University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

SPRING 2004 - PART I

Thursday, January 15, 2004 - 9:00 A.M. to 12:00 NOON

Part I of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed book examination. You may use a calculator. A list of some physical constants and properties that you may require is included; please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER**.

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put into the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	Values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ϵ_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m
Electron rest mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Proton rest mass	m_p	1.6726×10^{-27} kg
		0.93827 GeV/c^2
Neutron rest mass	m_n	1.6749×10^{-27} kg
		0.93957 GeV/c^2
Planck constant	h	6.63×10^{-34} J.s
		4.14×10^{-15} eV.s
Molar gas constant	R	8.31 J/mol.K
Avogadro's number	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
		8.62×10^{-5} eV/k
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2.K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Bohr radius	a_0	5.29×10^{-11} m
Gravitational constant	G	6.67×10^{-11} m^3/s^2.kg
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg
Mass of Sun		1.99×10^{30} kg
Mass of Moon		7.36×10^{22} kg

School of Physics and Astronomy
University of Minnesota
Graduate Written Examination. Part I. Short Problems

There are 12 problems. Your score for the 10 problems on which you do best will be used in determining your grade.

1. An electrical teapot has two heating coils. If only the first one is used the water is boiled in 30 min, if only the second one – in 15 minutes. How much time it will take to boil the water if both coils are used connected (i) in series; (ii) in parallel ?

2. An accelerator produces a beam of protons with the kinetic energy 10 000 MeV. Estimate what fraction of the speed of light is velocity of these protons?

3. A system is made up of $N = 10^{23}$ two level atoms. It has energy $E = 400J$. If the level spacing is $\Delta = 0.1eV$ find the temperature T (in K) of the system . Take the ground state to be the energy reference point and neglect all other degrees of freedom.

4. In 1916, Germany launched a project–Big Bertha, which was to invent a big gun to target at Paris $100kms$ away. Find the range of initial bullet speeds which would allow the bullet to reach Paris. Take the gravitational acceleration $g = 10m/s^2$ and the radius of the earth to be 6.4 x 10^6 m. Neglect air resistance.

5. Consider a system consisting of two non-interacting spin-1/2 particles. A measurement has been performed with the following result:

$$(S_1)_z = 1/2; (S_2)_x = 1/2,$$

where S_i is the spin operator of the i-th particle, S_z is the spin projection on the z-th axis, S_x on the x axis. What is the probability that the total spin of the system is 1 ?

6. Two 50 kg skaters, holding hands, revolve around each other so that their center of mass is at rest, with angular velocity ω =2 radians/sec. If they increase their angular velocity to 4 radians/sec by pulling each other closer, what increase in kinetic energy results? Show that the work done by the skaters in pulling each other closer is the same as the increase in kinetic energy.

7. The η meson has a mass mc^2 of 550MeV, spin 0, electric charge 0 and negative parity. Prove that it cannot decay via strong inteactions which conserve parity to either a pair of charged pions $\pi^{\pm}$ (mass mc^2=135 MeV each, spin 0, negative parity) or to a pair of neutral pions π^0(mass mc^2=140MeV spin 0 , negative parity).

8. A source of electromagnetic radiation of wavelength λ is a very large distance R away from the earth. One has two detectors on the earth separated by a distance d. One can measure the angle θ between a line between the two detectors and a line from the midpoint between the two detectors and the source. The signals from these two detectors are combined so that one detects the sum of the signals phase coherently. That is the final signal is proportional to $|\vec{E}_1 + \vec{E}_2|^2$ where $\vec{E}_{1,2}$ are the electric fields detected at detectors 1 and 2 respectively. How does the signal depend on d, λ and θ? If there were 2 sources of equal amplitude (but not coherent) at distance R, separated by a distance $b << R$ and if θ were near $\pi/2$, how large would b have to be (in terms of the other parameters) in order to assure that this pair of detectors could distinguish them?

9. If a gas bubble under water oscillates with a period $\tau = P^\alpha \rho^\beta E^\gamma$ where P is the pressure, ρ is the mass density of the water, E is the energy required to form the bubble and α, β, γ are dimensionless, find the values of α, β, γ .

10. A cylindrical solenoid containing N turns of wire , crossectional area A and having a removable iron core of magnetization M is attached in series to a resistor of resistance R. The iron core is removed. How much charge passes through the resistor?

11. In a fluid, Newton's second law (equivalent to the conservation of momentum) takes the form

$$\frac{\partial \vec{v}\rho}{\partial t} = -\nabla P$$

ρ is the mass density, $\vec{v}$ is the fluid velocity and P is the pressure. Use this, the equation of continuity and the equation of state to find an expression for the velocity of sound in an ideal gas under isothermal conditions in terms of the temperature T and the molecular mass m.

12. A cylinder has a membrane 1/2 way across it, dividing the volume inside into 2 equal parts. The cylinder and the membrane are impermeable to particles and to heat energy. Initially there is a monatomic ideal gas containing N atoms in one half of the cylinder and the other half of the cylinder is completely empty (a perfect vacuum). The initial temperature of the gas is T_1. Now the membrane is punctured by a remote control device which does negligible work on the gas and which causes a negligible heat leak. After requilibration by how much have the following quantities characterizing the gas changed?: Internal energy U, temperature T, entropy S and Helmholtz free energy F (sometimes called A).

University of Minnesota
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GRADUATE WRITTEN EXAMINATION

SPRING 2004 - PART 2

Friday, January 16, 2004 - 9:00 A.M. to 1:00 P.M.

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

This is a closed book examination. You may use a calculator. A list of some physical constants and properties that you may require is included; please take a moment to review its contents before starting the examination.

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University of Minnesota
Graduate Written Examination. Part II. Long Problems.

There are 6 problems. Your score will be determined from the 5 problems on which you do best.

1. A ship is docked in a harbor with the help of a thick rope wound around a cylindrical post with the radius $R = 25cm$. The ship pulls the rope with the force 1 000 000 N. How many revolutions of the rope around the post are needed, so a little boy, holding the end of rope, can prevent the ship from moving ? The boy can exert force of 100 N and the coefficient of the static friction between the rope and the post is, $\mu_s = 1$. (Hint: consider forces on an infinitesimal piece of the rope.)

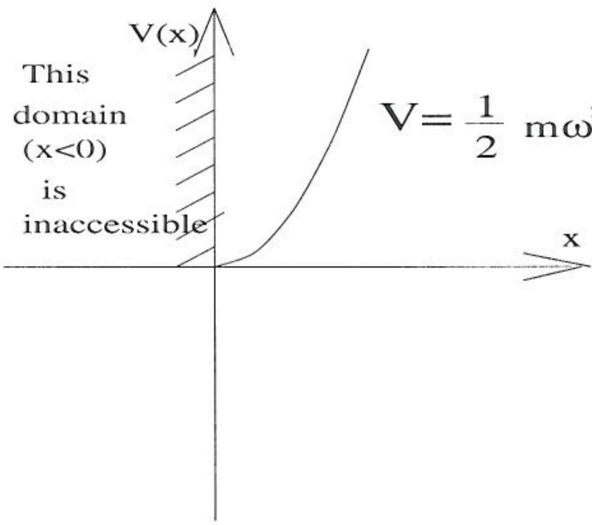
2. The oscillation frequency of atoms in a hydrogen molecule is $\omega = 8.25 \times 10^{14} rad/s$. The distance between the two hydrogen atoms is $d = 0.9 \times 10^{-10} m$.

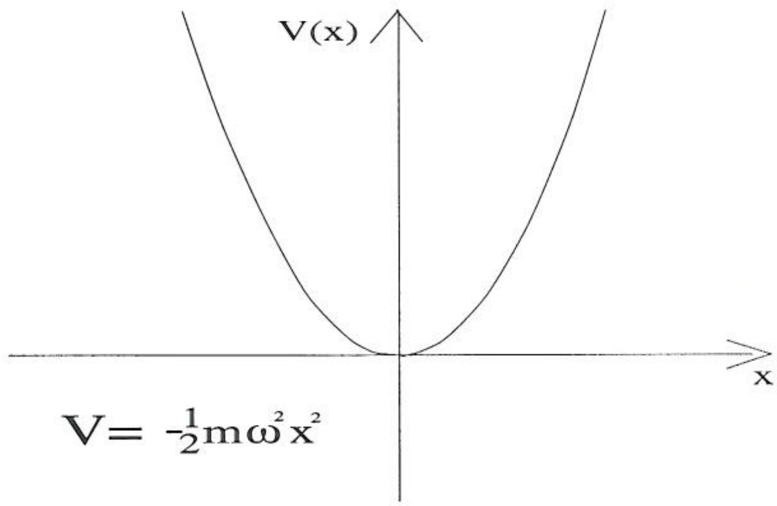
a. Write down the energy levels of both rotation and oscillation for the hydrogen molecule.

b. Find the heat capacitance (in units of J/K and including the contribution from the translational degrees of freedom) for a system consisting of $1kg$, $1m^3$ hydrogen molecules at $T = 20K$ and at $T = 300K$

3. Consider a one-dimensional harmonic oscillator with an infinite wall at $x = 0$ (figure at left below).

At $t < 0$ the system is in the ground state of the potential shown at the left , then, at $t = 0$ the wall suddenly disappears, so that all values of x become accessible (as in the figure at the right)





(a) Write down correctly normalized wave functions for the initial state and for the ground, first and second excited state wave functions of the harmonic oscillator potential shown at the right in the figure.

Find the probability that at $t > 0$ the system is

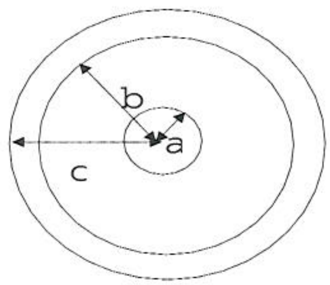
(b) in the ground state of the potential depicted at the right

(c) in the first excited level;

(d) in the second excited level;

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

4. Consider a coaxial cable of infinitely conducting metal. The outer radius is c, the inner radius of the outer conductor is b and the radius of the inner conductor is a:



a) Find the capacitance per unit length of the cable

b) Find the inductance per unit length of the cable.

c) Now suppose that this cable, say of length l is driven at one end by a potential of form (the real part of) $Ve^{j\omega t}$ $(j = \sqrt{-1})$ and is shorted at the other end. Find the impedance $Z = V/i$ of this circuit element in terms of the answers to parts a and b. (Hint: Consider the change in the potential drop and the charge on an infinitesimal length dx of the cable.)

5. The state $|\mu_\nu>$ of a muon neutrino created in the decay of a high energy pion can be written as

$$|\mu_\nu(t=0)>= cos\theta|p, E_1> +sin\theta|p, E_2>$$

where $|p, E_1>$ and $|p, E_2>$ can be regarded as eigenstates of a Hamiltonian with eigenvalues $E_{1,2} = \sqrt{(pc)^2 + (m_{1,2}c^2)^2}$. p is the momentum and $m_{1,2}$ are two slightly different masses. (All quantities are expressed in the laboratory frame.)

a) Write down this state at a later time t.

b)If a measurement is made at time t, calculate the probability that the state observed is the same as the one created at t=0.

c) Suppose that $pc >> m_{1,2}c^2$ and that the measurement is made 800km from the place where the muon is created. How big does the mass difference $\sqrt{|m_1^2 - m_2^2|}$ have to be in order to observe the muon in its original state if its initial energy is 10 GeV?

6. An elastic rod is fixed at one end and subjected to a force F at the other. We use the convention that when $F > 0$ the force is tensile, that is, the rod is stretched. The length L of the rod when $F = 0$ is L_0 and otherwise $F = \kappa(L - L_0)$ Suppose that κ has a negligible temperature dependence but that $L_0(T) = L_{00}(1 + \alpha T)$ where T is the absolute temperature and L_{00} and α are temperature independent. The heat capacity of the rod at fixed length is C_L which we suppose is temperature and length independent.

a) In terms of the specified quantities write expressions for the change in internal energy dU when the length changes by dL and the temperature changes by dT. (You will need to write the change dS in the entropy in terms of dL and dT.)

b) Suppose that, after equilibrating to temperature T in the presence of zero force, the rod is isolated so that no heat can enter or escape it (that is, the process is adiabatic) and the force increased so that the length slowly increases by an amount ΔL. Find the change in the temperature of the rod in terms of the specified quantities.