University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2000 - PART I

Friday, January 14, 2000 - 9:00 AM - 12:00 NOON

Part I of this exam consists of 15 problems of equal weight. You are required to do all of these questions: therefore, you should plan on spending an average of about 12 minutes for each question.

This is a closed book examination. You may use calculators. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

Please put your **CODE NUMBER** (not your name) in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant **PROBLEM NUMBER. BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER,** so that no sheet contains work for more than one problem. Use only one side of the paper: If some problems require more than one sheet for their solution, then be sure to indicate "page 1", "page 2",... etc. under the problem number entered on the sheets. All your completed problems should be put in the manila envelope provided.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Permittivity constant	ε_o	8.85×10^{-12} F/m
Permeability constant	μ_o	1.26×10^{-6} H/m
Electron charge to mass ratio	e/m_e	1.76×10^{11} C/kg
Proton rest mass	m_p	1.67×10^{-27} kg
Ratio of proton mass to electron mass	m_p/m_e	1840
Neutron rest mass	m_n	1.68×10^{-27} kg
Muon rest mass	m_μ	1.88×10^{-28} kg
Planck constant	h	6.63×10^{-34} J•s
Electron Compton wavelength	λ_c	2.43×10^{-12} m
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Standard atmosphere		1.01×10^{5} N/m^2
Faraday constant	F	9.65×10^{4} C/mol
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m^2•K^4
Rydberg constant	R	1.10×10^{7} m^{-1}
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Bohr radius	a_o	5.29×10^{-11} m
Electron magnetic moment	μ_e	9.28×10^{-24} J/T
Proton magnetic moment	μ_p	1.41×10^{-26} J/T
Bohr magneton	μ_B	9.27×10^{-24} J/T
Nuclear magneton	μ_N	5.05×10^{-27} J/T
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m
Earth-Moon distance		3.82×10^{8} m
Mass of Earth		5.98×10^{24} kg

GWE Winter 2000 - PART I

1. A satellite of mass $m = 500$ kg is in a circular orbit at an altitude $h = 150$ km above the Earth's surface. As a result of air friction, the satellite's orbit degrades. Protected by a heat shield, the satellite eventually impacts with a velocity 2 km/s. How much energy (in Joules) was released as heat in the process ?

2. What is the velocity of recoil of an Fe^{57} nucleus that emits a 100 keV photon, both in units of the speed of light in vacuum and in meters per second.

3. A resistance R and an inductance L are connected in series, and an alternating voltage $V_0 \cos \omega t$ is impressed across the combination. The resulting steady state voltage across the resistance can be written as $V_R \cos(\omega t + \beta)$. Find V_R and β, assuming both V_0 and V_R to be positive.

4. Find the magnetic flux through a square loop of side a due to a current I in a long straight wire. The geometry is as follows: the wire is coplanar with the loop, and runs parallel to the loop's closest side, at a distance b away. Write your result as a formula in SI units.

5. Estimate the energy released by the explosion of one ton of TNT, in electron-volts, to within an order of magnitude or so. Make sure you clearly spell out all of the assumptions you make in arriving at your estimate !

6. By actually evaluating the integral, show that

$$\int_0^\infty dx \frac{\cos x}{1+x^2} = \frac{\pi}{2e}$$

7. The drag force on a very high speed object of area A, passing through a gas of density ρ at a velocity v is expected to be of the form

$$\text{Force} \sim A^r \rho^s v^t$$

Determine the value of the exponents r, s and t.

8. The Fermi energy in metallic sodium corresponds to a temperature of approximately 4×10^4 K. Roughly, what fraction of the electrons are thermally excited at 10 K, and what contribution does this make to the heat capacity C_v of the metal ?

9. For waves in shallow water, the relation between frequency ν and wavelength λ is

$$\nu = \left(\frac{2\pi T}{\rho \lambda^3} \right)^{1/2}$$

where ρ and T are the density and surface tension of water. What is the group velocity of these waves ?

10. Find the eigenvalues and corresponding eigenvectors (which need not be normalized) of the following matrix,

$$M = \begin{pmatrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & -1 \end{pmatrix}$$

11. The mean free path of a 1 MeV neutrino in iron (A=56) is 0.2×10^{18} m. Given that the density of iron is 7.9 g/cm^3, what is the neutrino-nucleus cross-section at this energy ?

12. Suppose the electron were to have spin 3/2 instead of spin 1/2. What would then be the atomic numbers Z of the three lowest-mass noble gases, i.e. the equivalents of helium, neon and argon ?

13. Give a numerical value for each of the following, in appropriate units:

(a) the diffraction-limited resolution of a one-meter diameter telescope, in seconds of arc.
(b) the kinetic energy in eV of a 300 K thermal neutron.
(c) the mass difference, including sign, between the neutron and the proton, in MeV/c^2.
(d) the magnetic moment of the nucleus Pb^{208} (A=208, Z=82) in nuclear magnetons.

14. The Lyman-α transition in atomic hydrogen has a wavelength λ = 121.5 nm, and a transition rate of 0.6×10^9 sec^{-1}. Estimate the minimum value of $\Delta\lambda/\lambda$.

15. In atomic hydrogen, the hyperfine interaction gives rise to a splitting between the total spin (nuclear+electronic) F = 1 and F = 0 states, which leads to a transition characterized by the astrophysically famous 21 cm line. At what temperature of an atomic hydrogen gas cloud will the three F = 1 states have a total population equal to that of the F = 0 ground state ?

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GRADUATE WRITTEN EXAMINATION

WINTER 2000 - PART II

Saturday, January 15, 2000 - 9:00 AM - 1:00 PM

Part II of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

This is a closed book examination. You may use calculators. A list of some physical constants and properties that you may require is included: Please take a moment to review its contents before starting the examination.

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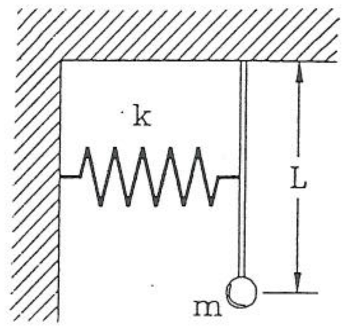
GWE Winter 2000 - PART II

1. Find the period T of small oscillations about equilibrium of the system shown in the diagram: a spring (of spring constant k) is attached to the mid-point of a light rigid rod of length L, at the end of which is fixed a mass, m. The spring is relaxed when the rod (whose mass can be neglected) is vertical, as shown.



2. The equation of state of a rubber band is

$$f = aT\{(\ell/\ell_o) - (\ell_o/\ell)^2\}$$

where T is the absolute temperature, f is the tension, and ℓ is the length. Use the following numerical values for the overall constant, $a = 1.3 \times 10^{-2}$ Newton/K, and for the unstretched length, $\ell_o = 1.0$ m.

(a) Show that, in fact, for the above equation of state,

$$\left(\frac{\partial U}{\partial \ell}\right)_T = 0$$

thereby establishing that U is a function of temperature only. You may assume this result for parts (b) and (c).

(b) Suppose the band is stretched reversibly and isothermally from its unstretched length of 1.0 m to a length of 2.0 m, at a temperature $T = 300$ K. Find the work done on the band and the heat absorbed or released by it (both in Joules).

(c) If the band had been stretched reversibly and adiabatically instead, what would its final temperature have been ? Assume that the heat capacity at constant length takes the value $C_\ell = 1.2$ Joule/K.

3. A quantum-mechanical model in which "mesons" are described as bound states of a quark and an anti-quark, each with spin 1/2, leads to a Hamiltonian for the mesonic state of the form

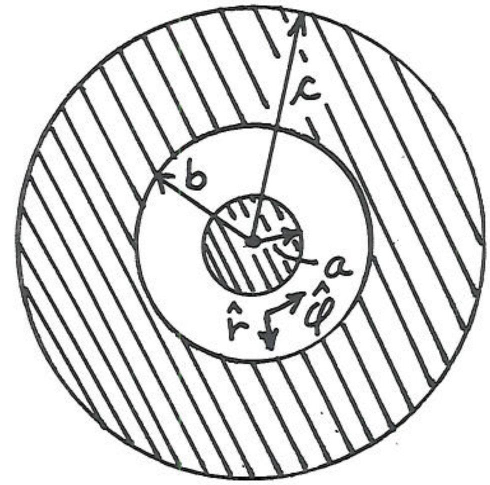
$$H = A + B\vec{L} \bullet \vec{S} + C\vec{s}_1 \bullet \vec{s}_2$$

where A, B and C are constants, $\vec{s}_1$ and $\vec{s}_2$ are the spins of the quark and antiquark respectively, $\vec{S}$ is the total spin and $\vec{L}$ is the orbital angular momentum of the quark-antiquark system: all of these are expressed in units of $\hbar$.

(a) Derive a general formula expressing the energy of a "meson" state with quantum numbers (J,L,S).

(b) Using the formula derived in (a), write down the energies for the specific cases of "mesons" in the $^1S_0$, $^1P_1$ and $^3P_2$ states.

4. A coaxial cable has an inner conductor of radius a, an air gap between $r = a$ and $r = b$, and another conductor from $r = b$ to $r = c$, as shown. The inner conductor is at a voltage V_0 with respect to the outer conductor, and the inner conductor carries a current I_0 (flowing in the $-z$ direction, into the page) which is returned by the outer conductor.



(a) Calculate the magnitude <u>and</u> direction of the electric field $\vec{E}$ and of the magnetic field $\vec{B}$ in the air gap, i.e. in the region $a<r<b$.

(b) Calculate the magnitude <u>and</u> direction of the Poynting vector $\vec{S}$, in the air gap, i.e. in the region $a<r<b$.

(c) Calculate the integral of $\vec{S}$ in the air gap over the surface of a plane perpendicular to the cable. What is the physical meaning of this quantity ?

5. A spaceship of proper length $L_0 = 600$ m is moving directly away from the Earth with uniform velocity. A light pulse is sent out from the Earth and is reflected from mirrors at the back end and at the front end of the spaceship. If the first reflected pulse is received back at Earth base 200 seconds after emission, while the second reflected pulse is received 17.4 microseconds later,

(a) How far was the spaceship from Earth when the light pulse sent from Earth reached its back end (in m) ?

(b) What is the velocity of the space-ship, in units of the speed of light ?

6. Consider a system of N independent <u>two</u>-dimensional simple harmonic oscillators, each of which has $(n+1)$-fold degenerate energy levels given by

$$\varepsilon_n = (n+1)\hbar\omega$$

with $n = 0,1,2,...$

(a) Calculate the partition function Z and internal energy U for this system.

(b) Use your result in (a) to calculate the heat capacity C_V of this system.

(c) What are U and C_V in the high-temperature limit $k_B T >> \hbar\omega$? You may get the answer either by direct use of the result in (a) and (b), or by appealing to some other argument.