University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2007 – PART I

Thursday, January 11, 2007 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m

Stirling's Approximation:

$\ln(N!) = N\ln(N) - N$ + (small corrections)

Short Problems (Best scores from 10 problems)

1. Suppose the electron were to have spin 3/2 instead of spin 1/2. What would then be the atomic numbers Z of the three lowest-mass noble gases, i.e. the equivalents of helium, neon and argon?

2. Assume that the Earth is a uniform rigid sphere. Find, as a function of latitude, the angle that a plumb line hung from a fixed point forms with the local vertical (radial from the center of the earth). How much is it in Minneapolis, latitude ≈ 45°?

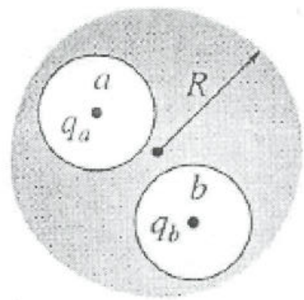
3. Ice on a pond initially has thickness 10 cm. The water just below the ice is at a constant temperature of 0 °C and the air temperature above the ice remains at constant -20 °C. By how much will the ice thickness increase in 1 hour? Assuming that the air temperature stays the same, how will the ice thickness increase with time? Describe the approximations that you make in your calculation.
   Density of ice (ρ) = 0.9 g/cm^3
   Thermal conductivity of ice (κ) = 0.001 *calorie/cm•sec•°C*
   Latent heat of fusion of water (L_H) = 80 *calorie/g*

4. Your job is to design an arrangement of identical polarizing filters (polaroids) so as to rotate the plane of polarization of an initially plane polarized beam of light by 90^0. The polaroids are not perfect, but each absorb 30% of the energy of the light, regardless of its polarization. By numerical inspection find the number N of polaroids that should be used to achieve a maximum final intensity. Hint: There exists no analytic solution. After some physically sound arguments, a simple numeric approach should be used to get the answer.

5. A closed loop of chain with mass m and length L rests horizontally on a smooth frictionless cone with half-angle α. The symmetry axis of the cone is vertical. What is the tension in the chain?

6. What is the back focal length (namely, measured from the second lens) of two thin lenses (having focal lengths f_1, f_2, respectively) separated by a distance d?

7. Two spherical cavities, of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed, call these charges q_a and q_b.



a) Find the surface charge densities σ_a, σ_b, and σ_R.

b) Describe the field outside the conductor.

c) What are the forces on q_a and q_b?

8. Consider a one-dimensional quantum simple harmonic oscillator at frequency ω. At time $t=0$ it is in some arbitrary state $|\Psi(t=0)\rangle \equiv |\Psi_O\rangle$ (do not assume $|\Psi_O\rangle$ is an eigenstate!). Show that at time $t = nT$, where n is an integer and T the period ($T=2\pi/\omega$), one has $|\Psi(nT)\rangle = (-1)^n |\Psi_O\rangle$.

9. An antiproton $\bar{p}$ of kinetic energy 2/3 GeV strikes a proton p which is at rest in the laboratory. They annihilate (reaction: $\bar{p} + p \rightarrow \gamma_1 + \gamma_2$), yielding only two photons which emerge from the reaction traveling forward or backward on the line along which the antiproton entered. Take the rest energy of the proton and the antiproton to be 1 GeV each.

   a) What are the energy and direction of each photon measured in the laboratory?

   b) As measured in the reference frame attached to the incoming $\bar{p}$, what energy does each of the photons have?

10. Two identical ideal gases with the same pressure P and the same number of particles N, but with different temperatures T_1 and T_2, are confined in two vessels, of volume V_1 and V_2, which are then connected. Find the change in entropy after the system has reached equilibrium.

11. With what speed would you have to approach a red light $\lambda_R = 620$ nm in order to have it appear green $\lambda_G = 540$ nm?

12. A rectangular box is divided by a membrane into two equal rectangular compartments. The membrane and the walls of the box are impenetrable for a particle. Initially the particle is in the ground state in one of the compartments and the other compartment is empty. If the membrane is abruptly removed, what is the probability of finding the particle in the ground state?

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GRADUATE WRITTEN EXAMINATION

WINTER 2007 – PART 2

Friday, January 12, 2007 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

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Stirling's Approximation:

$$\ln(N!) = N\ln(N) - N + (\text{small corrections})$$

Long Problems (Best scores from 5 problems)
Note: Some useful integrals are given at the end.

1. A mass $m = 20$ g is free to slide without friction on a wedge of mass $M = 500$ g with the inclination angle $\alpha = 40°$. The wedge itself can slide without friction on the horizontal flat surface of a table. Gravity acts vertically.

   What is the speed of the wedge (relative to the table, of course) when the mass m reaches the table surface after falling a vertical distance of $h = 50$ cm? Assume that both masses are initially at rest. Note that Lagrangian approach may be useful, but not necessary.

2. Consider a one-dimensional oscillator which is described by a position coordinate x and a momentum p, with energy given by

   $$E = p^2/(2m) + \lambda x^4$$

   Suppose this oscillator is in the thermal equilibrium with a heat reservoir at temperature T high enough so that classical mechanics may be used.

   a) What is the mean kinetic energy of the oscillator?
   b) What is the mean potential energy?
   c) What is the mean total energy?
   d) Consider an assembly of weakly interacting particles, each vibrating in one dimension, so that its energy is E as above. What is the specific heat per mole of the particles?

(Hint: It is possible to do the entire problem without doing any integral)

3. Two parallel conducting disks, each of area A, are separated by an air-gap d, are charged with charges $+Q$ and $-Q$.
   a) What is their potential difference?

   Ultraviolet light now shines between the plates, making the air between them into a conductor of total resistance R.
   b) What is the conduction current as function of time?
   c) Because of this current, the charges decrease. What is the electric field between the plates as a function of time?
   d) Describe the magnetic field between the plates due to the sum of the conduction current and the Maxwell displacement current? (Be careful with the signs)

4. A particle of mass m moves in a circular orbit in a hypothetical atom where the force acting on the particle is a "spring" force $F = -kr$ directed towards the center of the atom. Recalling the assumptions made by Bohr in his original derivation of the spectrum of hydrogen, and applying Bohr's method to the present case, derive, in terms of the quantum number n,
   (a) the radii r_n of the allowed orbits, and
   (b) the corresponding energies of these allowed orbits.

5. A particle of mass m moves in a potential $V(r) = -V_0$ when $r < a$, and $V(r) = 0$ when $r > a$. Find the smallest value of V_0 such that there is a bound state of zero energy and zero angular momentum.

6. Considering the effect of gravity and surface tension on water waves, the equation describing the time dependent part of wave motion ($T = A_o \exp(i\omega t)$) can be written as

$$\frac{d^2T}{dt^2} + [(gk + \sigma k^3 / \rho)\tanh(kh)]T = 0$$

Here g: gravity acceleration; σ: surface tension; k: wave number; h: depth of water; ρ: density of water.

a) What is the expression for the angular frequency describing this wave motion?

b) What is the proper approximation for the case of deep water waves?

c) What is the phase velocity for such a deep water wave?

d) What is the group velocity for such a deep water wave?

e) Will the surface tension effect be more important in the short or long wave length limit?

Some useful integrals:

$$\int_0^\infty \exp(-x^p)dx = \Gamma(1/p)/p$$

$$\int_0^\infty x^{n-1}\exp(-x)dx = \Gamma(n),$$ $\Gamma(n)$ is the Gamma function, $\Gamma(n+1) = n \bullet \Gamma(n)$

$$\Gamma(n+1/2) = 1 \bullet 3 \bullet 5 \cdots (2n-1)\sqrt{\pi}/2^n$$

$$\int_0^\infty xdx/[\exp(x)-1] = \pi^2/6. \qquad \int_0^\infty \sin(mx) \bullet \exp(-ax)dx = m/(a^2+m^2).$$