Green's Functions for Surface Waves

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Derivation on Blackboard

- Equation of motion + elastic constitutive eq.
- For plane wave propagation in x, depth z
- SH system (y disp.) and P-SV system (x-z disp.)
- Can derive eigenfunction equations and phase velocities (eigenvalues)

Surface Waves

- Surface wave Green's functions can be written as

$$G(\omega,z,h)=\frac{l_1(z)l_1(h)}{cUI}e^{i(kr+\pi/4)}\sqrt{\frac{2}{\pi kr}}\cdot f(\theta)$$

eigenfunctions: $l_1(z)l_1(h)$

phase: $e^{i(kr+\pi/4)}$

geometric decay: $\sqrt{\frac{2}{\pi kr}}$

azimuth dependence: $f(\theta)$

- So let's determine l_1, r_1, r_2

Eigenfunction Equations

- Assuming μ(z) and λ(z) then:

- Love: $$-\rho\omega^2 l_1 = -\mu(z)k^2 l_1 + \frac{\partial}{\partial z}\left[\mu(z)\frac{\partial l_1}{\partial z}\right]$$

- Rayleigh:

$$-\rho\omega^2 r_1 = -\lambda k^2 r_1 - \lambda k\frac{\partial r_2}{\partial z} - 2\mu k^2 r_1 - k\frac{\partial}{\partial z}\left[\mu r_2\right] + \frac{\partial}{\partial z}\left[\mu\frac{\partial r_1}{\partial z}\right]$$

$$-\rho\omega^2 r_2 = k\frac{\partial}{\partial z}\left[\lambda r_1\right] + \frac{\partial}{\partial z}\left[\lambda\frac{\partial r_2}{\partial z}\right] + \mu k\frac{\partial r_1}{\partial z} - \mu k^2 r_2 + \frac{\partial}{\partial z}\left[2\mu\frac{\partial r_2}{\partial z}\right]$$

Homogeneous Halfspace

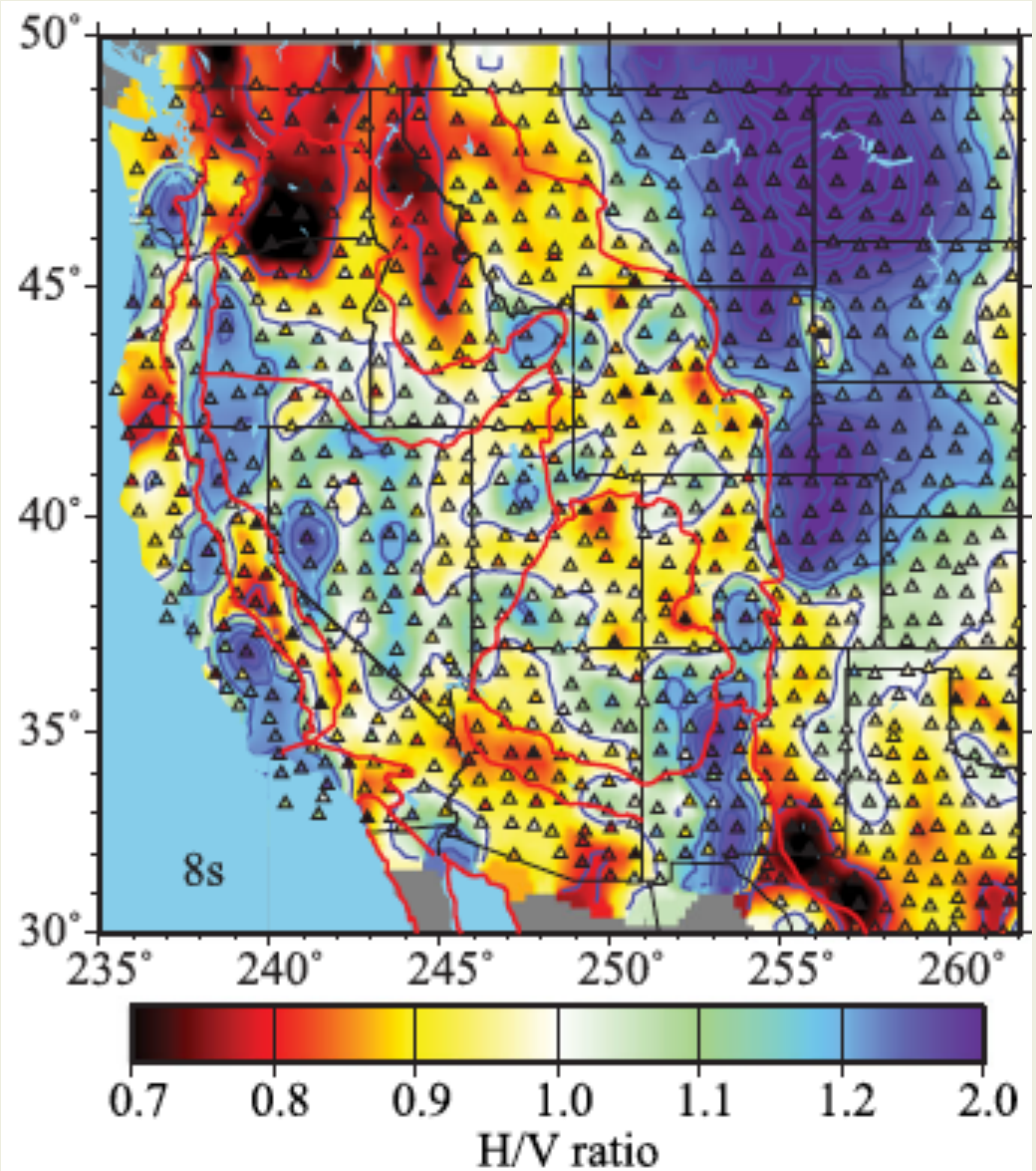
- Eigenfunctions are simple, analytic
- Love waves do not exist
- Rayleigh waves:

$$r_1 = e^{-0.8475kz} - 0.5773e^{-0.3933kz}$$

$$r_2 = 0.8475e^{-0.8475kz} - 1.4679e^{-0.3933kz}$$

- Implies: $$\frac{H}{V} \approx \frac{1 - .5773}{1.4679 - .8475} \approx 0.68$$

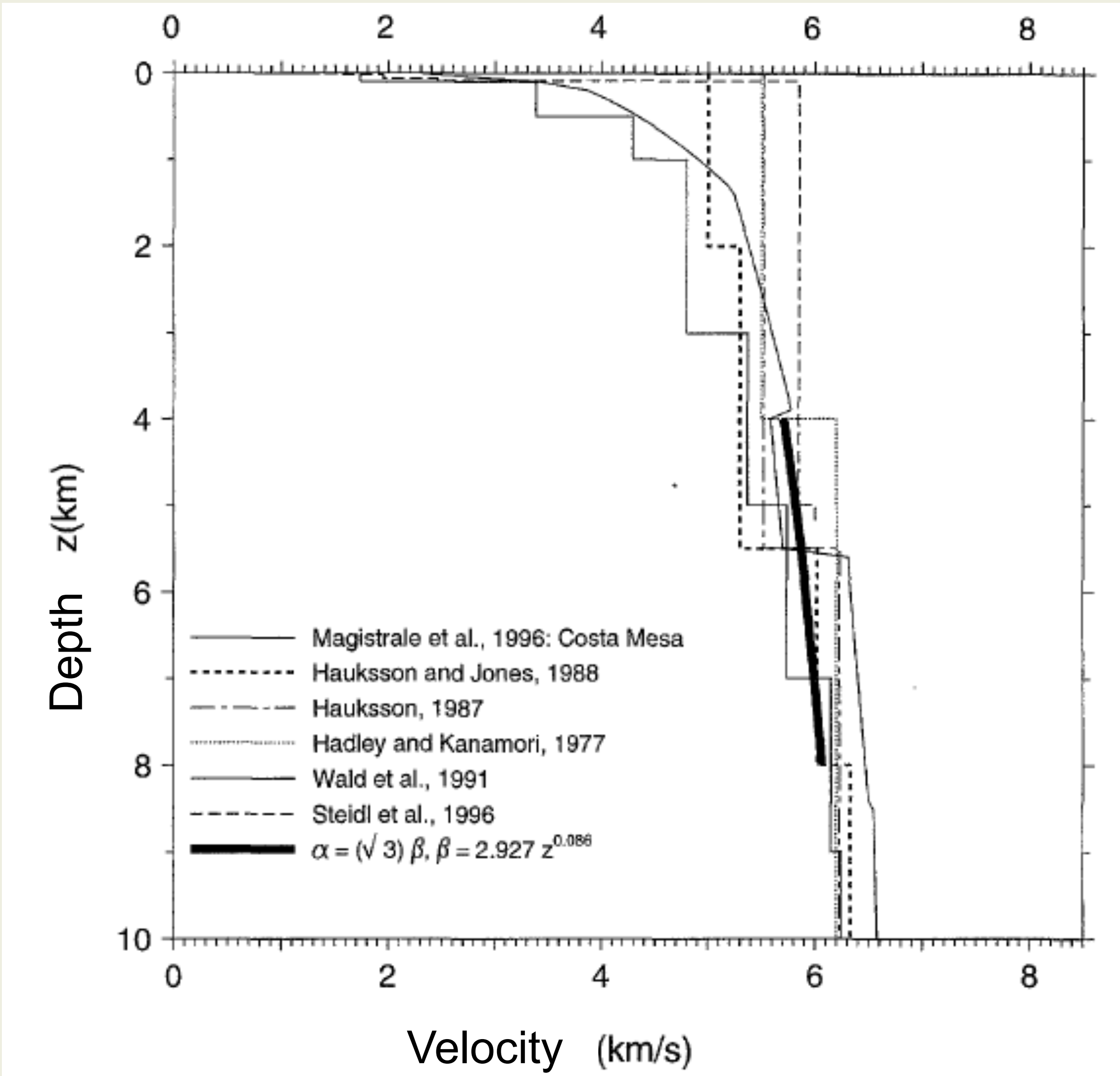
Q: Is H/V~0.7 realistic? Often no...



H/V ratios at 8s period

Lin, Tsai, Schmandt, GJI 2014

Near-Surface Green's Functions



Boore & Joyner, 1997

- Near-surface velocity structure is often decently approximated by a power-law function of depth

$$\beta \approx \beta_0 \left(\frac{z}{z_0} \right)^{\alpha}$$

Near-Surface Green's Functions

- Let's try to evaluate the near-surface Green's function in this case of a power-law structure $\beta \approx \beta_0 \left(z / z_0 \right)^{\alpha}$

- Surface-wave Green's function can be written as

eigenfunctions

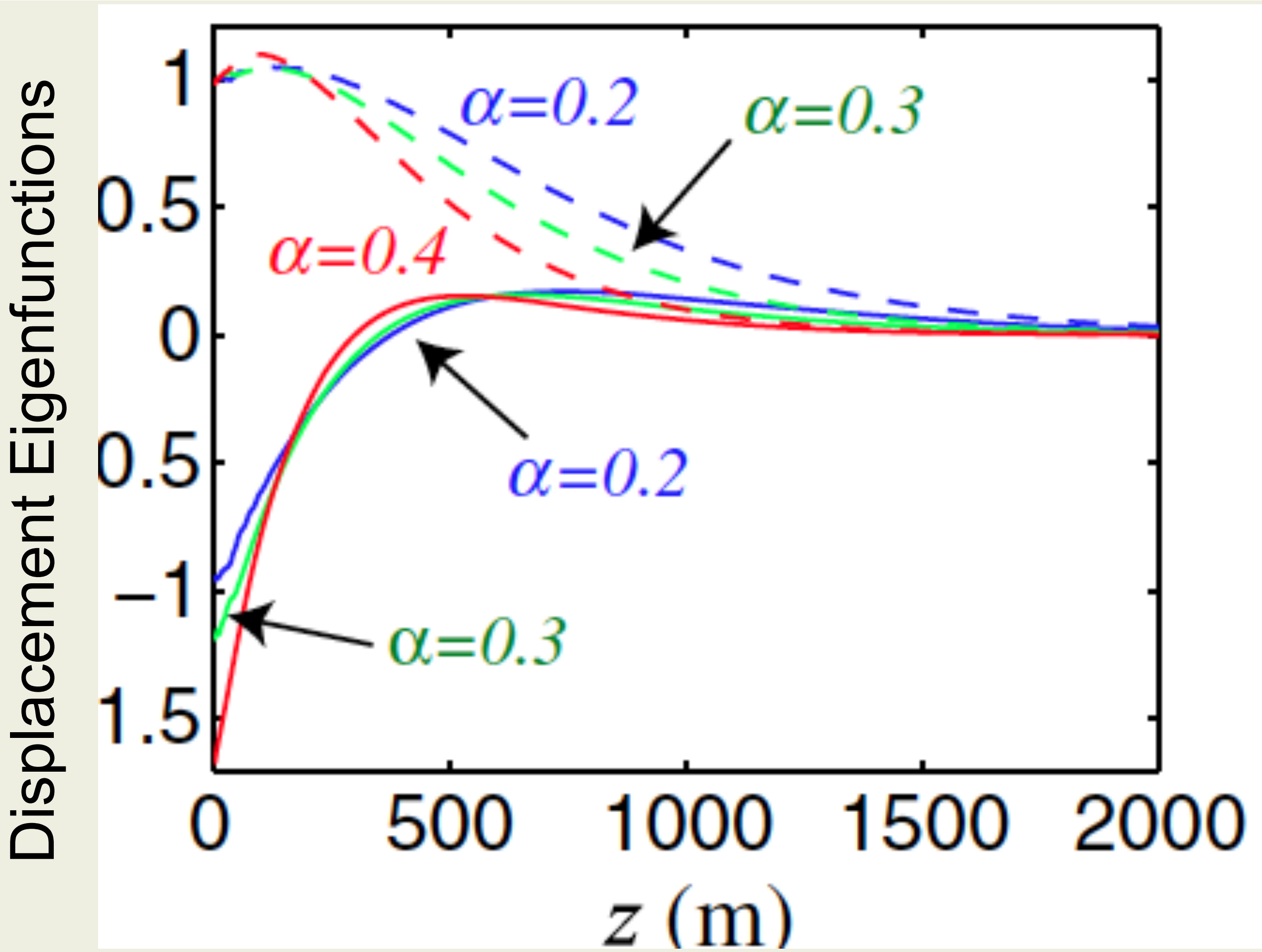
geometric decay

$$G(\omega, z, h) = \frac{l_1(z) l_1(h)}{cUI} e^{i(kr + \pi/4)} \sqrt{\frac{2}{\pi k r}} \cdot f(\theta)$$

phase

azimuth dependence

- Need to determine eigenfunctions and c(ω)…

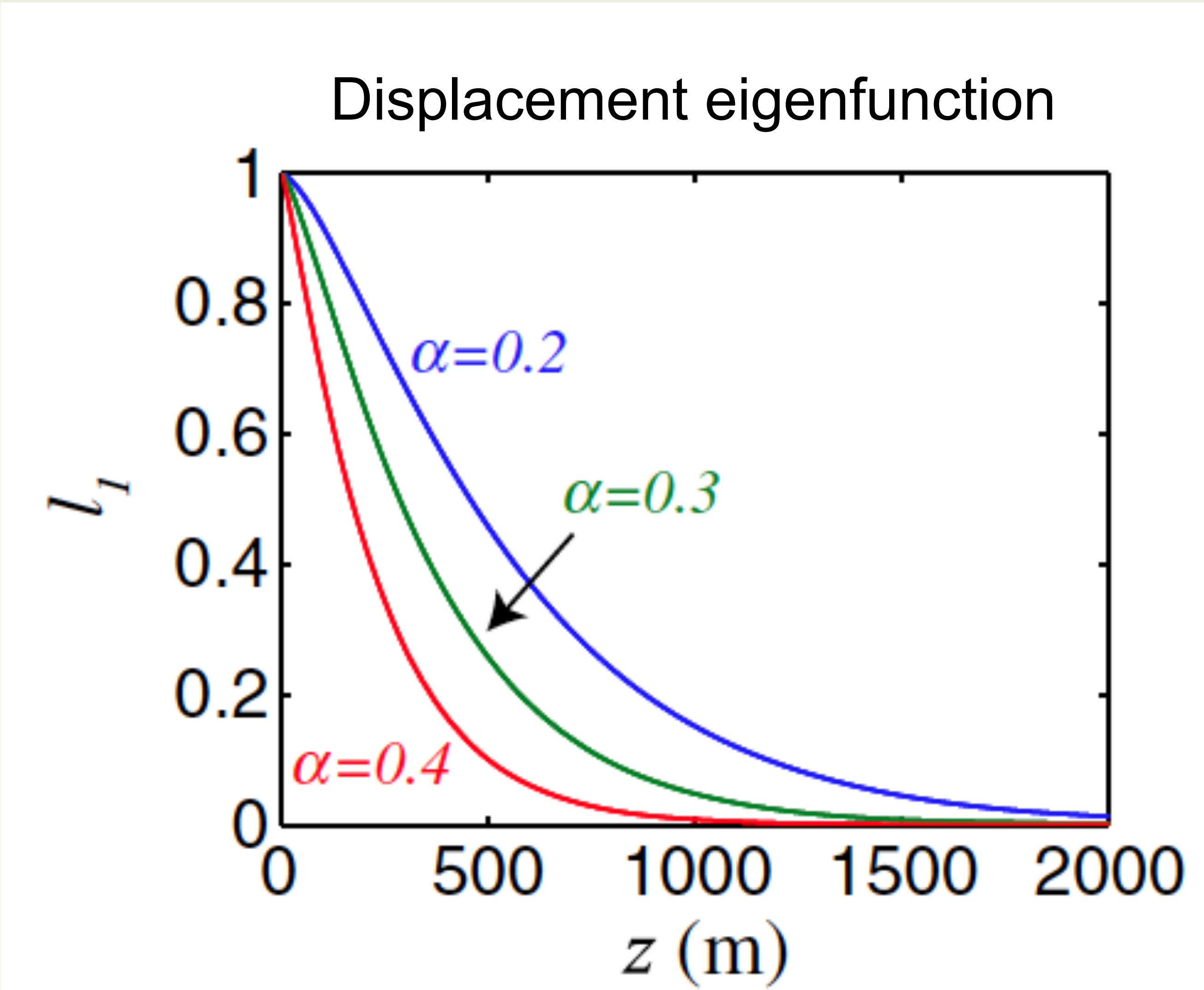


All power-law velocity eigenfunctions can be approx. as

$$r_1 \sim c_1 e^{-a_1 kz} + c_2 e^{-a_2 kz}$$

Eigenfunction	a_1	a_2	c_1	c_2
r_1	0.8421 ± 0.0426	0.7719 ± 0.0372	1.0000 ± 0.0000	-0.8929 ± 0.0101
r_2	0.9293 ± 0.0428	0.8262 ± 0.0316	0.8554 ± 0.0841	-0.9244 ± 0.0855

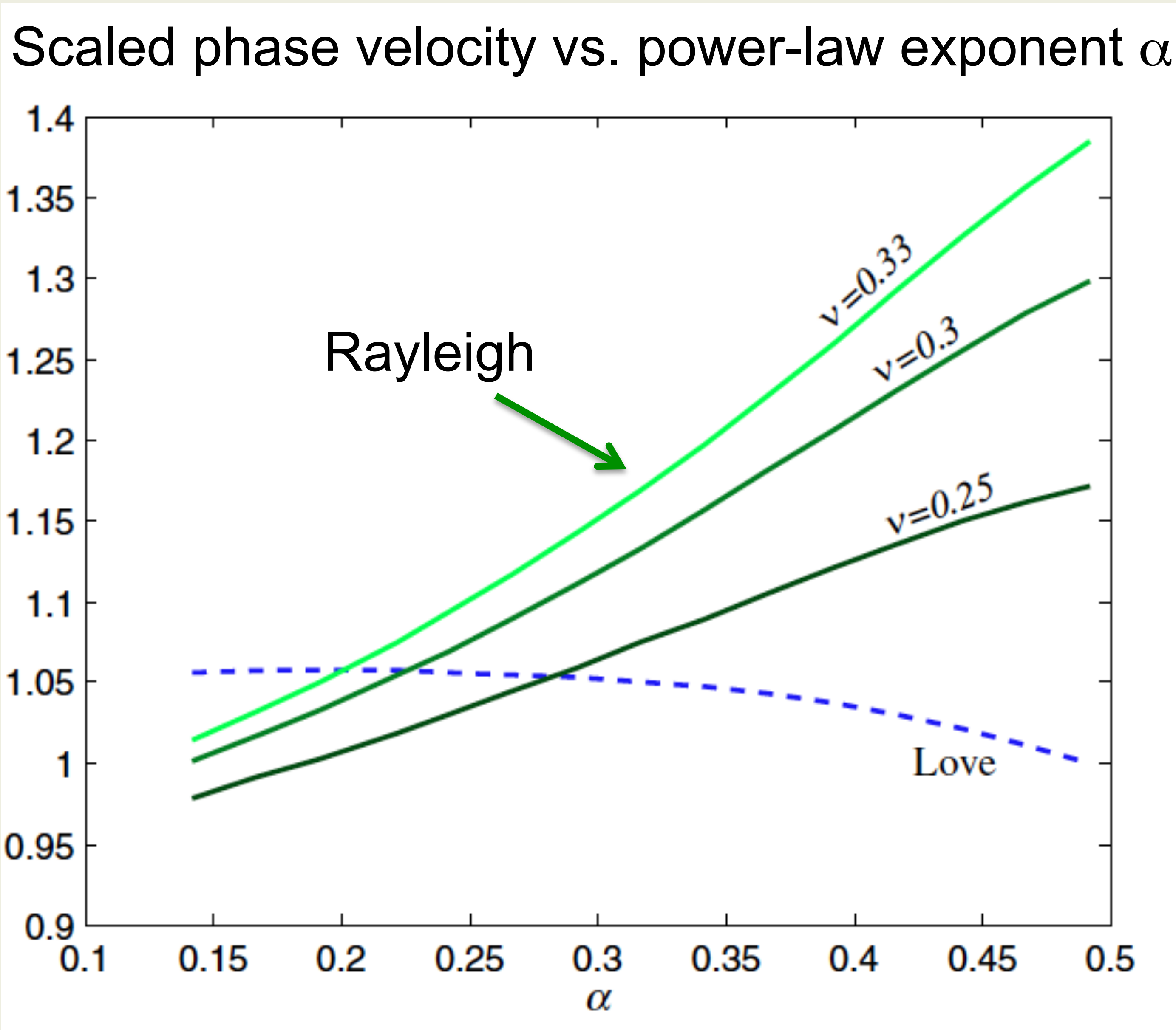
Eigenfunctions also determined for Love waves



- Eigenfunction approx. exponential decay
- $l_1 \sim e^{-0.85kz}$

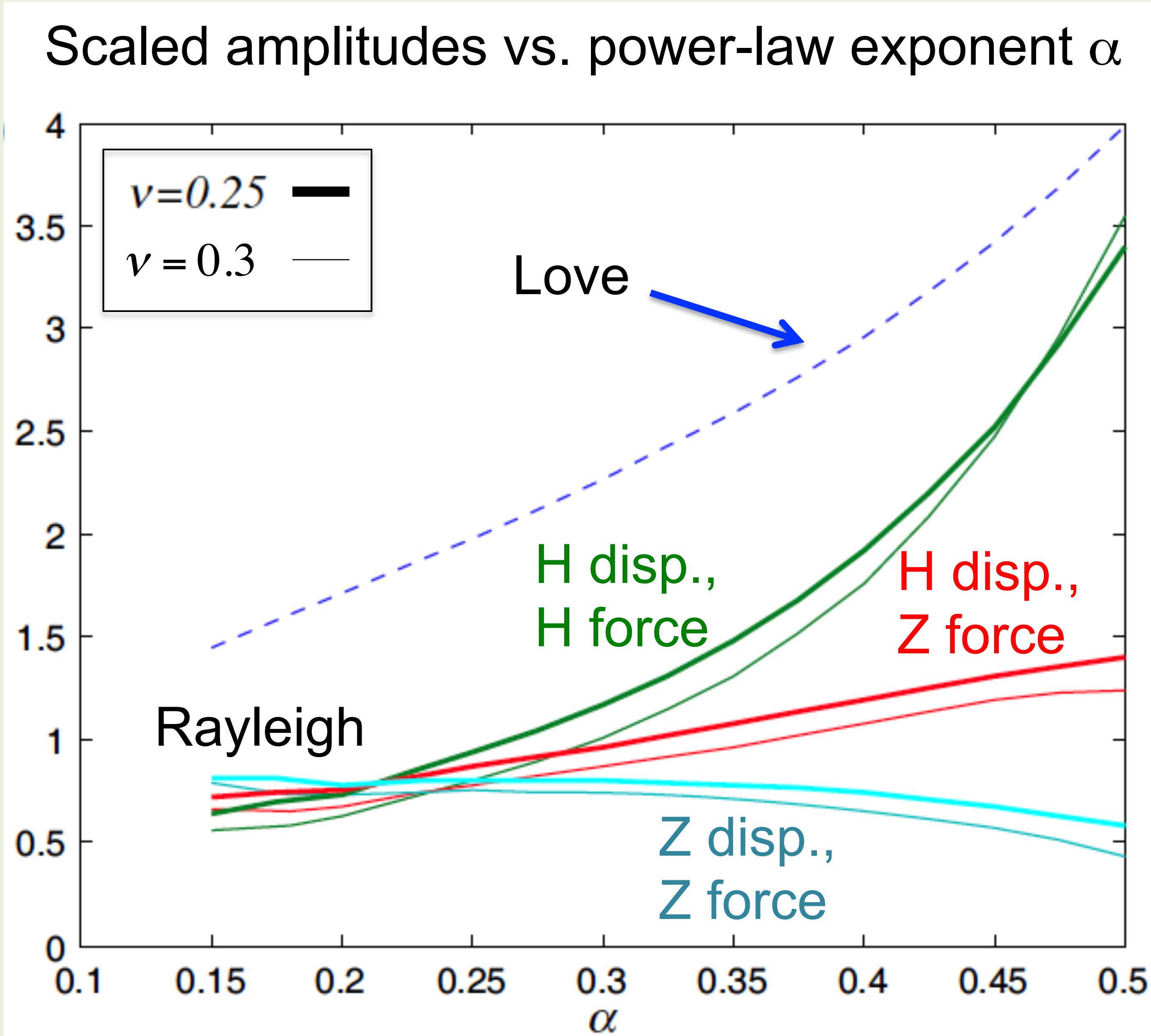
Tsai and Atiganyanun, BSSA 2014

Phase Velocity Coeff. for Rayleigh and Love Waves



Tsai and Atiganyanun, BSSA 2014

Amplitude Coeff. for Rayleigh and Love Waves



- Surface wave amplitudes are also analytically determined except for a coefficient:

$$A\Big|_{z=0,h=0} = \frac{A_0 k}{8\rho_0 c U}$$

Tsai and Atiganyanun, BSSA 2014

Long-Term Project Goals

- Use 3D array geometry to…
- Determine if noise correlations can be improved due to better noise (including coda time lapse)
- Determine if teleseismic earthquake analysis can be improved due to better noise
- Understand how wavefield changes with depth. How much scattering, etc?
  - Expectation that complexity decreases with depth
  - Use both noise/eqs to study the complexity