University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2001 - PART I

Friday, January 12, 2001 - 9:00 AM - 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your **10** best efforts.

This is a closed-book examination. You may use calculators. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant **PROBLEM NUMBER. BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER,** so that no sheet contains work for more than one problem. Use only one side of the paper: If some problems require more than one sheet for their solution, then be sure to indicate "page 1", "page 2",...,etc., under the problem number entered on the sheets. Once completed, all your work should be put, **IN ORDER**, in the manila envelope provided.

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GWE Winter 2001 - PART I

1. Find the change in entropy, ΔS (in J/K), of $n = 3.0$ moles of a monatomic ideal gas if, as a result of a certain process, the volume of the gas increases, from V_i to $V_f = 2V_i$, while the pressure decreases, from p_i to $p_f = p_i/3$.

2. Two unequal capacitors C_1 and C_2, charged to the same potential difference V, are connected in series (that is, "plus" to "minus"). A switch S that is initially open completes the circuit.
(a) What are the final charges Q_1 and Q_2 on each capacitor after the switch is closed ?
(b) Calculate the loss in stored electrostatic energy, and discuss (without calculation) what happens to this lost energy.

3. A neutral pion of kinetic energy $T = 270$ MeV decays in flight into two photons. Find the maximum energy (in MeV) that is kinematically allowed for either one of the photons produced in this decay. The rest energy of the neutral pion is $mc^2 = 135$ MeV.

4. Consider a simple model for radiation transfer within the Sun, in which photons propagate via a random walk process controlled by Thomson scattering from free electrons. The cross-section for Thomson scattering is given in terms of the classical electron radius r_e, and is given by $\sigma_T = \frac{8\pi}{3} r_e^2 = 6.7 \times 10^{-25}\, cm^2$. (a) Given the mean density of the Sun, $\rho = 1.42 g/cm^3$, estimate the (average) mean free path of the photons. (b) The radius of the Sun is 2.32 light-seconds: Estimate the time required for a photon to travel from the center of the Sun to the surface.

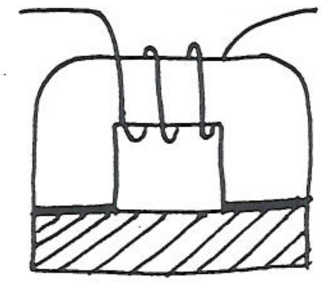
5. Your job is to design an arrangement of identical polarizing filters (polaroids) so as to rotate the plane of polarization of an initially plane polarized beam of light by 90°. The polaroids are not perfect, but absorb 30% of the light, regardless of its polarization. Find the number N of polaroids that should be used to achieve a maximum final intensity. Hint: for a given choice of N, each filter shall rotate the plane of polarization by an angle $\pi/2N$! Use trial and error, as N is expected to be a small number.

6. A neutron of energy $E = 2.0$ keV, incident from the left, is scattered by a one-dimensional repulsive delta function potential $V(x) = S\delta(x)$, of strength S and located at the origin $x = 0$. The appropriate solution of the Schrodinger equation is the wave-function

$$\psi(x) = \begin{cases} 25\exp(ikx) - (9+12i)\exp(-ikx) & ; \quad x < 0 \\ (16-12i)\exp(ikx) & ; \quad x > 0 \end{cases}$$

(a) Determine k (in m^{-1}), and the reflection and transmission probabilities, R and T, respectively.
(b) Determine the dimensionless number $S/\hbar c$ characteristic of the strength of the potential.

7. Estimate the total pole face area of a magnet (see sketch) designed to support the weight of a 10 ton iron block. You may use the fact that the magnetic field in soft iron saturates at a value of approximately 2 Tesla). Hint: think of virtual work !



8. The Earth's core is mostly iron and extends to about half of the Earth's radius. If all the iron in the Earth's core were made into a wire as long as the radius of the visible Universe, what would be the diameter of the wire ?

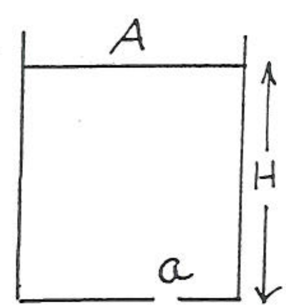
9. As a raindrop falls due to gravity, it increases its mass through deposition of vapor (at rest) on its surface. Assume the raindrop maintains a spherical shape throughout, and that its radius $R(t)$ increases linearly with time, i.e. that $\frac{dR}{dt} = K = const.$; this is equivalent to assuming that the rate of increase of its volume is proportional to its surface area. Neglecting air resistance, find the velocity $v(t)$ as a function of time of a raindrop that starts at rest and with an initial radius $R(0) = a$, at $t = 0$. Check your answer by verifying that $v(t) \cong gt$ when $Kt << a$, i.e. a very short time into the raindrop's fall. What becomes of $v(t)$ in the opposite limit, $Kt >> a$?

10. A particle of mass m and charge e is constrained to move around a circular orbit of radius a. A constant magnetic field of magnitude B is applied perpendicular to the orbit. The resulting Schrodinger equation is

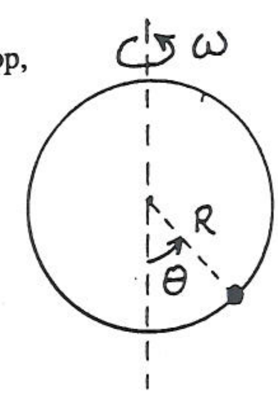
$$\frac{1}{2m}\left(\frac{-i\hbar}{a}\frac{d}{d\phi} - e\frac{aB}{2}\right)^2 \psi(\phi) = E\psi(\phi)$$

where ϕ is the angular position of the particle along the orbit. Find the eigenfunctions (there are two possible types) and the corresponding eigenvalues of the energy. Hint: both types of solution must satisfy the periodicity condition $\psi(\phi) = \psi(\phi + 2\pi)$.

11. An open-topped cylindrical vessel of cross-sectional area A is filled to a height H with a liquid of negligible viscosity. At $t = 0$, a small hole of area a is opened at the bottom of the vessel. Assuming ideal flow and that $a << A$, how long does it take for the vessel to empty ?



12. A bead of mass m is constrained to move (without friction) along the circumference of a vertically oriented circular hoop of radius R. The hoop, in turn, rotates about its vertical diameter at an angular velocity ω. As ω varies, the bead will have different equilibrium positions $\theta_{eq,i}$: What are these equilibrium positions ?



University of Minnesota
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GRADUATE WRITTEN EXAMINATION

WINTER 2001 - PART II

Saturday, January 13, 2001 - 9:00 AM - 1:00 PM

Part II of this exam consists of 6 problems of equal weight. You will be graded on your **5** best efforts.

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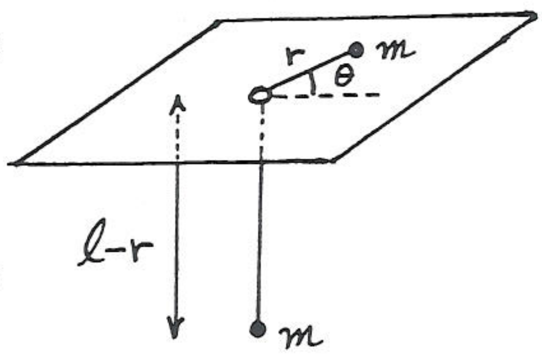
GWE Winter 2001 - PART II

1. Two equal masses m are connected by a string of fixed length ℓ. One of the masses moves without friction on a horizontal surface, while the other is suspended through a hole in the surface. The string hangs vertically, and there is no friction between the string and the lip of the aperture in the surface (see the rough sketch shown).

(a) Set up the Lagrangian of the system in terms of the coordinates r and θ of the upper particle, and derive the equations of motion.



(b) Show that for any value ω_0 of the angular velocity $\omega = \dot{\theta}$, there exists an equilibrium solution with $r = r_0$ and $\dot{r} = 0$. Use the equations of motion to describe what happens to the system when this equilibrium is slightly perturbed radially, i.e. when $r_0 \to r_0 + \delta$, with $\delta << r_0$.

2. The matrices representing the Cartesian components of the spin operator $\vec{S}$ of a spin-1 particle are given by (choosing S_z to be diagonal):

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad , \quad S_y = \frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} \quad , \quad S_z = \hbar\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

A beam of spin-1 particles, each prepared in the spin state

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}|s_z = +\hbar\rangle - \frac{1}{\sqrt{2}}|s_z = 0\rangle,$$

is passed through a certain apparatus that separates out the various components of this state corresponding to eigenstates of S_x, the spin in the x-direction.

(a) Calculate the average value of S_x in the state $|\psi_i\rangle$.

(b) Using the standard representation above, construct the normalized eigenstates of S_x, i.e. $|s_x = +\hbar\rangle, |s_x = 0\rangle, |s_x = -\hbar\rangle$, with $S_x|s_x\rangle = s_x|s_x\rangle$.

(c) Calculate the respective probabilities $p(s_x)$ that a measurement of S_x done on the state $|\psi_i\rangle$ would yield the eigenvalues $s_x = +\hbar$, 0 and $-\hbar$. As a check, use these results to recompute the average you obtained in part (a).

3. A rocket of proper (rest) length $L = 10^3 m$ moves with respect to us at a speed $v = 0.6c$. There are two clocks on this ship behind transparent windows at the nose and the tail of the ship, which have been previously synchronized with each other before departure. We, on the ground, have a number of clocks also synchronized with each other. Just as the nose of the ship reaches us, say at $x = 0$, we can take a photograph showing that both our clock (at $x = 0$) and the clock in the nose of the ship indicate the same time $t = 0$.

(a) At this time, $t = 0$, to us, what time does the clock in the tail of the ship read, in seconds ?

(b) How long (in seconds) does it take for the tail of the ship to reach us ?

(c) When the tail of the ship is at our position $(x = 0)$, what time does the clock in the tail of the ship read, in seconds ?

(d) When the tail of the ship is at our position, what time does the clock in the nose of the ship read, in seconds ?

4. Consider a cubical box of volume $V = L^3$, with perfectly conducting walls, in equilibrium at temperature T. Initially, this box contains N photons, with the equation of state for radiation $p = \frac{1}{3}\rho$, where ρ is the energy density.

(a) Suppose now that the box were to expand adiabatically, to a new side length $L' = 2L$. What would be the new temperature T', and how much work was done by the radiation during this expansion ?

(b) Now suppose, contrary to part (a), that the box expands isothermally to a new side length $L' = 2L$. Find the heat input required for this process.

(c) What are the numbers of photons in the box as a result of the respective expansions considered in parts (a) and (b) ?

5. The binary system Cygnus X-1, at a distance of about ten thousand light-years from Earth, consists of two stellar objects orbiting about their common center-of-mass under the influence of their mutual gravitational forces. For what follows, assume that the orbits of both objects are circular: The orbital period of motion is $T = 5.6$ days. One of the objects is a blue supergiant star with a mass M equal to 25 solar masses, while the other is believed to be a black hole, with a mass m equal to 10 times the mass of the Sun. Make a sketch of this system and from the information given, determine:

(a) The orbital radii R and r of the supergiant star and black-hole candidate, respectively, in meters.

(b) The orbital speeds V and v of the supergiant star and black-hole candidate, respectively, in meters per second.

6. Many physicists, following a suggestion made by Dirac in 1931, have considered the possible existence of magnetic monopoles as a way of imposing a symmetry between electric and magnetic fields and their sources in Maxwell's equations. Two of Maxwell's equations then take on new forms, namely

$$\vec{\nabla}\cdot\vec{B} = \mu_o \rho_m$$

and

$$\vec{\nabla}\times\vec{E} = -\mu_o \vec{J}_m - \frac{\partial \vec{B}}{\partial t},$$

where ρ_m and $\vec{J}_m$ are magnetic charge and current densities, respectively. A point magnetic monopole of magnetic charge g at the origin, gives rise, by the first equation, to a radial magnetic field $\vec{B} = \frac{\mu_o}{4\pi}\frac{g}{r^3}\vec{r}$ which is the exact analogue of the electric field of a point charge e, with the replacement $e/\varepsilon_o \to \mu_o g$. Further, Dirac showed by a quantum-mechanical argument that the unit of magnetic charge g should be related to the unit of electric charge e by the formula

$$g = ec/2\alpha \cong 68.5ec,$$

where $\alpha \cong 1/137$ is the fine structure constant and c is the speed of light. To detect such hypothetical magnetic monopoles, one could use superconducting loops, as follows.

(a) Suppose that a monopole of Dirac magnetic charge g travels at a constant speed $v << c$ along the z-axis, going through the origin O at $x = y = z = 0$ at time $t = 0$, so that its position at all times is given by $x = y = 0$ and $z = vt$. At the origin O, the monopole passes through the center of a superconducting loop of radius a which lies in the $x - y$ plane. Calculate the magnetic flux $\Phi_B(t)$ through the flat circular surface of the loop as a function of time, and make a sketch of your result.

(b) Integrating the modified Faraday's law given above over the surface of the loop gives the emf induced in the loop, in terms of the magnetic current i_m and the time derivative of magnetic flux through the surface. Write this emf equation out assuming that the loop has inductance L and zero resistance (the loop is superconducting!).

(c) Use the result of part (b) to show that the motion of the monopole from $-\infty$ to $+\infty$ induces an electric current $i = \mu_o g / L$, and find the value of this current (in Amperes) for a Dirac monopole of unit magnetic charge (see above), and given a self-inductance of $L = 2.0$ μH for the loop.