University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2002 - PART 1

Thursday, January 17, 2002 - 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

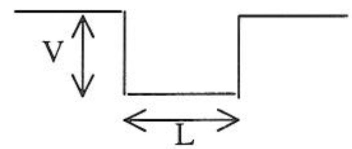
USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	Values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ϵ_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m
Electron rest mass	m_e	9.11×10^{-31} kg 0.511 MeV/c^2
Proton rest mass	m_p	1.67×10^{-27} kg 0.938 GeV/c^2
Neutron rest mass	m_n	1.68×10^{-27} kg 0.940 GeV/c^2
Planck constant	h	6.63×10^{-34} J.s 4.14×10^{-15} eV.s
Molar gas constant	R	8.31 J/mol.K
Avogadro's number	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K 8.62×10^{-5} eV/k
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2.K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Bohr radius	a_0	5.29×10^{-11} m
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg
Mass of Sun		1.99×10^{30} kg
Mass of Moon		7.36×10^{22} kg

1. A1000 kg automobile has ground clearance of 18 cm but when loaded with an extra 500 kg from its 4 passengers it only clears the ground by 12 cm. The car's shock absorbers are ineffective. At what speed (in miles per hour) will the car bounce in resonance when it travels along a smooth road containing transverse tar patches every 15 m? Assume that the front and rear suspensions have the same bouncing frequency.

2. Suppose a particle of mass m moves in a 1-dimensional square potential well of width *L* and depth *V*. What is the minimum depth of the well such that the particle will have two bound states?



3. The cross-section for collision between helium atoms is about 10^{-16} cm^2. Estimate the mean free path of helium atoms in helium gas at atmospheric pressure and temperature.

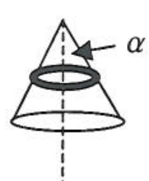
4. Consider an infinitely long cylindrical coaxial capacitor. The outer conductor has radius *R* and the applied voltage is *V*. For what radius of the inner conductor will the field strength at its surface be a minimum?

5. A rigid container is filled with a classical perfect gas of molecular mass m. The temperature inside the container is T and the pressure is P. Outside there is a vacuum. Suppose that a small hole of area A is punched in the outer wall. At what rate (molecules per unit time per unit area) will gas leave the container? Write the result as a function of m, P, and T.

Some possibly useful information:

$$\int_0^\infty x^n e^{-\lambda x^2}dx = \frac{1}{2}\frac{\Gamma(\frac{n+1}{2})}{\lambda^{\frac{n+1}{2}}};\Gamma\left(\frac{1}{2}\right)=\pi^{\frac{1}{2}}$$

6. A single closed loop of chain with mass *m* and length *L* rests horizontally on a smooth frictionless cone with half-angle α. What is the tension in the chain?



7. A laser beam (photon energy 1 eV) collides head-on with a 50 GeV ultra-relativistic electron beam. What is the energy of the photons reflected backwards in the collision?

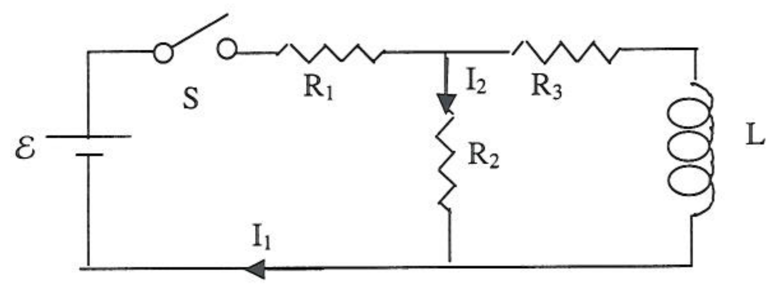
8. In the figure below, $\mathcal{E} = 100$ V, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 15\Omega$, and L = 1.0 H. Find the values of the currents I_1 and I_2

a) immediately after the switch S is closed;

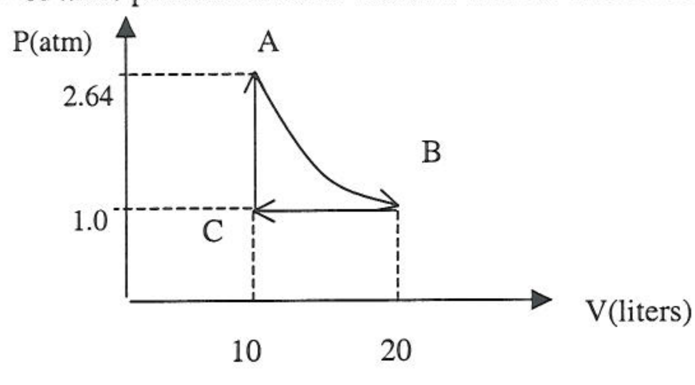
b) a long time later;

c) immediately after switch S is opened again;

d) and then how long must you wait, after the switch is opened, before I_2 falls by a factor e?



9. An engine using 1 mole of an ideal diatomic gas performs the cycle A→B→C→A as shown in the diagram below. A→B is an adiabatic expansion, B→C occurs at constant pressure and C→A takes place at constant volume. What is the efficiency of the cycle?



10. A thin circular hoop rolls down an inclined plane under the influence of gravity. What minimum coefficient of friction is required to ensure that it rolls rather than slides?

11. A particle is confined within a cubical box with sides of length L and is initially in the ground state. If the length of one side of the box (along the x-direction) is abruptly increased to a length $2L$, what is the probability that the particle remains in the ground state?

12. The frequency f of a deep water gravity wave (i.e. an ordinary ocean wave) is given by

$$f = \rho^a g^b \lambda^c / \sqrt{2\pi}$$

where ρ, g, and λ are the water density, gravitational acceleration, and wavelength of the wave, respectively. What are the values of the exponents a, b, c and what is the ratio of the wave group velocity:phase velocity?

University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2002 - PART 2

Friday, January 18, 2002 - 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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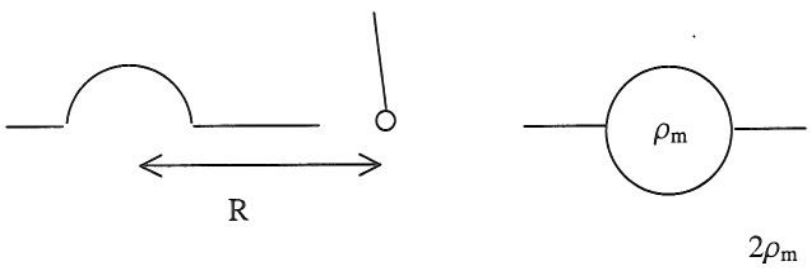
1. The gravitational attraction of a nearby mountain range might be expected to cause a plum-bob to be deflected at a slight angle θ to the vertical.

a) Determine θ for the case where the mountain range is approximated by an infinite half-cylinder of radius a and density ρ_m. Consider distances both far and near to the mountain.

b) And determine the deflection when the mountain is approximated by an infinite cylinder of radius a and density ρ_m floating in a medium of density $2\rho_m$.



2. A zipper has N links; each link has a closed state with zero energy and an open state with energy ϵ. We require, however, that the zipper can only unzip from the left end, and that the link number s can only open if all links to the left (i.e. 1,2,...s-1) are already open.

a) Find an explicit expression for the partition function by doing the appropriate summation.

b) In the limit $\epsilon \gg k_B T$ find the average number of open links. This model is a very simplified model of the unwinding of two-stranded DNA molecules.

3. Find the equation for the energies E at which a particle is not reflected by a potential consisting of two delta functions of the form:

$$U(x) = \alpha[\delta(x) + \delta(x-a)]$$

Assume a is positive and solve this equation in the limit $E \ll m\alpha^2/2\hbar^2$

Hint: Note that the discontinuity of the first derivative of a wave function across a delta-function follows from integrating the Schroedinger equation from a - ϵ to a + ϵ.

4. A particle of magnetic material with magnetic moment μ moves with speed v along the axis of a circular loop of conducting wire. The loop has radius a. If μ points in the direction of motion, what force does the particle experience when it is a distance z ($\gg a$) from the center of the loop? What is the direction of this force? You may assume that $v \ll c$ and that the loop has negligible self-inductance.

5. A doped semiconductor has a concentration of N_d donor atoms. If ϵ_C is the energy of the bottom of the conduction band, ϵ_d is the donor level energy, and ϵ_F is the Fermi energy, obtain an expression for N_d in terms of ϵ_C, ϵ_d, ϵ_F, the electron mass m, and the temperature T.

You may assume the following:

1) No excitation from the valence band.

2) The level density in the conduction band is given by:

$$N(\varepsilon)d\varepsilon = \frac{8\pi(2m^3)^{1/2}}{h^3}\varepsilon^{\frac{1}{2}}d\varepsilon$$

3) $(\epsilon_C - \epsilon_F) \gg kT_B$

4) $\int_0^\infty e^{-\beta x^2}dx = \sqrt{\frac{\pi}{4\beta}}$

6. The following simplified model portrays certain mechanical aspects of a star:

The mass M of the star is represented by a thin spherical shell of radius R.which may expand or contract without any perceptible tension or rigidity.

The pressure supporting the star against gravitational collapse is represented by a uniform ideal gas trapped within the shell and this gas expands or contracts adiabatically so that pV^γ=constant. The mass of this gas is much less than M.

The radius of the star may oscillate about some equilibrium value R_0.

a) Find the gravitational potential energy of the massive shell, assuming that its thickness is much smaller than R, and also the internal energy of the gas as a function of R/R_0, γ, and U_0 (the equilibrium internal energy).

b) Using R as the time-dependent variable, write a Lagrangian for radial motions of the model, including both gravity and gas internal energy in the " potential" part. (The mass of gas is negligible).

c) Write the equation of motion for $R(t)$. Eliminate U_0 from the equation using its dependence on M, R_0, and γ, and find the frequency of small oscillations about R_0 as a function of M, R_0, and γ.

d) What does your result suggest about stars that are supported by radiation pressure?