University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2013 – PART I

Thursday, January 17, 2013 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	m_ec^2	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	m_pc^2	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	m_nc^2	940 MeV
Planck constant	h	6.63×10^{-34} J•s
Gravitational constant	G	6.67×10^{-11} m^3/s^2•kg
Molar gas constant	R	8.31 J/mol•K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius		6.37×10^{6} m
Earth-Sun distance		1.50×10^{11} m

Stirling's Approximation:

$\ln(N!) = N\ln(N) - N$ + (small corrections)

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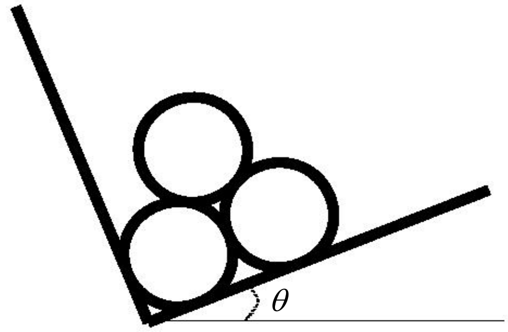
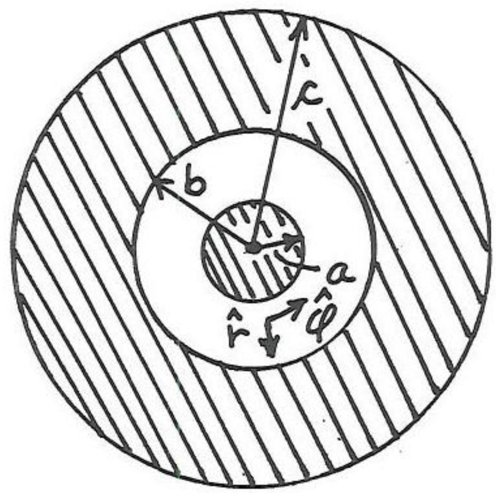


1. Three perfectly cylindrical frictionless oil pipes are packed inside an inclined railway car as shown on the figure. Find the range of angles θ, where such a packing is stable.

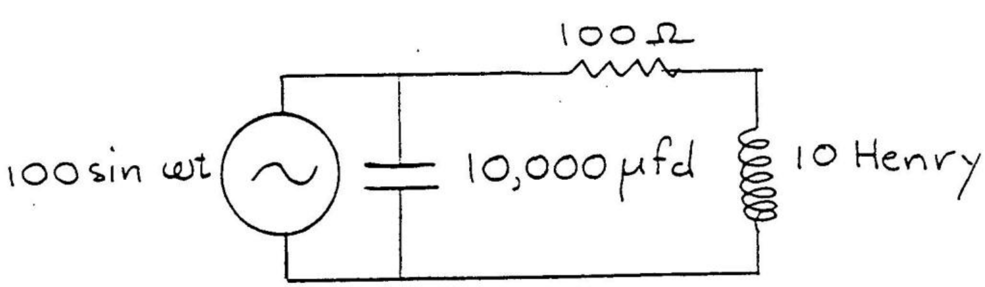
2. An "air molecule" at room temperature and atmospheric pressure is moving with the average speed of 450 m/sec. It travels about 7×10^{-6} cm between collisions. Estimate how much time it takes for a molecule to travel 1cm?

3. A particle of mass m, moving with velocity v, crosses a boundary between the region where its potential energy is equal to U_1 to a region where its potential energy is equal to U_2. Derive a "Snell's law" of refraction for this boundary.

4. A coaxial cable made of ideal conductor (no resistance) has an inner cylindrical conductor of radius a, and air gap between $r = a$ and $r = b$, and another conductor between $r = b$ and $r = c$, as shown. The inner conductor is connected to a voltage source at one of its ends such that its voltage with respect to the outer conductor is V_0. The inner conductor carries a current I_0 in the $-z$ direction (into the page) which is returned along the outer conductor via a resister which connects the inner and outer conductors at the other end. Calculate the magnitude and direction of the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ in the region in the air gap where $a < r < b$.



5. An ideal voltage generator with output voltage $V = V_0\sin(\omega t)$ where $V_0 = 100$ Volts is connected to the circuit shown below. Calculate the time-averaged power dissipated in the 100 Ω resistor as a function of the generator frequency ω.



6. A beam of monoenergetic π^+ of kinetic energy T = 140 MeV and rest mass m_0 = 140 MeV/c^2 is to traverse a total flight path of length D = 20 m. Calculate the fraction of pions that survive the 20 m trip, provided that the mean lifetime of charged pions is $\tau = 2.6 \times 10^{-8}$ sec (in the rest frame of the pion).

7. Some organic molecules have a spin triplet (S = 1) excited state at an energy $k_B\Delta$ above a singlet (S = 0) ground state. Find an expression for the magnetic moment, $\langle\mu\rangle$, in a field B. What is the susceptibility at high temperature?

8. Two identical perfect gases with the same pressure P and the same number of particles N, but with different temperatures, T_1 and T_2 are confined in two vessels, of volume V_1 and V_2 that are then connected. Find the change in entropy after the system has reached equilibrium.

9. A charged particle is residing in an infinite one-dimensional square well potential.
    a. Write down an expression for the matrix element of the electric dipole moment for the particle when making a transition from one quantized level to another.
    b. From consideration of your answer in part (a), what would be the corresponding selection rules governing the allowed transitions for this particle? Provide a physical justification for your answer.

10. Consider a diatomic molecule of two dissimilar nuclei that have the following properties: a reduced mass μ = 20 m_p (where m_p is the mass of a proton), interatomic spacing, $r_0 = 3.0\times10^{-10}$ m, and the force constant, $C = 8\times10^{18}$ eV/cm^2.
    a. What is the energy difference, in eV, between the r = 0 and r = 1 rotational levels for this molecule in the vibrational ground state?
    b. What is the energy difference, in eV, between the $\nu = 0$ and $\nu = 1$ vibrational levels for this molecule? [Assume that the rotational state does not change.]

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GRADUATE WRITTEN EXAMINATION

WINTER 2013 – PART 2

Friday, January 18, 2013 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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Stirling's Approximation:

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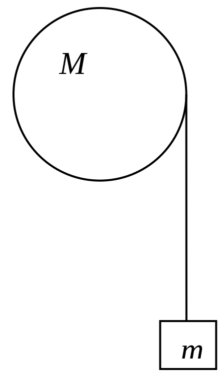
1. A uniform cylindrical drum of mass M and radius R is free to rotate about its axis, which is horizontal. An elastic cable with spring constant κ and negligible mass is wound on the drum. (In real life, κ will decrease as the cable unwinds, but ignore this effect and assume that it is a constant.) On its free end, it carries a mass m, which is allowed to fall down unwinding the elastic cable.



   (i) Write down a Lagrangian function in terms of the drum rotation angle θ and the vertical displacement x of the mass m. Derive the corresponding equations of motions.
   (ii) Find proper normal modes and determine oscillation frequency (or frequencies) of the system.

2. An electron is oscillating in a simple harmonic oscillator potential with an angular frequency $\omega = 10^{15}$ rad/sec and amplitude $x_0 = 10^{-10}$ m.
   (i) Calculate the amount of energy radiated per cycle. If you don't remember the radiation equation, you may want to think about what quantities should be included, and dimensional analysis may be useful.
   (ii) What is the ratio of the radiated energy per cycle to the average mechanical energy?
   (iii) How long will it take the system to radiate away half of its energy?

3. An engine, that uses a photon gas as the working substance, operates in accordance with the Carnot cycle. The energy of a photon gas is given by Stefan-Boltzmann law $U = \alpha V\tau^4$, and the entropy, σ, is given by $4U/3\tau$, where U is the energy, α is the Stefan-Boltzmann constant, V is the volume of the gas, and τ is the temperature.

   Given τ_h, τ_l, the high and low temperatures of the cycle, and starting with isothermal compression at (V_l, τ_l) calculate the work done by the gas for each stage of the cycle and compute the total work. Use this information to calculate the efficiency of the engine.

4. Consider a one-particle system capable of three states ($-\varepsilon$, 0, and ε) in thermal contact with a reservoir at temperature τ. Find:

   (i) Partition function
   (ii) Average Energy
   (iii) Heat capacity at constant volume
   (iv) Entropy
   (v) Free energy

   In addition, find the leading temperature dependence of (ii), (iii), and (iv) when $\tau \gg \varepsilon$ and $\tau \ll \varepsilon$.

5. The nucleus of a hydrogen atom isotope of mass 3 is radioactive, and changes suddenly into a helium nucleus of mass 3 with the emission of an electron that escapes the nucleus. If the initial hydrogen atom was in its ground state, what is the probability that the single-electron helium ion formed by this radioactive decay is in the 1s state? Which other state(s) will it be found in, other than the 1s state?
Useful information:

$$\psi_{nlm} = R_{nl}(r)Y_l^m(\theta,\varphi); R_{10}(r) = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-\frac{Z}{a_0}r}, \int_0^\infty e^{-x}x^n dx = n!.$$