GRADUATE WRITTEN EXAMINATION, JANUARY 2008

First Part — *Best 10 of 12 solutions*

1. Using Gauss's law, show that there are no points of stable equilibrium in any electrostatic field, except exactly at the position of another charge.

2. In many areas of physics one runs into the integral:

$$I = \int_{-\infty}^{\infty} \frac{dx}{x - i\omega} \tag{1}$$

where x and ω are real. Prove that $I = i\pi \mathrm{sgn}(\omega)$, where $\mathrm{sgn}(\omega)$ means the sign of ω.

3. The pressure of water vapor over ice is 3.88 torr at a temperature of $-2°C$ and 4.58 torr at $0°C$. Estimate the heat of vaporization of ice at -1 $°C$. Express your results in Joules/mole.

4. Consider a system for which the relation between pressure, volume and energy is of the form $pV = \alpha E$ where α is some number. Show that the equation of the adiabatic lines in the $p - V$ phase diagram of this system is $pV^{\alpha+1} = C$ where C is an arbitrary constant.

5. Consider a small dental mirror, of the kind dentists use to examine teeth. Typically, when held at about 3 cm from a tooth such a mirror should produce an upright image, enlarged five times. Draw a picture showing mirror, object and image: trace the rays. What is the radius of curvature of the mirror? What kind of a mirror is it? Is the image real or virtual?

6. An electron is traveling at a speed of 3×10^7 m/s. a) If it attained this speed by being accelerated by a constant voltage, what was the voltage? b) If the above speed is known with one percent uncertainty, what is the uncertainty in the electron's position? c) What is the wavelength of the electron?

7. The surface tension of water is about 7×10^{-2} N/m. An open ended glass capillary one millimeter in internal diameter is stuck vertically in water. How far will the water go up the capillary due to surface tension?

8. A radioactive source emits gamma rays (energy ~ 1 MeV) at a rate of 10^5 per second. Assume these gamma rays strike a scintillator and are converted into photons of green light (wavelength ~ 530 nm) at 100% efficiency. The green light then strikes a photocathode made of Cs-Sb which has a work function of 1.5 eV. You measure the current of electrons emitted by the photocathode. Estimate the value of this current.

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Second Part — *Best 5 of 6 solutions*

1. A particle with unit mass situated at a distance a from the origin has an initial velocity v_0 in the azimuthal direction. It moves under the action of an attractive force:

$$f(r) = -k(\frac{4}{r^3} + \frac{a^2}{r^5}) \tag{1}$$

   a) Does the total energy or the angular momentum depend on time?

   b) Define suitable generalized coordinates and write down the Lagrangian describing the motion of this particle.

   c) Show that if $v_0^2 = 9k/(2a^2)$, then the polar equation of the resulting orbit satisfies:

$$(dr/d\theta)^2 + r^2/9 = a^2/9 \tag{2}$$

2. Consider a model of a hydrogen atom in which the electron moves around the proton in a circular orbit of radius $a_B = 4\pi\epsilon_0\hbar^2/me^2$ (the Bohr radius) and orbital angular momentum $mvr = \hbar$. Classically, what fraction of the electron's kinetic energy would be radiated away in one orbit? Express your result in terms of the fine structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c)$.

3. Consider a quantum mechanical one dimensional simple harmonic oscillator of frequency ω. Assume that at time $t = 0$ the wavefunction $\psi(x, 0) = f(x)$ where $f(x)$ is normalizable but otherwise arbitrary (do **not** assume that it is an eigenfunction). Show that the probability density $P(x, t) \equiv |\psi(x, t)|^2$ is periodic in time with period $T = 2\pi/\omega$.

4. In Minneapolis the average annual temperature is $5°C$. Assume the annual variation is sinusoidal with an amplitude of $30°C$. The typical thermal diffusivity of soil is $10^{-3}cm^2/s$. At what depth must water pipes be buried so that they do not freeze in the winter?

5. a) A tuning fork has a frequency $f = 440$ Hz. When the fork is hit impulsively, it takes two seconds for the amplitude to decay to $1/e$ times its initial value. The fork is being driven by a source with a completely flat frequency spectrum. You measure the discrete Fourier transform ("FFT") of the response. What is the minimum number of points required in your discrete Fourier transform to verify from your data both the peak frequency (f) of the spectrum and the linewidth (Δf)?

   b) Consider a compact disk. Assume that the pits used to record one bit of data are 10^{-4} cm on each side and that they cover about a half of the area of the disk, which is 12 cm in diameter (do not worry about the hole). You want to record in stereo (two channels of separate data) and want the CD to faithfully represent music with frequencies up to 12 kHz. Assume that the CD player uses a 16 bit digital to analog converter. How many minutes of music can be recorded?

6. A quantum mechanical system is characterized by a Hamiltonian $H = H_0 + V$ where V is a time-independent perturbation that needs to be considered only to lowest order. At time $t = 0$ the system is known to be in a state $|1\rangle$ which is one of two degenerate eigenstates of H_0 ($|1\rangle$ and $|2\rangle$). The matrix elements of V in the subspace spanned by these two states are $V_{1,1} = V_{2,2} \equiv V_d$ and $V_{1,2} = V_{2,1} \equiv V_o$, where V_d and V_o are known real numbers. Determine, as a function of time, the probability of finding the system in state $|2\rangle$.

9. A composite state is formed from two spin 1/2 fermions. The mass for the composite state is given by:

$$M = m_1 + m_2 + \frac{A}{m_1 m_2}\mathbf{S_1} \cdot \mathbf{S_2} \tag{2}$$

What is the difference in mass between the composite states with $S = 1$ and $S = 0$?

10. The Ψ meson has a mass of 3.1 GeV/c^2 and can decay to a muon pair ($\Psi \rightarrow \mu^- + \mu^+$). The muon has a mass of 0.105 GeV/c^2 and a lifetime $\tau = 2.0 \times 10^{-6}$ s. After decay, what is the average distance traveled by a muon in the rest frame of Ψ?

11. A nasty virus has infected, at random, 1% of the population. A simple screening test has been invented which is found to be 99% accurate, meaning that detailed and expensive medical studies have shown that 99% of those who are indeed infected test positive while 99% of those who are in fact free of the virus test negative. A random individual is given the screening test and the result is positive. What is the probability that this person is actually infected by the virus? How useful is the screening test?

12. A set of data is fit with a quadratic function, resulting in the figure shown. a) Reading the error bars from the plot, estimate the reduced χ^2 for this fit. b) Is it a good fit? Assume that the errors are statistical and have been estimated properly.