University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2005 – PART I

Thursday, January 13, 2005 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER**.

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

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Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

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Speed of light in vacuum	c	3.00×10^{8} m/s
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Rydberg constant	R	1.10×10^{7} m^{-1}
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Earth radius		6.37×10^{6} m
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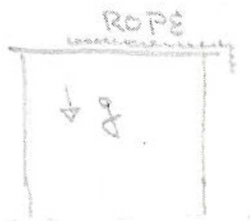
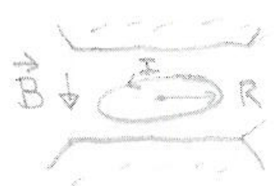
1. 

A 1-meter infinitely flexible rope lies on a (frictionless) table with 1cm of its length hanging over the edge. At t=0 the rope is released (from rest). When will the entire rope leave the table? (The table is more than 1m high!)

2. Consider sodium metal. Its atomic density is 2.65×10^{23} atoms/cc and it has one conduction electron per atom. (a) Write an approximate expression, correct in order of magnitude, for the electronic specific heat C_V per atom, valid in the range of temperatures from 0 to room temperature. Your expression should depend on the variables ρ (atomic density), T, m_e, $\hbar$ and Boltzmann's constant k_B. (b) Evaluate (using your approximate expression) C_V/k_B numerically at room temperature (300 degrees). Also give the value this would have if the electrons in the metal were acting like a classical ideal gas. (c) Provide a brief qualitative explanation of the physical reason why this "classical" C_V/k_B value is much larger than the correct order of magnitude.

3. 

A circular loop of wire of radius R is at rest in a uniform constant magnetic field **B**, perpendicular to the plane of the loop. Calculate the tension in the wire if it carries a small current I. (Ignore gravity and any contribution to the **B**-field due to I.)

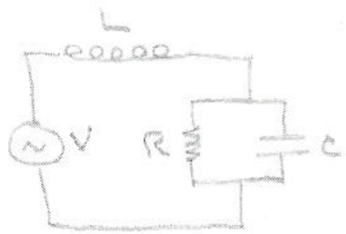
4. Briefly describe what apparatus you would use to measure distances of 1m, 1mm, 1μ, 1nm and 1fm (within a few percent) and briefly describe how the apparatus would function.

5. $\Theta(x)$ is defined by: $\Theta(x) = 1$ for $x>0$ and $\Theta(x) = 0$ for $x < 0$ (do not worry about $x=0$). Show that:

$$\Theta(x) = (1/2\pi)\int e^{izx}/(z-i\varepsilon)\, dz$$

with $\varepsilon>0$, real, and very small. The integral is from $-\infty$ to ∞.

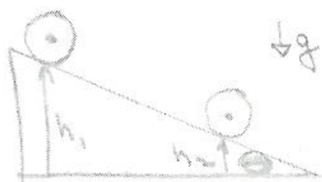
6. Consider an operator H with the properties that $H^\dagger = H$ (hermitian) and $H^4 = I$ (the unit operator). Find the eigenvalues of this operator.

7. 

An oscillator provides a voltage $V(t) = V_0 \sin(\omega t)$ to the circuit diagrammed. (a) Find the (complex) impedance seen by V(t). (b) Find the magnitude and the phase of the current through the inductor at frequency $\omega = 1/(LC)^{1/2}$.

8. An ideal gas consists of atoms of mass M which have only 2 internal states with energies 0 and ε. (a) Write a formula for the average internal energy per particle as a function of the absolute temperature T (ignore the translational energy of the atoms). (b) Draw a graph illustrating your result from part (a) and another graph showing the heat capacity at constant volume as a function of T.

9. When a kitchen stove becomes useless, it is now (very) often moved to the street to await recycling. Estimate the amount of time you would have to bicycle around Minneapolis to spot such a kitchen stove. List your assumptions.

10. 

A bicycle wheel of mass M (assumed to be concentrated at the rim) and radius R is released at height h_1, at t=0 on a ramp inclined at θ. It rolls down without slipping. How long does it take to get to height h_2?

11. The matrix elements of **J**, angular momentum, are often represented by matrices. Suppose we have a set of matrices for **J** and $\mathbf{J}^2$, and that $\mathbf{J}^2$ is diagonal with each element equal to $12\,\hbar^2$. (a) What are the dimensions of the matrices? (b) What are the eigenvalues of the operator

$$(J_x - J_z)/\sqrt{2} \ ?$$

12. A nucleus A is in an excited state with total angular momentum 1 (h) and even parity. The emission of an alpha particle (which has spin 0 and even parity) is energetically allowed if the final state of the daughter nucleus is its ground state (with 0 angular momentum and even parity). This reaction does not occur. Why? [Hint: consider the wave function of the 2-body state.]

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School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2005 – PART II

Friday, January 14, 2005 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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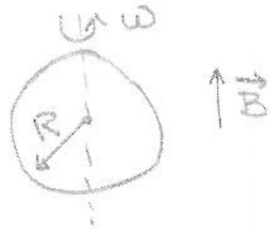
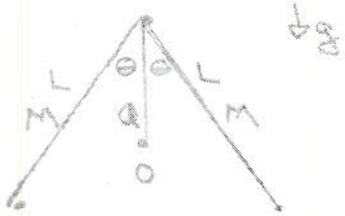
1. Consider a box containing non-interacting bosons with mass m. They are non-relativistic, so that their energy ε is $\mathbf{p}^2/2m$. Show that a container of volume V containing N of these bosons at temperature T will experience Bose-Einstein condensation below temperature T_0. (a) Write an expression for this temperature, T_0. (This may involve a finite dimensionless integral which is not evaluated.) (b) Find the specific heat at constant volume below this transition temperature, T_0. [Hint: T_0 is determined by the condition that the chemical potential is zero there.]

2.



An uncharged normally conducting metal spherical shell of radius R slowly rotates at ω with its axis parallel to a constant magnetic field **B**. (a) Find the electrostatic potential on the sphere. (b) Which multipole is the lowest non-zero contributor to the potential? [Possible hint: consider one of the slowly moving electrons on the sphere.]

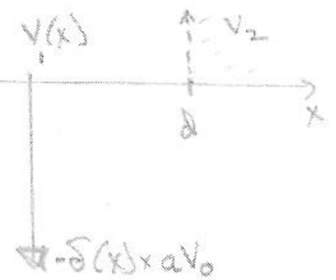
3.



Two uniform rods of length L and mass M are rigidly connected (in a plane) to a massless rod of length a. The fixed angle between each of the rods of length L and the rod of length a is θ. The lower end of the rod of length a is fixed and acts as a pivot for the system at O. (a) Find the conditions on L, M, a, and θ that ensure that the system is stable with respect to small oscillations in the plane about the pivot point O. (b) Find the frequency of these small oscillations.

4.

A mixed beam of pions and kaons (π^+/K^+) with a (common) momentum of 7 GeV/c exists. If 2 thin scintillation counters, each with a time resolution (accuracy) of 0.1 nsec, separated by a distance of L are to be used to identify each particle in the beam as a π^+ or K^+, what separation L is needed? [m_π = 140 MeV/c^2, m_K = 500 MeV/c^2 are exact enough]. Given the (at rest) lifetimes of the π^+ (26 nsec) and K^+ (12 nsec), is this an appropriate technique? Explain.

5. 

A particle of mass m moves in a 1-d potential given by $V(x) = -aV_0\delta(x)$. (a) Find the bound-state energy eigenvalue. (b) If a 2[nd] potential $V_2(x)$ is added to the original potential where $V_2(x) = 0$ for $x<d$ and $V_2(x)= \infty$ for $x \geq d$, find an algebraic equation whose solution determines the new bound-state energy.

6. Consider a 3-dimensional potential well whose potential is $V(\mathbf{r}) = \frac{1}{2}M\omega^2\mathbf{r}^2$. The energy eigenvalues are given by: $E(n_r,l) = \hbar\omega(2n_r+l+3/2)$, for both n_r and l = 0,1,2,3,... The levels are occupied by spin-½ fermions. (a) Sketch the spectrum for n_r ranging between 0 and 2 and l ranging from 0 to 6. Label each level by (n_r,l) and label the level with its degeneracy, assuming that spin-½ fermions occupy the levels. (b) What are the "magic numbers", i.e. the total occupancy up to filled shells, for nuclei with this potential?