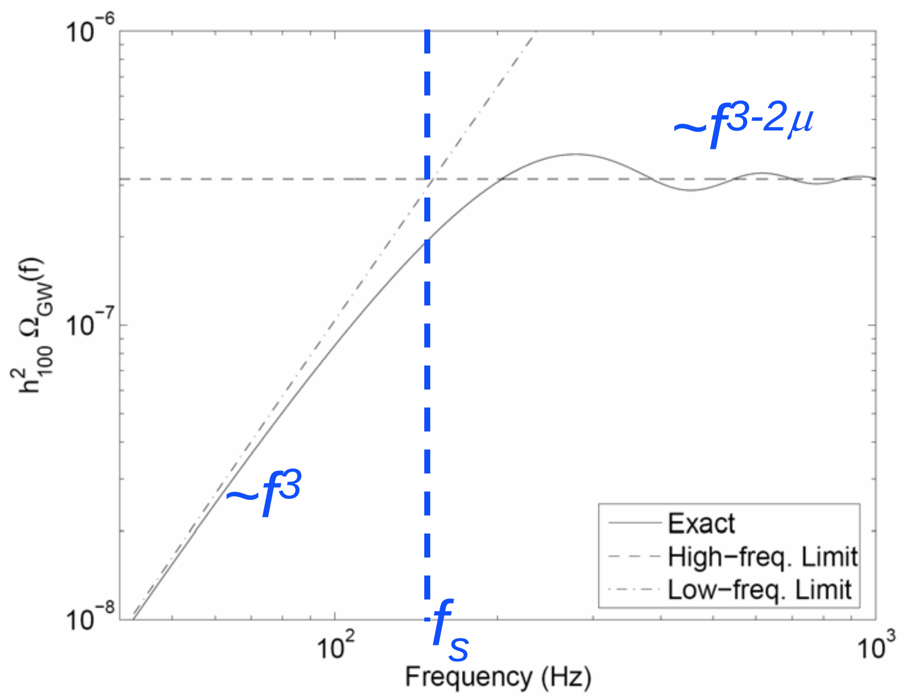
Large Loop Case



- Recent simulations indicate that loops could be large at formation, and therefore long-lived.
- Loop distribution more complex.
  - » Larger amplitudes of gravitational-wave spectra.
- Free parameters:
  - » String tension: $10^{-12} < G\mu < 10^{-6}$
  - » Reconnection probability: $10^{-4} < p < 1$
- Assuming that loop-size is 10% of the horizon at the formation time.
  - » Some simulations indicate that a more complicated distribution would be more accurate, involving both small and large loops.

Pre-Big-Bang: Model



Mandic & Buonanno, PRD73, 063008, (2006).

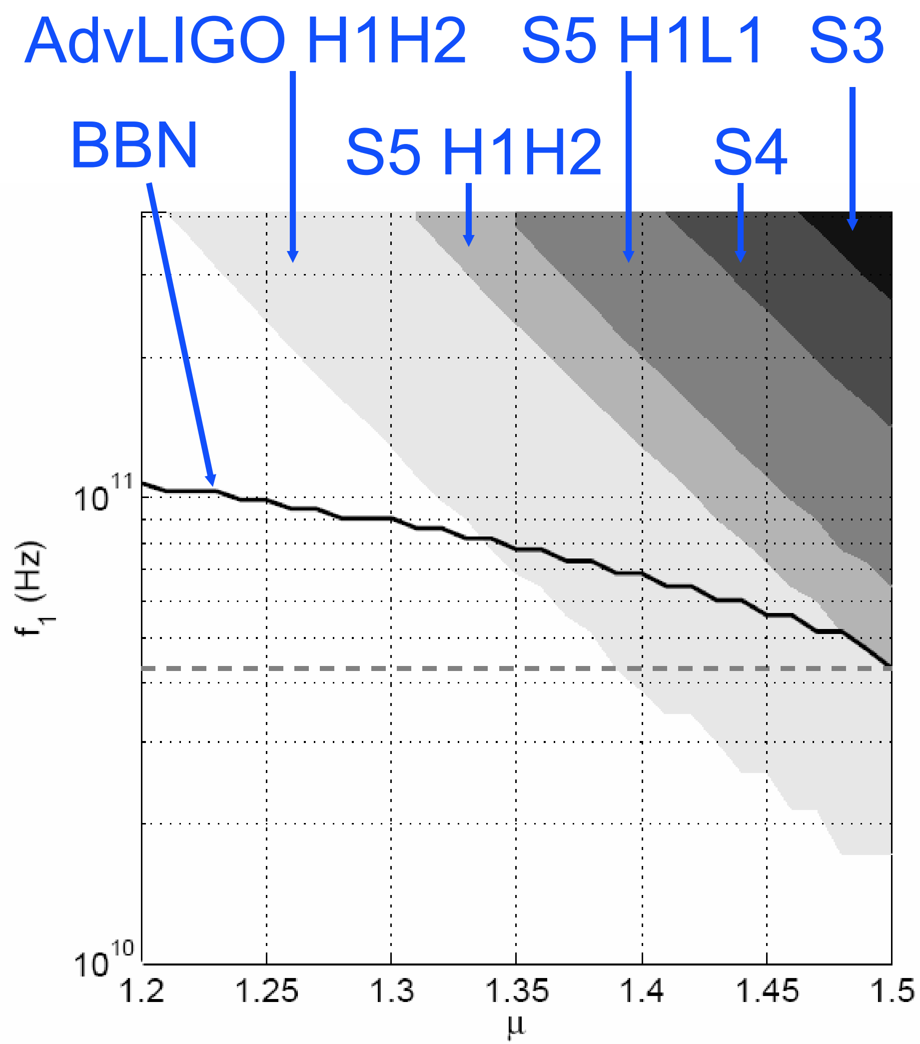
$$f_1 \simeq 4.3 \times 10^{10}\ \text{Hz} \left(\frac{H_s}{0.15 M_{Pl}}\right)\left(\frac{t_1}{\lambda_s}\right)^{1/2}$$

- Mechanism similar to inflation:
  - Amplification of vacuum fluctuations
  - Super-horizon modes are amplified during transitions between phases.
- Universe goes through several phases:
  - Dilaton-dominated phase
  - Stringy phase
  - Radiation, followed by matter phase.
- 2 free parameters:
  - $1 < \mu < 1.5$
  - f_s – essentially unconstrained
- But: High-frequency amplitude goes as f_1^4.
  - f_1 depends on string related parameters, which are not well known.
  - We vary it by factor of 10 around the most "natural" value.

Pre-Big-Bang: Results

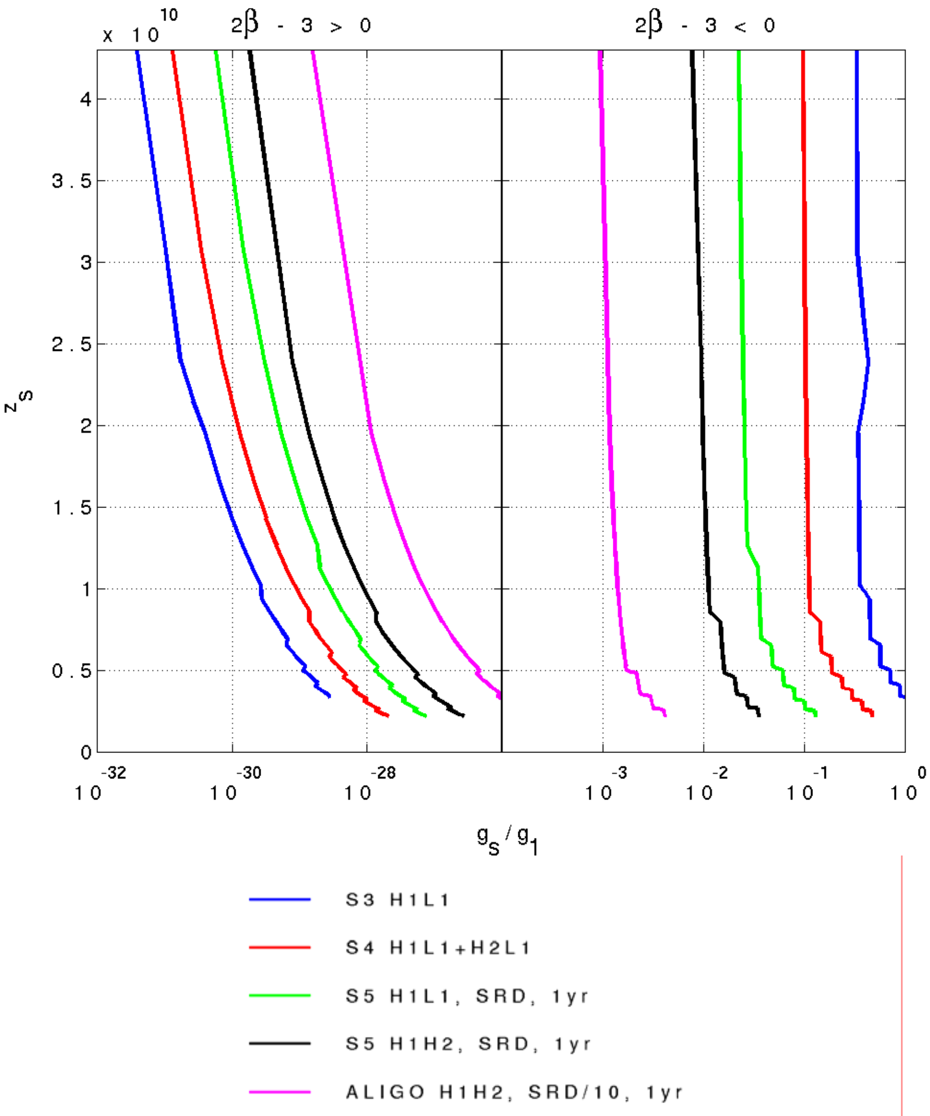
- Scan f_1 - μ plane for f_s=30 Hz.
- For each model, calculate $\Omega_{GW}(f)$ and check if it is within reach of current or future expected LIGO results.
- Beginning to probe the allowed parameter space.
- Currently sensitive only to large values of f_1.
- Sensitive only to spectra close to flat at high-frequency.
- But, not yet as sensitive as the BBN bound:

$$\int \Omega_{\rm GW}(f) h_{100}^2 d(\ln f) < 6.3 \times 10^{-6}$$



Pre-Big-Bang: Results

- Can also define:
  - » $z_s = f_1/f_s$ is the total redshift in the stringy phase.
  - » $g_s/g_1 = (f_s/f_1)^\beta$, where $2\mu = |2\beta - 3|$
    - – g_s (g_1) are string couplings at the beginning (end) of the stringy phase
- Probe fundamental, string-related parameters, in the framework of PBB models.
- Assumed $f_1 = 4.3 \times 10^{11}$ Hz (relatively large).



Conclusion

- S4 stochastic upper limit:
  Ω_{GW} = 6.5×10^{-5} for flat spectrum.
  - » Significant improvements expected in the coming years:
    - – S5: 10-100x
    - – AdvLIGO: 100-1000x
  - » Eventually should reach **Ω_{GW} ~ 10^{-9} or 10^{-10}**
- Beginning to explore models of stochastic GW background.
  - » Cosmic strings, PBB...
- 1-year long run at design sensitivity nearly finished!