Problem 1

A classical particle of mass m is subject to a potential

$$U(x) = V \cos \alpha x - F\, x$$

where the constants V and α are both positive.

(a) Find the maximum value that F can take such that the particle will be in equilibrium at some value of x.

(b) Assuming that F is less than the maximum value calculated in (a), determine the frequency of small oscillations about the point of equilibrium.

Problem 2

Calculate the radius of the orbit of a cosmic ray proton of kinetic energy 10 GeV as it propagates through the 1 nT magnetic field of the Milky Way galaxy. You may assume that the magnetic field is perpendicular to the motion of the proton.

Problem 3

The two electrons in a helium atom can be in a spin singlet or a spin triplet state.

(a) If the electron-electron repulsion could be neglected, what would be the energy difference (in eV) between the lowest energy spin singlet and the lowest energy spin triplet states?

(b) Which type of state would lie lowest in energy?

(c) The actual energy difference between these states is 19.7 eV. Explain briefly why the effect of electron-electron repulsion is to *reduce* the energy separation between these states.

Problem 4

A batch of 1000 components of the same type for use in the Ash River neutrino detector is believed to include 5% which are faulty.

(a) If 5 components are selected at random, what is the probability that no defective component will be chosen?

(b) What is the probability that exactly 2 out of the 5 will be defective?

Problem 5

The CMS detector observes two photons of energies $E_1 = 321$ MeV and $E_2 = 370$ MeV emerging from a single point with an angle $\theta = 105^o$ between them. They are inferred to be the decay products of a neutral meson M^o. From the data given, find the rest mass and the kinetic energy of the meson in MeV.

Problem 6

A hydrogen atom is prepared in the superposition

$$|\psi\rangle = \frac{1}{6}\left[4|1,0,0\rangle + 3|2,1,1\rangle - |2,1,0\rangle + \sqrt{10}|2,1,-1\rangle\right]$$

where the states are labeled by the principal, angular momentum and magnetic quantum numbers of the electron $|n, \ell, m\rangle$. In terms of the ground state energy,

(a) What is the expectation value of the energy $\langle E\rangle$ in this state?

(b) What the energy uncertainty ΔE in this state?

Problem 7

A tunnel leading straight through a hill is found to greatly amplify pure tones at 135 Hz and 138 Hz. With the speed of sound given as $c_s = 343$ m/s, find the minimum length of the tunnel that can accommodate this phenomenon, in meters.

Problem 8

In atomic hydrogen, the hyperfine interaction gives rise to a splitting of the ground state level into two states of respective total (nuclear+electronic) spin F = 1 and F = 0. The transition $1 \rightarrow 0$ between these states gives rise to the astrophysically famous 21 cm line. At what temperature of an atomic hydrogen gas cloud will the three F = 1 states have a total population equal to that of the F = 0 ground state?

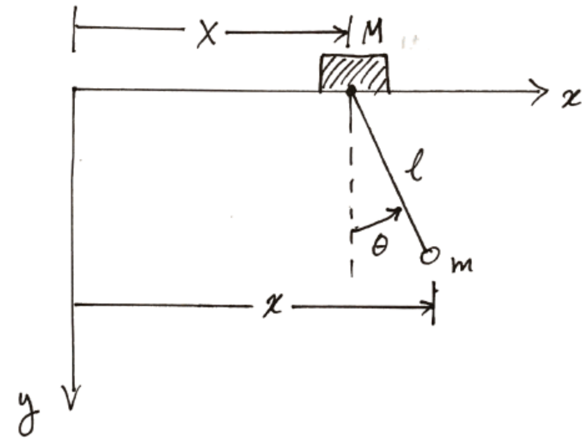
Problem 9

Find the change in entropy ΔS (in J/K) of n = 3.0 moles of a monoatomic ideal gas if, as a result of a certain process, the volume of the gas increases, from V_i to $V_f = 2V_i$ and the pressure decreases, from p_i to $p_f = {}^{p_i}/_3$

Problem 10

A set of four point charges are arranged collinearly along the z-axis, as follows: q_1 at the origin, $q_2 = +2e$ at $z = a$, $q_3 = +4e$ at $z = 2a$ and q_4 at $z = 4a$ (draw a picture!). Determine the values of q_1 and q_4 such that the electric field will fall *more rapidly* than $1/r^3$ at great distances from the charges.

Problem 1



A mass M is free to slide on a frictionless air track (which you may take to be along the x-axis, at $y = 0$). Suspended to this mass by a pivot and a very light rod of length ℓ is another mass m which swings freely in the plane of the accompanying figure. Both masses are initially at rest when m is released at some non-zero angle θ_o.

(a) Construct the Lagrangian for this system. Hint: Use X and θ as shown in the figure as generalized coordinates to describe the locations of the two masses.
(b) Derive the equations of motion and identify two conserved quantities for this system.
(c) Determine the angular frequency of small oscillations $|\theta| \ll 1$. Check your result in the limit $m \ll M$.

Problem 2

The matrices representing the Cartesian components of the spin operator $\boldsymbol{S}$ of a spin-1 particle are given by (choosing S_z to be diagonal):

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad S_z = \hbar\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

A beam of spin-1 particles, each prepared in the spin state

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}|s_z = +\hbar\rangle - \frac{1}{\sqrt{2}}|s_z = 0\rangle$$

is passed through a certain (Stern-Gehrlach) apparatus that separates out the various components of this state corresponding to the eigenvalues of S_x, the spin in the x-direction.

(a) Calculate the average value of S_x in the state $|\Psi_i\rangle$.

(b) Using the standard representation above for the spin-1 matrices, construct the normalized eigenstates of S_x i.e. $|s_x = +\hbar\rangle, |s_x = -\hbar\rangle$ and $|s_x = 0\rangle$.
(c) Calculate the respective probabilities $p(s_x)$ that a measurement of S_x done on the state $|\Psi_i\rangle$ would yield the eigenvalues $s_x = +\hbar, 0$ and $-\hbar$. As a check, use these results to recompute and confirm the average you obtained in part (a).

Problem 3

A plane electromagnetic wave, with wavelength λ =3.0 m, travels in free space with its electric field vector $\mathbf{E} = E_o \sin(kx - \omega t)$ $\mathbf{j}$ directed along the +y direction and with amplitude E_o = 300 V/m.

(a) What are the values of k (in m^{-1}) and of the frequency ν (in Hz) of this electromagnetic wave?

(b) What are the direction and amplitude B_o (in Tesla) of the magnetic field **B** associated with this electromagnetic wave?

(c) What is the time-averaged rate of energy flow per unit area for this wave in W/m^2 ?

(d) If this wave falls perpendicularly on a perfectly absorbing sheet of area 200 cm^2, what is the force exerted on the sheet (in Newton) ?

Problem 4

A spaceship travels at constant velocity v = 0.8 c with respect to Earth. Denote spaceship-frame coordinates by a prime (‘). At $t = t'$ = 0 by Earth and spaceship clocks respectively, a light signal is sent from the tail (back end) of the spaceship towards the nose (front end) of the spaceship. The length of the spaceship, as measured in a frame in which it is at rest, is L . The answers to the following questions should be expressed in terms of L and c, the speed of light.

(a) At what time, by *spaceship* clocks, does the light signal reach the nose of the spaceship?

(b) At what time, by *Earth* clocks, does the light signal reach the nose of the spaceship?

Now suppose that there is a mirror at the nose of the spaceship which instantaneously reflects the light signal back to the tail of the spaceship.

(c) At what time, by *spaceship* clocks, does the light signal finally return to the tail of the spaceship?

(d) At what time, by *Earth* clocks, does the light signal finally return to the tail of the spaceship?

Problem 5

Consider a cubic box of volume $V_i = L^3$ with perfectly conducting walls in equilibrium at a temperature T_i. Initially, this box contains N_i photons, with the equation of state for radiation being $p = \frac{1}{3}\rho$, where $\rho = U/V$ is the energy density. Give your answers in terms of the initial values T_i , U_i and N_i .

(a) Suppose now that the box were to expand adiabatically, so that its new side is of length $L_f = 2\,L$. What would then be the new temperature T_f and how much work was done by the radiation during this expansion?

(b) Now suppose, contrary to (a), that the box expands isothermally to a new side length $L_f = 2\,L$. Find the heat input required for this process.

(c) What are the numbers N_f of photons in the box as a result of the respective expansions considered in parts (a) and (b)?