University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Spring 2019 - PART I

Thursday, May 30th 2019 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a non-programmable, scientific calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	$m_e c^2$	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	$m_p c^2$	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	$m_n c^2$	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	$\ln(N!) = N\ln(N) - N +$ (small corrections)	

Problem 1

Consider the following one-dimensional Hamiltonian

$$H = \frac{p^2}{2m} - a\left[\delta(x+b) + \delta(x-b)\right] ,$$

where a is a positive constant.
a) Find a discrete symmetry for this system, and state its implications for the eigenfunctions of the Hamiltonian.

b) How many bound states are there in this problem? Assume that the two delta functions in the potential are far apart. Sketch the wavefunction(s) for the bound state(s) and comment on any symmetry properties. [*Hint:* You may assume that an attractive delta-function potential in one dimension produces a single bound state.]

c) Consider the normalized wave function $\psi(x) = \frac{1}{\sqrt{\lambda}} e^{-|x|/\lambda}$ where λ has dimensions of length. Use this wave function to compute the expectation value of the energy of the system when $b = 0$.

Problem 2

A quantum mechanical model in which "mesons" are described as bound states of a quark and anti-quark, each with spin $\frac{1}{2}$, leads to a Hamiltonian for the mesonic state of the form

$$H = A + B\vec{L} \cdot \vec{S} + C\vec{s}_1 \cdot \vec{s}_2 ,$$

where A, B, C are constants, $\vec{s}_1, \vec{s}_2$ are the spins of the quark and antiquark, respectively, $\vec{S} = \vec{s}_1 + \vec{s}_2$ is the total spin, and $\vec{L}$ is the orbital angular momentum of the quark-antiquark system. All angular momenta are expressed in units of $\hbar$.

a) Derive a general expression for the energy of a "meson" state with quantum numbers J, L, S, where $\vec{J} = \vec{L} + \vec{S}$ is the total angular momentum with associated quantum number J.

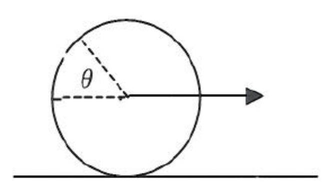
b) Using the formula derived in a) write down the energies for the specific cases of "mesons" in the $^1S_0$, $^1P_1$ and $^3P_2$ states. Note the spectroscopic notation: $^{2S+1}L_J$.

Problem 3

Raindrops of mass m are falling vertically with speed v, and with a flux density of n drops per unit area per unit time. An athlete runs through this rain on a horizontal track with speed w. Her cross-sectional area perpendicular to her velocity is A. Assume that every drop she intersects, sticks to her. At what rate will she accumulate mass, and what retarding force will the rain exert on her? You can neglect the rain that falls on her head and shoulders. [*Hint:* Consider the volume swept out per unit time by the area A. The flux density equals the number density (drops per unit volume) times the speed of the drops.]

Problem 4

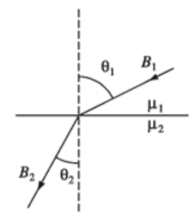
Mud is thrown tangentially from various points on the rim of a circular wheel (radius R) of a bicycle travelling along a horizontal road at a uniform speed v ($> \sqrt{gR}$). The wheel rolls without slipping. Neglecting any dissipative effects, show that none of the mud can rise to a height greater than



$$R + \frac{v^2}{2g} + \frac{gR^2}{2v^2} .$$

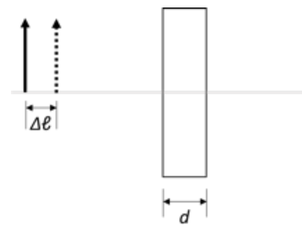
Problem 5

The magnetic field lines bend at the interface between two linear magnetic materials. Assuming there are no free currents at the boundary, derive the relation between angles θ_1 and θ_2 as a function of the magnetic permeabilities μ_1 and μ_2.



Problem 6

An object viewed through a glass slide of thickness d and refractive index n appears to be closer to the observer by a distance $\Delta\ell$ than its actual location. Determine the shift in distance $\Delta\ell$ as a function of d and n. Assume that the object is small and located far from the glass slide so that the small-angle approximation can be used to simplify the calculation.



Problem 7

A current loop of radius R is located in the $x-y$ plane, centered around the origin, and carries a current, I. What is the magnetic field at an arbitrary point $\vec{r}_f = (x, 0, z)$ in the $x-z$ plane? (Note: Your answer can be left as an integral.) Evaluate your answer when $x = 0$, and explain why it makes sense.

Problem 8

At what temperature will a non-interacting 3D gas of nonrelativistic fermions with number density n have zero chemical potential? Express your answer in terms of the Fermi energy ε_F, i.e. the chemical potential of the same gas at zero temperature. [*Hint:* The following integral may be useful: $\int_0^\infty dx \frac{\sqrt{x}}{e^x+1} = 0.678$.]

Problem 9

One question in early planetary science was whether each of the rings of Saturn was solid or was, instead, composed of individual chunks, each in its own orbit. Let the radial width of a given ring (there are many) be Δr, the average distance of that ring from the center of Saturn be represented by R, and designate the average speed of that ring by $\bar{v}$. The difference in speed between its outermost and innermost portions can then be written as $\Delta v = v_{out} - v_{in} = \alpha\,\bar{v}\frac{\Delta r}{R}$, where $v_{out}(v_{in})$ is the speed of the outermost (innermost) portion, and α is a constant. Determine the value of α when the ring is **a)** solid, and **b)** composed of many small chunks (assuming $\Delta r \ll R$).

Problem 10

A spherical satellite of radius r, painted black, travels around the sun at a distance D from its center. The sun, a sphere of radius R, radiates as a blackbody at a temperature $T_0 = 6000$ K. Obtain an expression for the equilibrium temperature of the satellite, assuming there is a uniform temperature across the entire surface of the satellite. What is the satellite temperature if $D = 1.6 \times 10^{11}$ m? (Assume the solar radius is $R \simeq 7 \times 10^8$ m.)

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GRADUATE WRITTEN EXAMINATION

Spring 2019 - PART II

Friday, May 31st 2019 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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Problem 1

Consider a particle of mass m which can only move along the x axis, constrained to lie in a one-dimensional box of size L. Take the box to have one edge at $x = 0$ and the other at $x = L$.

a) Solve for the spectrum and wavefunctions of the particle starting from the Schrodinger equation.

b) Next, suppose that there is a delta-function potential spike in the middle of the box given by

$$V = a\,\delta(x - L/2)\ ,$$

where a is a positive constant. Treating this potential as a time-independent perturbation, evaluate the first-order energy shift, and determine the condition on a for your perturbative calculation to be a good approximation for the ground state.

c) Suppose that at time $t = -\infty$ the system is in the ground state of the unperturbed Hamiltonian ($V = 0$). Let the perturbation be given by

$$V = a\delta(x - L/2)e^{-t^2/T^2}\ ,$$

where a is a positive constant. Working to first order in time-dependent perturbation theory, deduce the transition probabilities to go to the first excited energy level and the second excited energy level at $t = \infty$. (Note: You do not have to derive the basic formula for the transition rate.) The following integral identity may be useful: $\int_{-\infty}^{+\infty} dy\, e^{-\alpha y^2 + \beta y} = e^{\beta^2/(4\alpha)}\sqrt{\pi/\alpha}$.

Problem 2

A half-cylinder of radius R and mass M rests on a rough, horizontal surface as shown in the above figure. Calculate the frequency of small amplitude, rolling oscillations about the equilibrium position.

Problem 3

A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density $\rho(r)$ is given by:

$$\rho(r) = \begin{cases} \alpha & r < \frac{R}{2}\ , \\ 2\alpha\left(1 - \frac{r}{R}\right) & \frac{R}{2} \le r \le R\ , \\ 0 & r > R\ , \end{cases}$$

where α is a positive constant. Express your answers in terms of Q and R.

a) Determine the value of α.

b) Find the magnitude of the electric field everywhere.

c) At $t = 0$ an electron with charge $-e$ and mass m is placed at rest somewhere in the region $0 < r < \frac{R}{2}$. Describe qualitatively the motion of the electron and find the time T, when the electron first returns to its starting position.

Problem 4

Consider a system of five non-interacting binary magnets (with magnetic moment $\pm m$) in a magnetic field B, and in thermal contact with a reservoir at a temperature T. Determine the following:

a) partition function Z.

b) probability to measure a maximum *magnitude* of magnetization.

c) entropy of the system.

d) energy of the system.

For parts **b)** to **d)** investigate *and explain* the limits $T \to 0$ and $T \to \infty$.

Problem 5

Grand-unified theories in particle physics generically predict that the proton will decay. The proposed Hyper-Kamiokande experiment in Japan consists of cylindrical tanks each containing ultra-pure water which are lined with ultra-sensitive photo sensors to detect the Cerenkov light produced by the proton decay products.

a) The tanks at the Hyper-Kamiokande experiment will contain 260,000 metric tons (1 ton = 1000 kg) of water. If the mean proton lifetime is 10^{34} years, how many decays would you expect to observe in one year? Assume that the detector is 100% efficient and that protons bound in nuclei and free protons decay at the same rate.

b) A possible proton decay channel is $p \to \pi^0 + e^+$, where π^0 is a neutral pion and e^+ is a positron. Calculate the positron energy if the proton decays at rest. How does the positron speed compare to the speed of light in water (refractive index $n = 1.33$)?

c) The π^0 immediately decays to photons (in 10^{-16} sec), $\pi^0 \to \gamma + \gamma$. What is the minimum and maximum photon energies to be expected from a proton decaying at rest. (Note π^0 mass = 135 MeV/c^2).