University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

SPRING 2006 – PART I

Thursday, January 12, 2006 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER (not your name)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m

Stirling's Approximation:

$\ln(N!) = N\ln(N) - N + \text{(small corrections)}$

Spring 2006 GWE Short Problems

1) Consider a particle which has only two energy states, $E_1 = 0$, $E_2 = \varepsilon$.
   a) Compute the average energy $\langle E \rangle$ of such particle in a reservoir with temperature T.
   b) Calculate the heat capacity C_v of a system of N such non-interacting particles.

2) Find the commutator of the operators of coordinate $\hat{x}$ and kinetic energy $K = \frac{\hat{p}^2}{2m}$ and the corresponding uncertainty relation for Δx and ΔK .

3) Consider a satellite that is initially in a circular orbit around the Sun at the distance of the Earth's orbit ($r = 1$ AU $= 1.50 \times 10^8$ km). A rocket is fired in the direction opposite to its velocity and adds 10 km/s to its velocity. How far from the Sun will the satellite go?

4) Consider a charged, insulating slab that has a thickness $2L$ in the z direction and is very large in the x and y direction that contains a charge per unit volume that varies linearly from $-\rho_0$ to ρ_0 from one side of the slab to the other, i.e., $\rho = \rho_0 z / L$ with z going from $-L$ to L. Find the electric field everywhere inside the slab (magnitude and direction), and the potential difference between the two edges of the slab.

5) You are walking on the ice, in the middle of a large Minnesota lake in winter, when you come across a hole cut straight through the ice, probably by ice fishermen. Looking down the hole, you notice that the water level is 6 cm below the surface of the ice. What is the thickness of the ice covering the lake? Density of ice = 0.9 g/cm^3.

6) An electron in a hydrogen atom does not fall to the proton because of quantum motion (which may be accounted for by the Heisenberg uncertainty relation for an electron localized in the volume with size r). This is true because the value of the Coulomb potential energy goes to minus infinity (with decreasing distance to the center r) relatively slowly, like $-1/r$. For any potential behaving as ($-1/r^s$) is such an "atom" stable against collapse? If not, find the range of values of s for which the "atom" is stable, so that "the electron" does not fall to the center.

7) Estimate the average velocity (in m/s) and the mean free path (in m) of nitrogen molecules in this room.

8) What is the velocity of the recoil of an Fe^{57} nucleus that emits a 100 keV photon, both in units of speed of light and meters per second.

9) A recently discovered effect (called the Quantum Hall effect) measures with the ten digit accuracy "quantum resistance", which is the combination of the charge of the electron e and the Planck constant h.

a) Find this unique combination of e and h and express this resistance in Ohms.
b) Show that fine structure constant $e^2 / [4\pi\varepsilon_0 \hbar c] = 1/137$
c) How many formulas with resistance dimensionality can one write adding the light velocity c to e and h
d) Find the simplest (shortest) of these formulas and calculate the corresponding resistance in Ohms. (This resistance is called "the resistance of vacuum".)

10) A wire cube is welded from 12 identical metallic wires. The resistance of each wire is 1 Ohm. How large is the resistance of the cube measured between contacts attached to the ends of a bulk diagonal?

11) In a plasma with equal concentration of free positive and negative ions the Coulomb potential $e^2 / 4\pi\varepsilon_0 r$ of a point charge e is screened and acquires the Yukawa form $\exp(-\mu r) e^2 / 4\pi\varepsilon_0 r$. Here r is the distance from the point charge to the observation point and μ is the inverse screening radius.
a) Calculate the Fourier transform of the Yukawa potential.
b) What is Fourier transform of the Coulomb potential?

12) A coaxial cable of length l=1 m is made of two thin coaxial copper cylinders with diameters a = 1 cm and b=2 cm separated by air. At one end of the cable the internal and external cylinders are connected by a short wire. At the other end internal and external cylinders are connected to opposite poles of a battery. The current runs on the external cylinder to the opposite end of the cable and then returns back to the battery via the internal cylinder. Calculate the cable inductance L in H (henry).

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GRADUATE WRITTEN EXAMINATION

SPRING 2006 – PART 2

Friday, January 13, 2006 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

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Stirling's Approximation:

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Spring 2006 GWE LONG PROBLEMS

1) Consider a nucleus of ^{235}U, which has 92 protons. Protons are distinguishable from neutrons and both are fermions, so two of each particle can be put into each energy state (spin up, spin down). Assume that attraction of nucleons creates for each of them a large negative constant potential energy -V inside the sphere with the radius of $1.3\ A^{1/3}\ 10^{-15}$ m, where A is the atomic weight. Outside the sphere the nucleon potential energy vanishes.

a) Assuming that both protons and neutrons are ideal gases estimate the Fermi energies of protons and neutrons and the total kinetic energy of ^{235}U nucleons in MeV.

b) Estimate the Coulomb energy of ^{235}U in MeV.

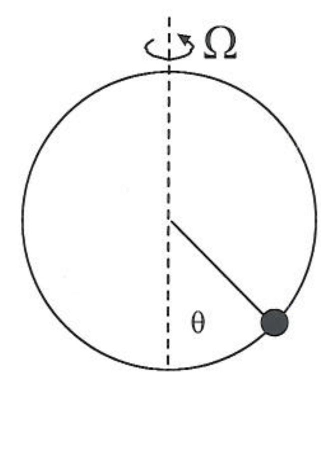
2) In semiconductors electron states of some donors are similar to those of the hydrogen atom because a donor has the same charge as a proton. The only difference is that the Coulomb potential of an electron in the field of a donor $V(r) = -e^2/(4\pi K\varepsilon_0 r)$ contains a large dielectric constant K of a semiconductor and the electron has an effective mass m^* which is usually much smaller than the free electron mass m.

a) Calculate the corresponding ground state energy in eV of a hydrogen-like donor in GaAs ($K = 12.5$ and $m^* = 0.07\ m$).

b) Write the wave function of such a state. Calculate corresponding Bohr radius a.

c) Consider a donor located at the interface $z = 0$ of the semiconductor (GaAs), which occupies half space $z > 0$. We can assume that at $z < 0$ the electron potential energy $V(r) = \infty$, so that an electron cannot penetrate there. On the other hand, at $z > 0$ the electron is still subject to the three-dimensional Coulomb potential $V(r) = -e^2/(4\pi K\varepsilon_0 r)$ (the donor is at $\mathbf{r} = 0$). If we direct the polar axis of the spherical system of coordinates along z this means that the wave function vanishes at $\theta = \pi/2$. Find the ground state energy of such a surface donor using your knowledge of properties of wave functions of excited states of the conventional hydrogen atom.

3) A particle with the mass M is constrained to move (with gravity but without friction) on a circular wire with the radius R rotating with constant angular velocity Ω about a vertical diameter. The position of the particle can be characterized by an angle θ, between the radius vector from the center of the circle to the particle and direction to the "south pole". Find the equilibrium position θ of the particle and the frequency of small oscillations about this equilibrium. Show that the behavior of the system is different above and below a critical angular velocity Ω_c.



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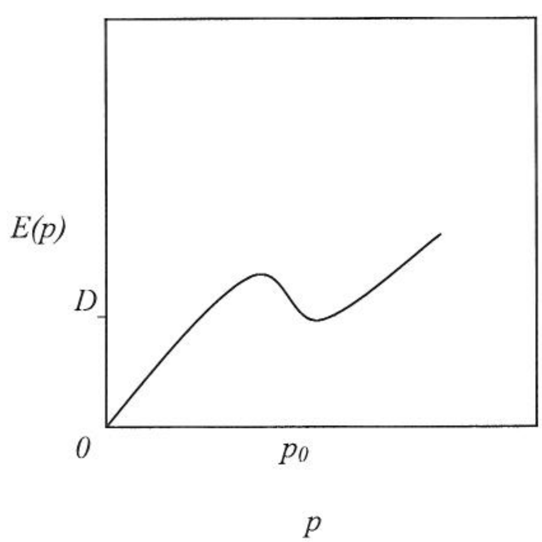
4) A metallic ball with radius R is immersed and suspended in a weakly conducting medium with conductivity σ in the middle of a large metallic vessel (say, salty water in a metallic bathtub).

a) One wire from a battery is attached to the ball and the second wire is attached to the bathtub. Calculate the resistance of the media. (This is how a standard plasma probe test the ionization degree of plasma.)
b) The ball is charged, the battery is disconnected at $t = 0$ and the ball discharges with time. Find how the ball charge Q depends on time and the characteristic time of this discharge writing a simple differential equation for $Q(t)$ (this characteristic time is called the Maxwell time) .
c) If you can not do b) try to estimate the Maxwell time from the point of view of dimensionality analysis.

5) In order to explain the experimental data on the low temperature specific heat of super-fluid helium, Landau (1938) conjectured that the low energy spectrum of its Bose excitations, $E(p)$, has a peculiar form. Namely at $p \ll p_0$ it goes as $E(p) = sp$, reaches a maximum and then goes through a minimum at $p = p_0$, where $E(p_0) = D$. Excitations in the first (linear) part of the spectrum are called phonons. They are similar to acoustic phonons in solids. Excitations in the parabolic minimum near $p = p_0$ at are called rotons.
a) Calculate energy and the specific heat of unit volume of super-fluid helium at very low temperatures. Use some notation for the dimensional integrals if you do not know their values.
b) Formulate strong inequality on temperature, which guarantees that above results are valid



6) A futuristic starship with mass M (including fuel) equal to one million metric tons departs from a base in outer space. The starship is propelled by converting, with 100% efficiency, its fuel into light which is emitted exactly opposite to the thrust. The starship accelerates in a straight line and reaches the cruising speed and then decelerates (also in straight line) reaching its destination near a star in a distant galaxy, which moves very slowly with respect to the ship's home base. On the way back the ship again accelerates to its cruising speed and then decelerates returning to the base. The cruising speed of the starship corresponds to the time dilation (as observed from the base) by 10 times. In other words, at cruising speed its energy is 10 times the rest energy.

a) Find the fraction of mass which reaches cruising speed after first acceleration.
b) What fraction of remaining mass is left after the first deceleration, when it arrives at the distant galaxy?
c) What is maximum mass m of the starship (excluding the fuel) which can make this trip?