The University of Minnesota.

School of Physics and Astronomy

Graduate Written Exam, Part 1

Thursday January 12th 2012

1. For a many particle system of weakly interacting particles will quantum effects be more important for **a)** high densities or low densities and **b)** high temperatures or low temperatures for a system. Explain your answers in terms of the de Broglie wavelength λ defined as $\lambda^2 \equiv h^2/(3mk_BT)$ where m is the mass of the particles and k_B Boltzmann's constant.

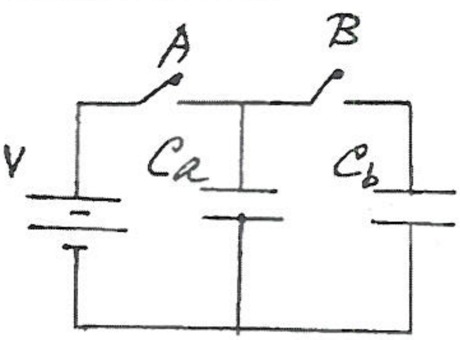
2. The ground state energy of Helium is -79 eV, what is its ionization energy, which is the energy required to remove just one electron?

3. It is known that the force per unit area (F/A) between two neutral conducting plates due to polarization fluctuations of the vacuum (namely, the Casimir force) is a function of h (Planck's constant), c (speed of light) and z (distance between the plates) only. Using only dimensional analysis, obtain F/A as a function of h, c, and z.

4) In the circuit diagram opposite, initially the two identical capacitors with capacitance C are uncharged. The connections between the components are all made with short copper wires. The battery is an ideal EMF and supplies a voltage V.

a) At first Switch A is closed and Switch B is kept open, what is the final stored energy on capacitor C_a?
b) Switch A is opened and afterwards Switch B is closed. What is the final energy stored in both capacitors?

c) Provide a physical explanation for any difference between the results of parts **a)** and **b)**, if there is one.



5) A planet of mass m moves around the sun, mass M, in an elliptical orbit with minimum and maximum distances of r_1 and r_2, respectively. Find the angular momentum of the planet relative to the center of the sun in terms of these quantities and the gravitational constant G.

6) A particle moves in a circular orbit under the influence of a central force that varies as the n-th power of the distance. Show that this motion is unstable if $n < -3$. (Hint: Consider the centrifugal potential.)

7) A classical, ideal monatomic gas of N particles is reversibly compressed *isentropically*, *i.e.* with the entropy kept constant, from an initial temperature T_0 and pressure P to a pressure $2P$. Find **(a)** the work done on the system, and **(b)** the net change in entropy of the system and its surroundings.

8) For an ideal Fermi gas of N neutral spin-½ particles in a volume V at $T = 0$, calculate the following:

- **(a)** The chemical potential
- **(b)** The average energy per particle
- **(c)** The pressure

9) A transparent spherical lens, with a uniform index of refraction n, has the image of a distant object on the opposite surface of the lens when it is in a vacuum. What is the index of refraction n?

10) A piece of p-doped silicon has a carrier density $n = 10^{15}$ cm^{-3} and dimensions $\Delta x = 10$ mm, $\Delta y = 2$ mm, $\Delta z = 1$ mm. A magnetic field $B_z = 1$T is applied in the z-direction and a current $I_x = 1$A flows in the x-direction, and the voltage V_y is measured.

- **a)** Express the current density j_x in terms of the carrier density n and the carrier velocity v_x.
- **b)** Write down the equilibrium force condition that determines V_y.
- **c)** Find V_y in volts.

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Graduate Written Exam, Part 2

Friday January 13th 2012

Answer as many questions as you can.

1) An electron in a hydrogen atom occupies a state:

$$|\psi\rangle = \sqrt{(1/3)}\,|3,1,0,+\rangle + \sqrt{(2/3)}\,|2,1,1,-\rangle$$

where the properly normalized states are specified by the quantum numbers $|n,\ell,m,\pm\rangle$, where the '±' specifies whether the spin is up or down.

a) What is the expectation value of the energy in terms of the ground state energy?

b) If you measured the expectation values of the orbital momentum squared, $\langle L^2\rangle$ the square of the spin $\langle S^2\rangle$, and their z-components $\langle L_z\rangle$ and $\langle S_z\rangle$ what would be the result?

c) Show that, if you measure the position of the electron, the probability density for finding it at an angle specified by θ and ϕ integrated over all values of r, is independent of θ and ϕ Note: for this part you will need: $Y_1^0 = (3/4\pi)^{1/2}\cos\theta$ and $Y_1^1 = -(3/8\pi)^{1/2}\sin\theta\exp(i\phi)$ You do **not**, however, need to know the radial functions, only that they are properly normalized and orthogonal to each other.

d) List all additional possible states that are degenerate with the first state in the linear combination above. Note: this part can be done even if you have not answered the previous parts.

Assume now that the state $|\psi\rangle$, given above, is the initial state of an electron in a hydrogen atom.

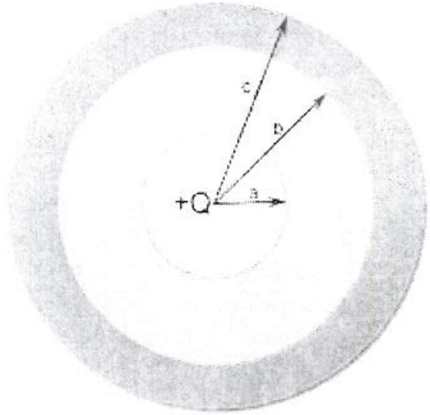
e) Write down the electron's state as a function of time for all t > 0.

f) Go through the results you obtained in parts (a) through (c) and determine which of them are time independent.

2) The Heat Equation $\dfrac{\partial T}{\partial t} = \dfrac{\lambda}{c\rho}\dfrac{\partial^2 T}{\partial x^2}$ describes the flow of heat in one dimension, where λ is the thermal conductivity, ρ is the density and c is the specific heat. Derive the dispersion relation for harmonic solutions that obey this equation. For harmonic seasonal variations in surface temperature, described by $T = T_0 + T_1\cos(\omega t)$, where $\omega = 2\pi$ radians per year, at what depth will the temperature phase lag be 6 months if $\lambda = 1$ W/m·K, $c = 10^3$ J/kg·K and $\rho = 2\times10^3$ kg/m^3?

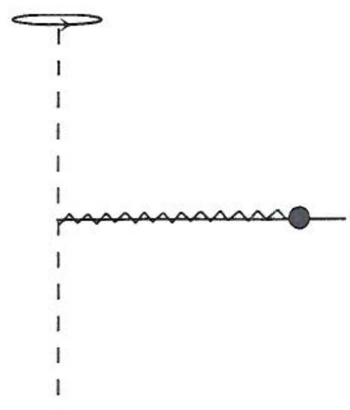
It may be useful to remember that $\sqrt{i} = \dfrac{1}{\sqrt{2}}(1+i)$

3) A charge $+Q$ is located at the center of a concentric hollow cavity (radius a) inside a spherical conductor with radius b. The conductor is surrounded by a linear dielectric material of permittivity ε out to radius c.



a) Derive expressions for electric fields in the four regions: $0 < r < a$, $a < r < b$, $b < r < c$, and $r > c$.
b) Find the potential at $r = a$ (relative to the potential at infinity).
c) Show that the polarization density in the dielectric can be expressed as $\vec{P} = [(\varepsilon - \varepsilon_o)\,Q/(4\pi\varepsilon r^2)]\,\hat{r}$.
d) What is the bound charge in the dielectric?
e) What is the bound surface charge (at $r = b$ and $r = c$) in the dielectric?

4) A bead of mass m slides on a straight wire. The wire rotates with an angular frequency ω around an axis perpendicular to the wire. The bead is connected to the axis by a spring of natural length L and spring constant k.



a) Give the Lagrangian of the system.
b) Derive the equation of motion of the bead.
c) Under what conditions will the bead have a stable equilibrium position along the wire?

5) Consider N non-interacting, stationary particles, each with magnetic moment $\vec{\mu}$ at temperature T in a uniform external magnetic field $\vec{B}$. Their energy is $-\vec{\mu}.\vec{B}$. Calculate the partition function (Z), the internal energy, and magnetization for two distinct cases (a and b below):

a) The magnetic moment of each particle can be oriented only parallel or anti-parallel to the magnetic field;
b) The magnetic moment of each particle can rotate freely.
c) Show that, in both cases, the total magnetization, $\vec{M}$, can be written as a derivative of the partition function.
d) In each case, calculate the fluctuations of the magnetization $\langle (\Delta\vec{\mu})^2 \rangle$.