Seismic Radiometer and Rayleigh eigenfunction PE

pat

Seismic Radiometer

- See note by Vuk et. al.
  - http://zzz.physics.umn.edu/_media/groups/homestake/analysis/directional_analysis_v4.pdf

Field amplitude in direction d (or basis element for sky decomposition)

$$\log(\mathcal{L}) \propto -\frac{1}{2}\left(\left(Y_i^* - \gamma_{i,d}^* S_d\right) N^{-1} \left(Y_i - \gamma_{i,d} S_d\right)\right)$$

coherence of i[th] channel pair

Direction, d: **ϕ,θ**

$$\gamma_{P,\mathrm{d}} = \int d\hat{\Omega}\, Q_{\mathrm{d}}(\hat{\Omega})(\hat{\Omega}\cdot\alpha)(\hat{\Omega}\cdot\beta)e^{2\pi i f \hat{\Omega}\Delta\vec{x}/v_p}$$

$$Q_{\hat{\Omega}'}(\hat{\Omega}) \equiv \delta(\hat{\Omega} - \hat{\Omega}')$$

channels we're cross correlating

Seismic Radiometer Results

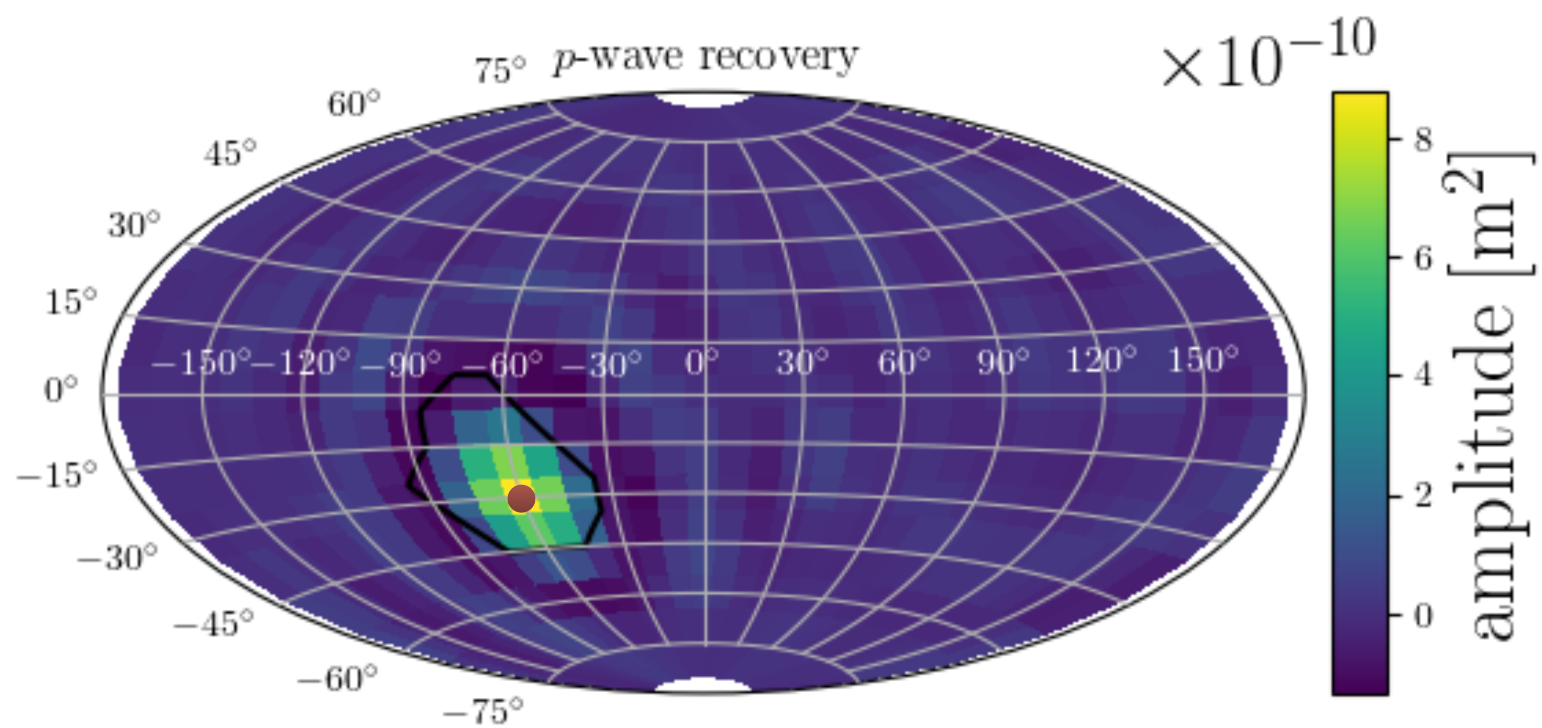
- p-wave recovery. Recovers power well. Previous issues were with spectral leakage.

INJECTION PARAMETERS:

Duration : 500
P Amp : **0.0001**
P Phi,Theta : -120.0,-30.0
Noise Amp : 1e-07
Recovery String : p

RECOVERY

P PARAMS
phi low 100.0
phi recovered 120.0
phi max 140.0
theta low 100.0
theta recovered [120.]
theta high 130.0
Recovered amplitude 7.99969526736e-05 m
Total Map Power 4.99001114717e-09 m2



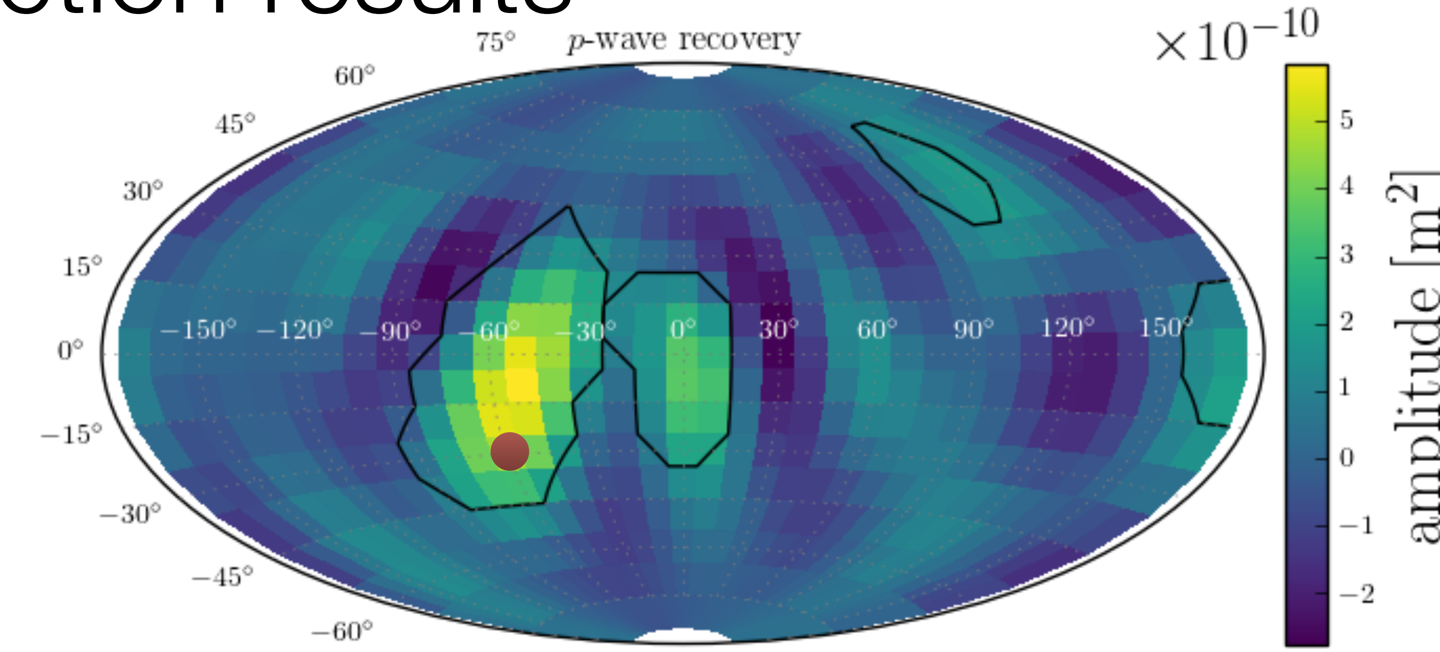
Seismic Radiometer

- We can try to recover all at the same time
- We can inject p, s, and r-waves and recover most combinations with close to the correct power

$$\gamma_{tot} = \begin{pmatrix} \gamma_{i,s1d}^{S1} \\ \gamma_{i,s2d}^{S2} \\ \gamma_{i,pd}^{P} \\ \gamma_{i,rd}^{R} \end{pmatrix}$$

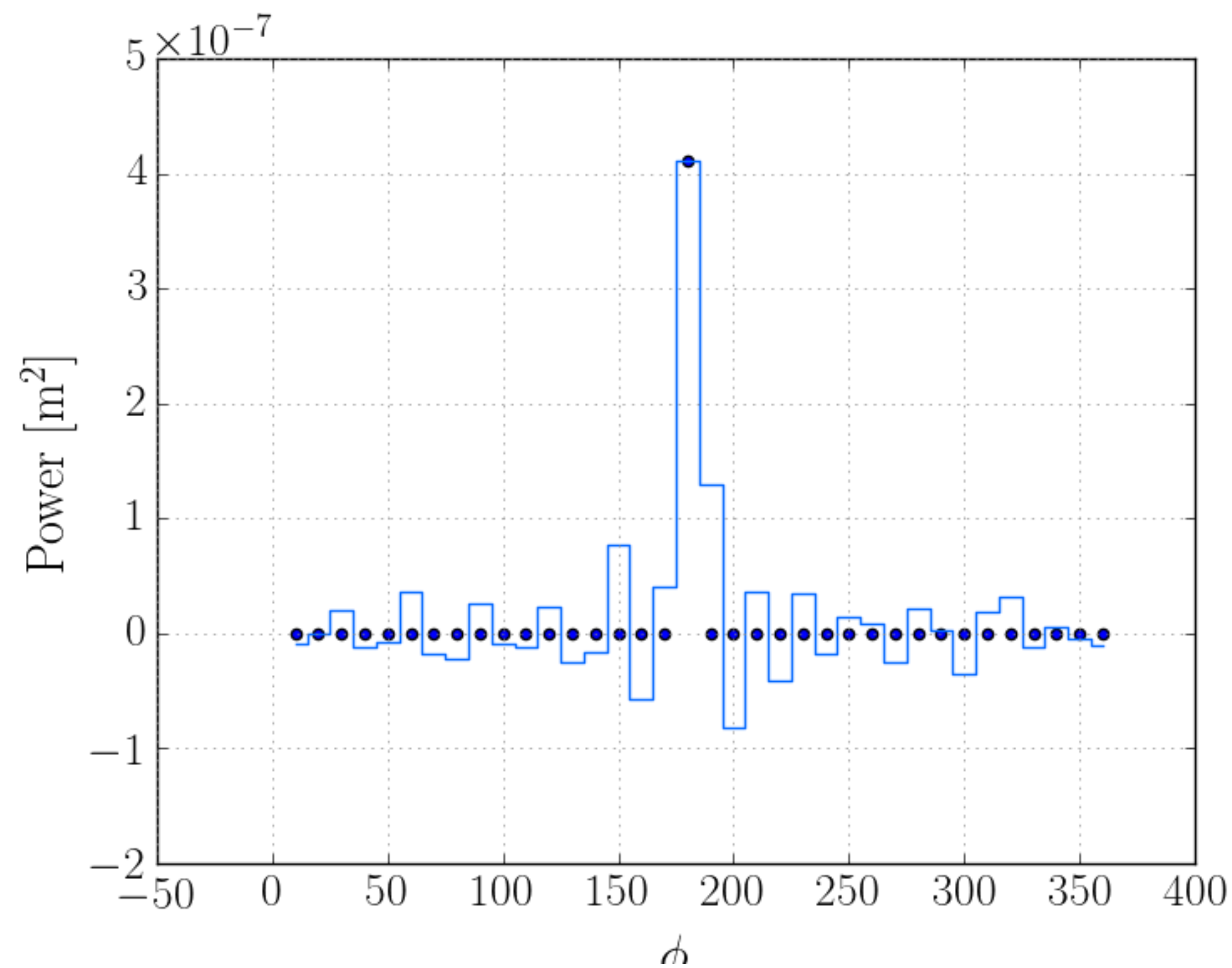
Seismic Radiometer injection results

- Injected **p & r** waves at 1 Hz
  - Amplitude of 1e-4 m for each
  - **p** (**ϕ,θ**): 120,120
    - (-60,-30 in map)
  - **r** (**θ**): 180



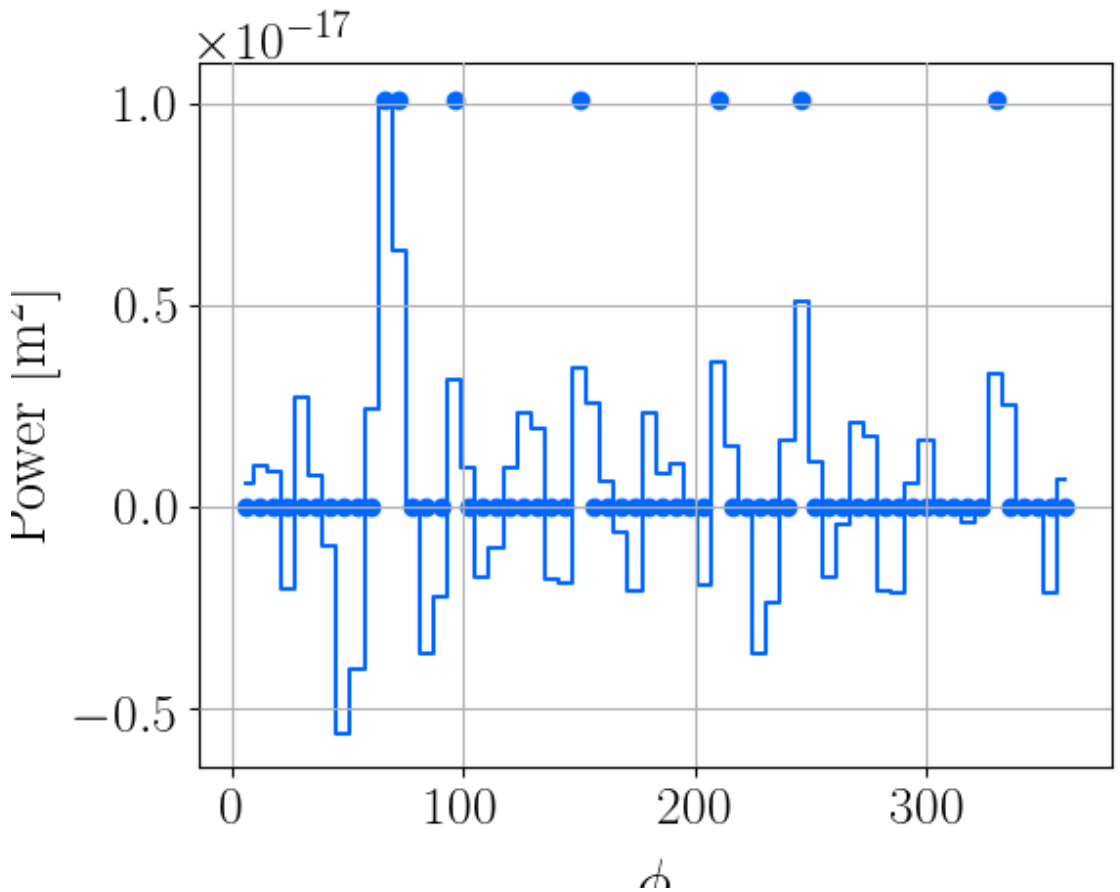
P PARAMS
phi low 100.0
phi recovered **130.0**
phi max 360.0
theta low 40.0
theta recovered [**100**.]
theta high 130.0
Recovered amplitude 0.000123607590389 m
Total Map Power 1.0013563127e-08 m2

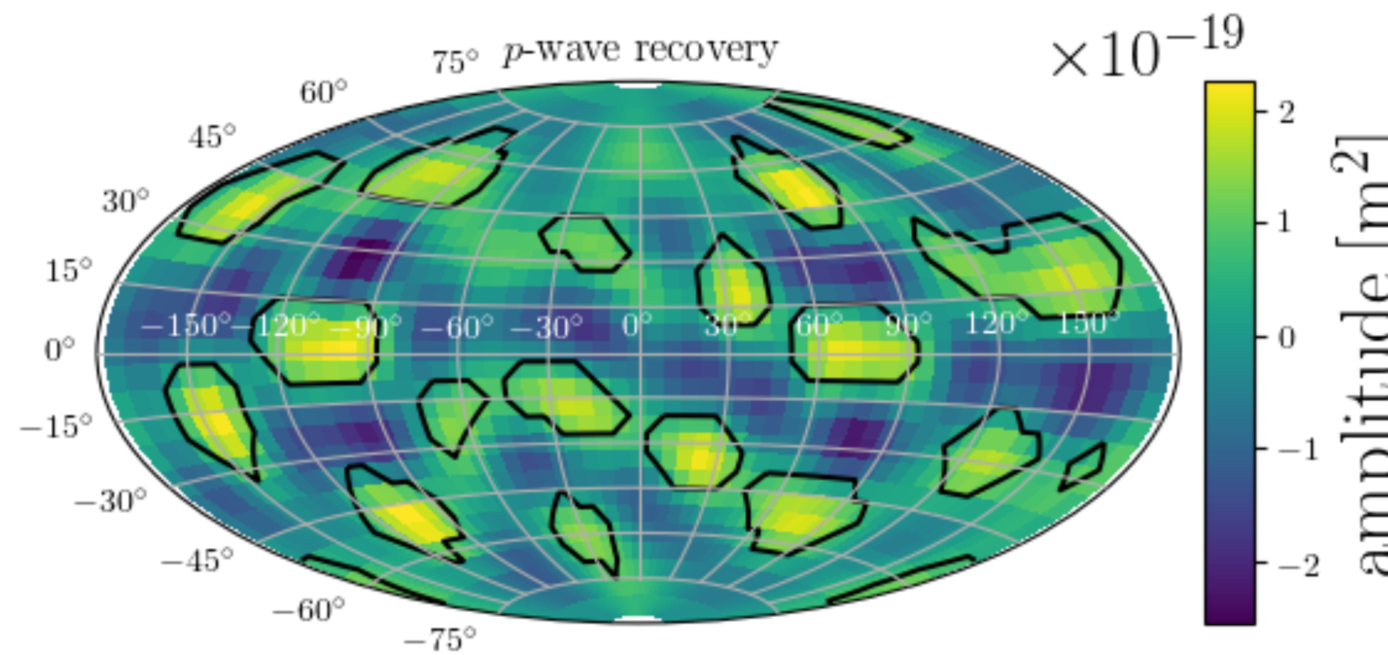
R PARAMS
phi: min, recovered, max[180. 180. 180.]
Total map power 5.23354661106e-09 m2
recovered amplitude 6.41166825101e-05 m



Seismic radiometer on real data

- Simultaneous P & R-wave recovery on 1000 s of data in 2015
  - P-wave map looks like noise
  - R-wave looks peaked at ~80 degrees





Seismic radiometer

- More benchmarking tests in progress.
  - **Q:** Which is better: Simultaneous recovery or individual recovery? Generally way better to use simultaneous recovery.
  - **A:** Recovering p & r separately for dual injection tricks the p-wave recovery (it recovers in direction of r-wave injection). **Use simultaneous recovery.**
- What is general beam-spot size? Is it roughly what we expect?
  - Yes. Spot size expected: v/(2 * d * f) where “d” is typical size of our array.
  - For p-waves: 5700m/s / (2 * 5000m * 1 Hz) ~ 0.5 rad ~ 30 degrees
    - (in p-wave recovery we had 40 degrees in phi, 30 degrees in theta)
- Would we just use horizontal - vertical correlations if we’re doing p and r recovery?
  - Takagi paper suggests “yes” (in that case p-wave correlations are purely real, r-wave correlations are purely imaginary) (doi: 10.1002/2013JB010824)
  - Could speed things up. Could give us better recovery.

Seismic Radiometer

- **Another option/something to think about:**
  - Actually calculate maximum likelihood.
    - **Model**: Assume power is dominant in one direction. Parameters:
    - $S_{\phi,\theta},\ \phi,\ \theta$
      - Invert with ORF this to get what cross-correlation should be.
      - Can also use a noise-only model and calculate a bayes-factor this way as well…are we **actually** dominated by one direction?
      - Could potentially even due this by brute-force…dimensionality is not particularly large, is thread-safe so could easily calculate many realizations quickly by parallelization.
  - NOT a priority…but something that might be interesting in the future.

Rayleigh wave eigenfunctions

- We use a bi-exponential model

$$r_1 = e^{-a_1 kz} + C_2 e^{-a_2 kz}$$
$$r_2 = C_3 e^{-a_3 kz} + C_4 e^{-a_4 kz}$$
$$k = 2\pi f / v$$

- The top is for horizontal the bottom is for vertical. We're set things up such that the raleigh-wave displacement field is:

$$\vec{r}(\vec{x}, t) = r_1 \cos(\omega t - \vec{k} \cdot \vec{x})\hat{k} + r_2 \sin(\omega t - \vec{k} \cdot \vec{x})\hat{z}$$

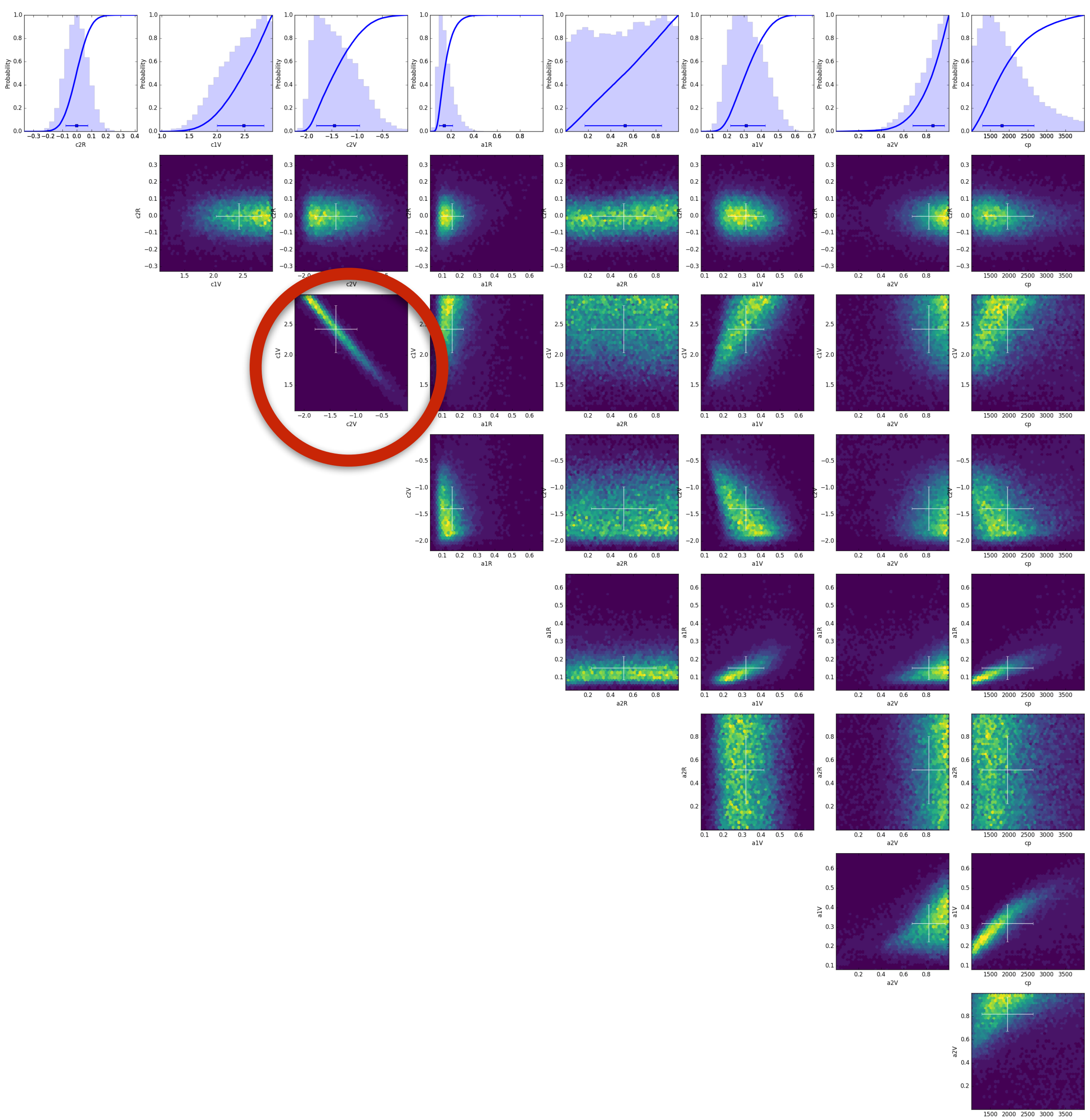
Transient based estimation

- We calculate normalized rayleigh-wave amplitudes at several depths for transient events.
    - Normalized such that radial amplitude at surface is 1.
- We generate a chi^2:

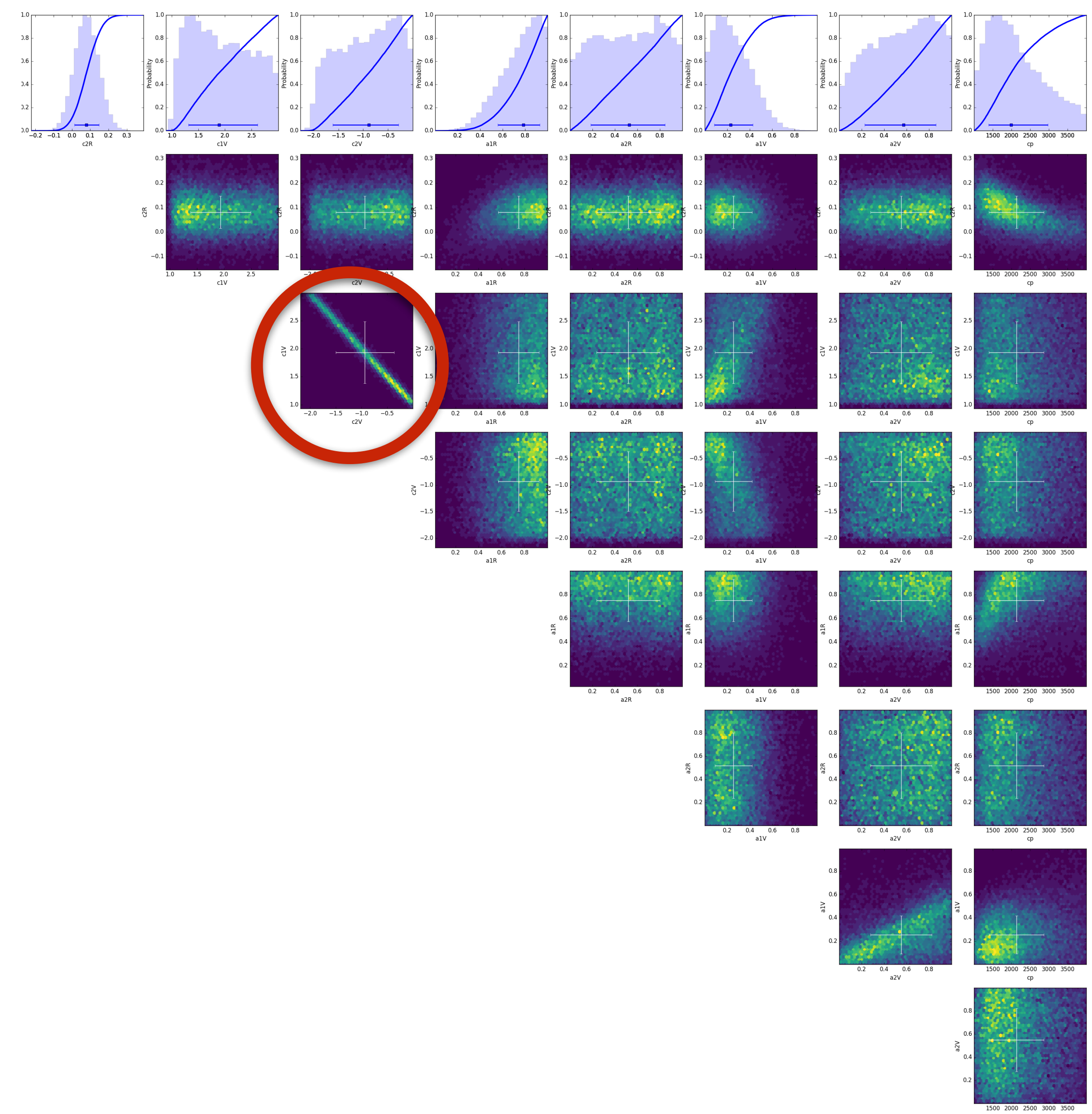
$$\chi^2 = \sum_i \left[(r_1(\vec{\theta}, z_i) - R(z_i))^2 + (r_2(\vec{\theta}, z_i) - V(z_i))^2 \right]$$

Rayleigh-wave eigenfunctions results

1 Hz

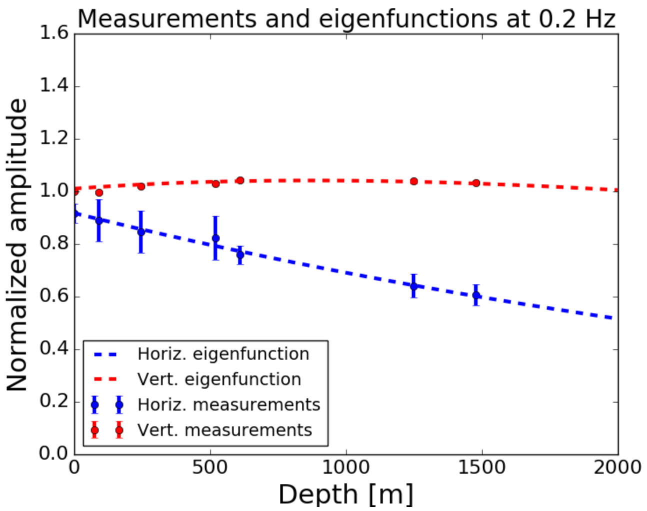


0.1 Hz

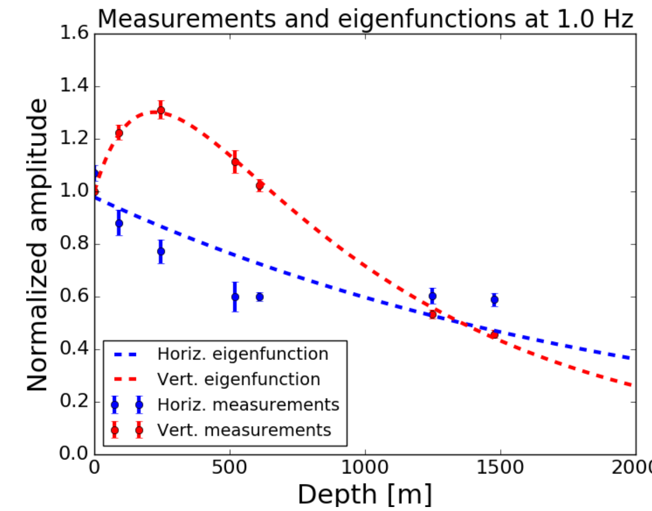


Notes

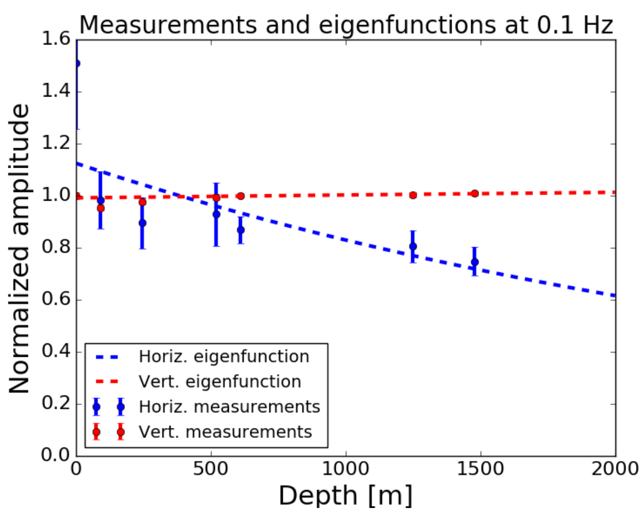
- C3 and C4 seem to be degenerate.
    - It looks like C3-C4 is fixed.
- I only went down to 1000 m/s in velocity. Need to let prior go lower.
- There are several other variables that look degenerate but their combinations are generally well-localized. I wonder if there is a different “basis” we could use that would be better so that parameters are more orthogonal to one another.
- We may be able to reduce dimensionality
- a2R and a2V are basically not localized at for 0.1 Hz.
    - => more like single exponential or a constant here.
- Railing up against the prior in a few cases.

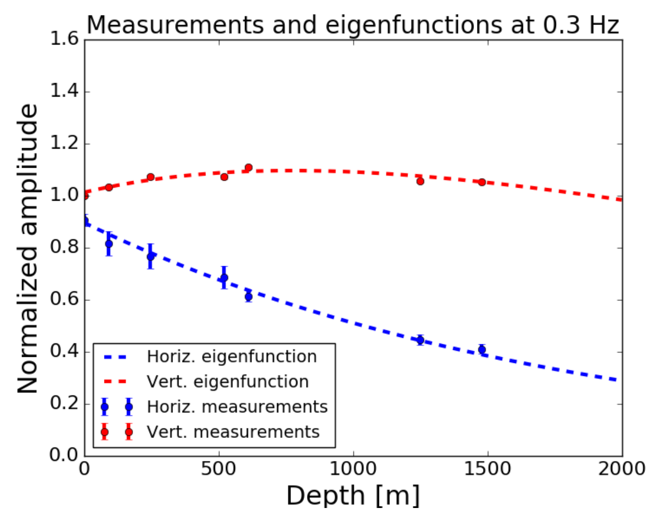


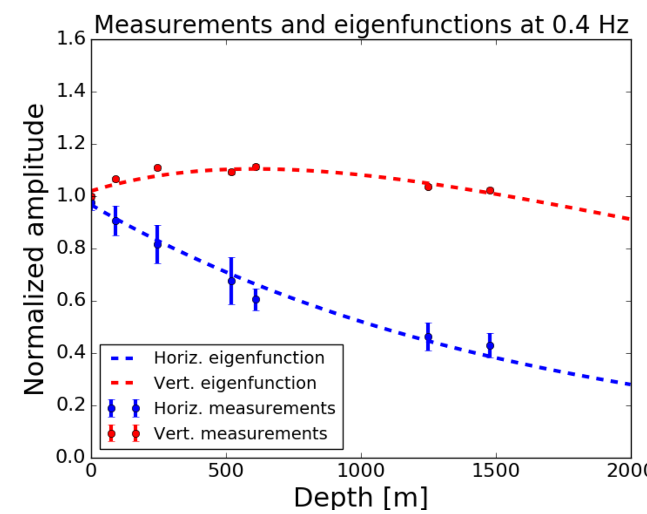
Data to estimate comparisons

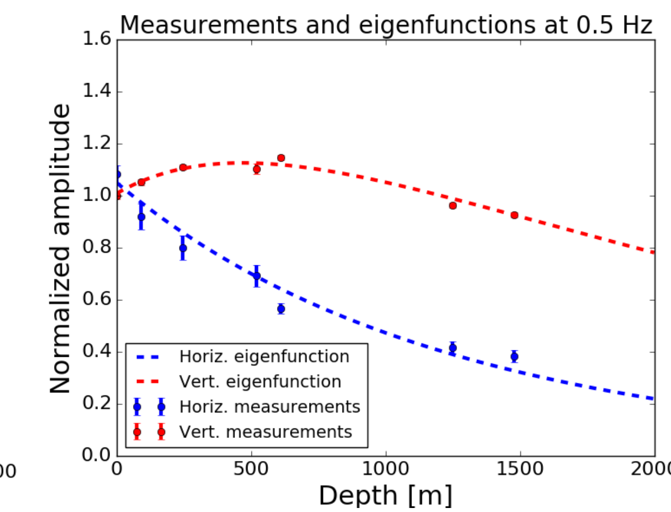


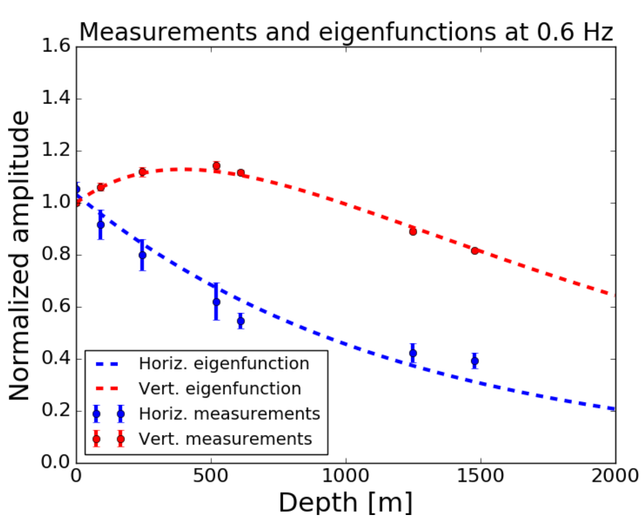
- These actually seem almost better than Tanner's. I wonder if using different prior on velocity is what did it?

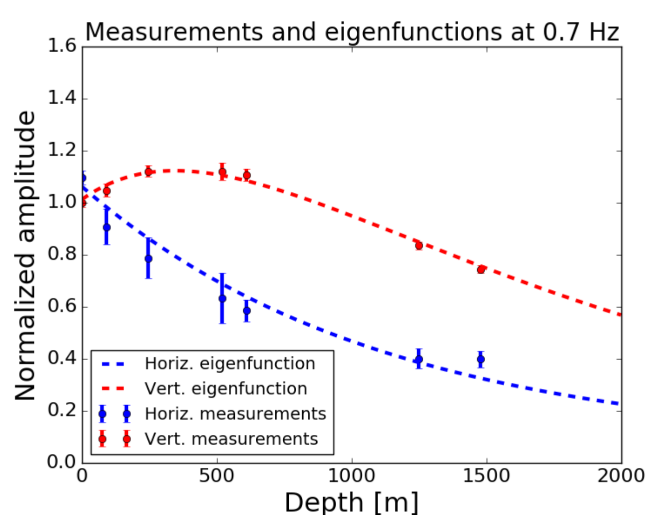


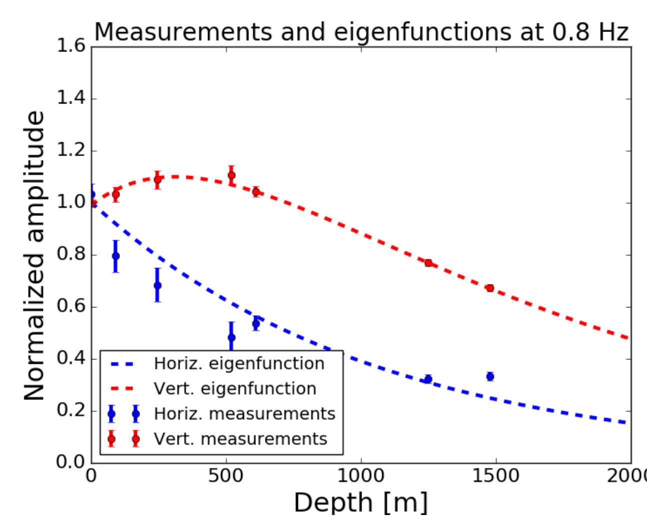


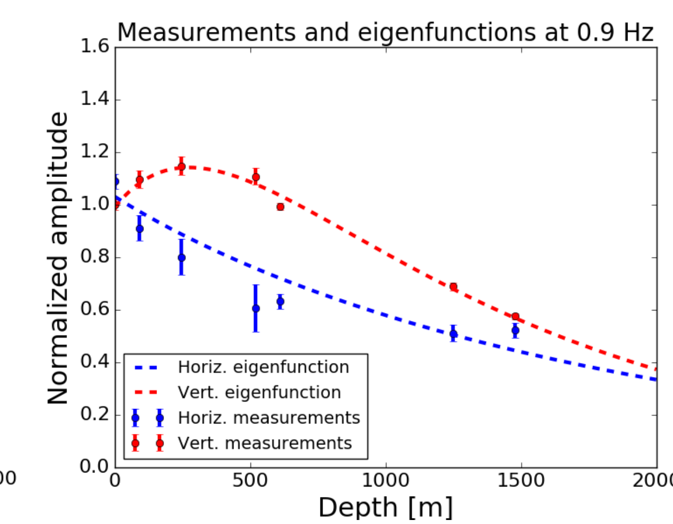






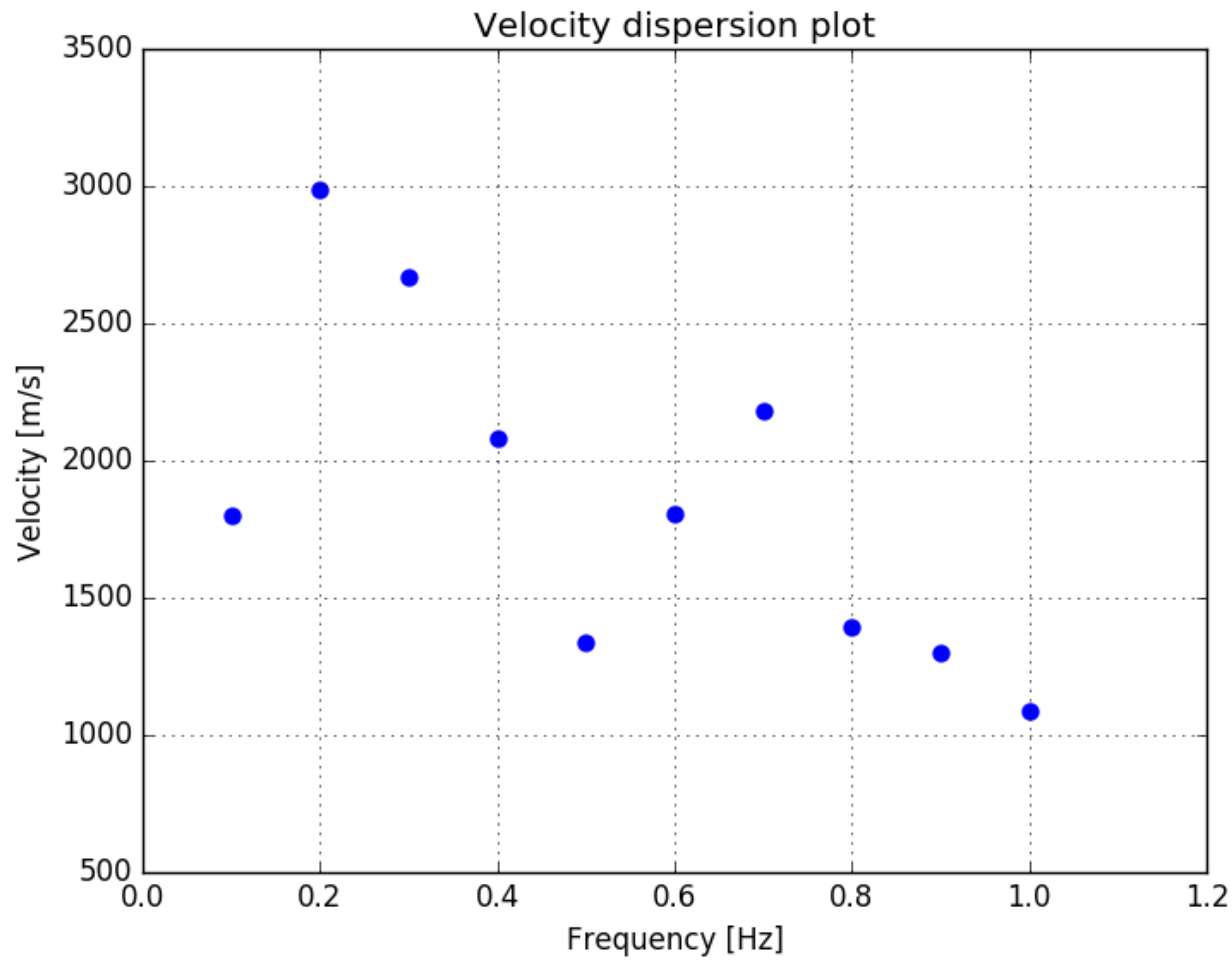






Rayleigh velocity dispersion

- Still need to add error bars (a few 100 m/s usually)
- Need to rerun these to allow for <1000 m/s velocities.



Love-wave eigenfunctions

- Just single exponential.
- a and v are degenerate.
- Could we assume that velocity depth profile is the same for group and phase velocity??
    - If so, then we could impose some sort of velocity depth profile gained from transient analysis which might help break this degeneracy.
    - We could also impose a power-law velocity depth profile here and measure the power-law index.

Love-wave eigenfunction results

- Velocity and “a” are degenerate.
- a/v ~ 1e-4…that’s something!
- log(amp(depth)) ~ -1e-4 * z