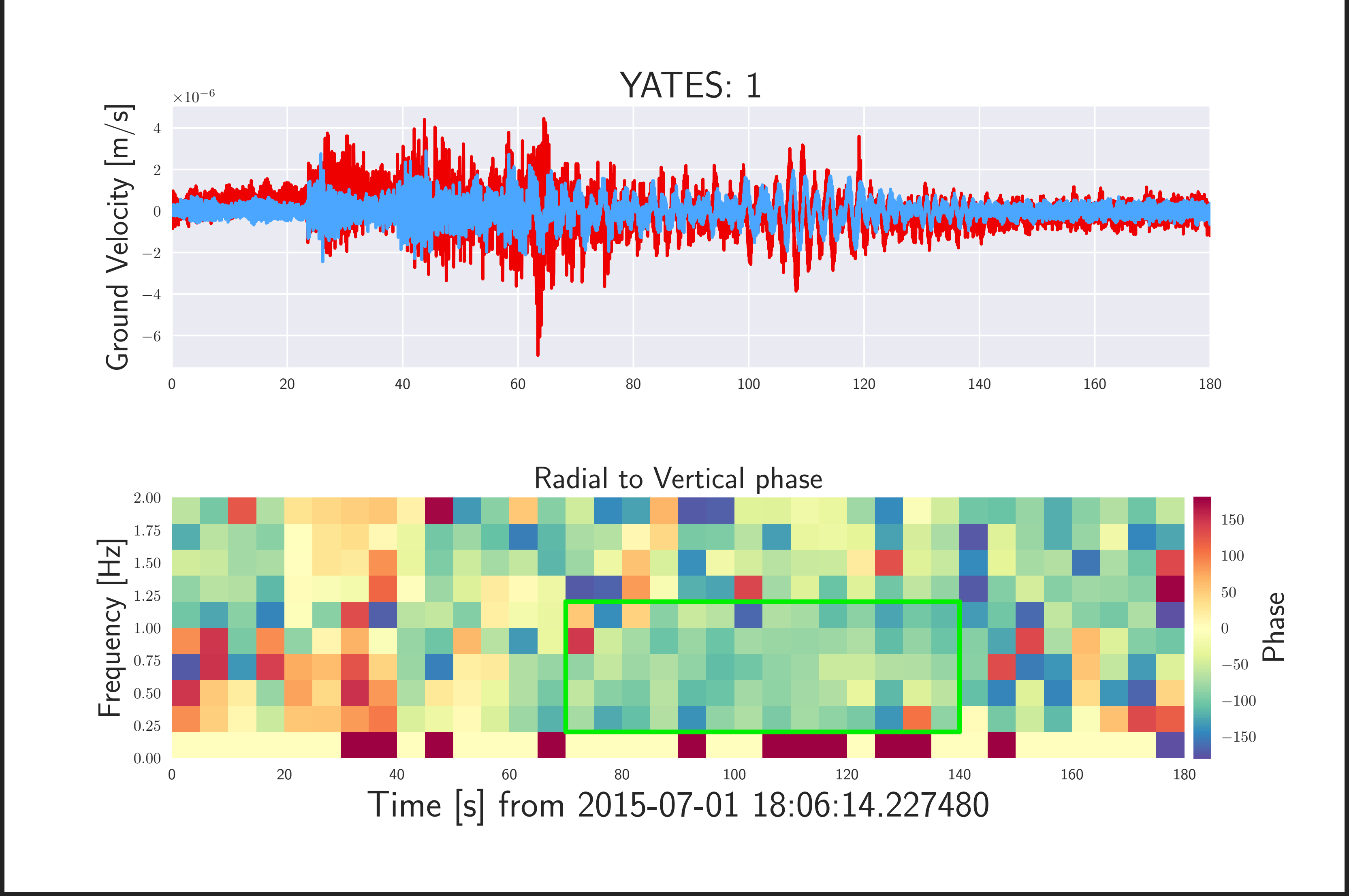
PAT MEYERS

R-WAVE EIGENFUNCTION ANALYSIS [REDUX]

EVENT SELECTION

- Use transient events from Gary/Ross with obvious R-wave regions
- Choose those “r-wave regions” by eye, and only use that data.



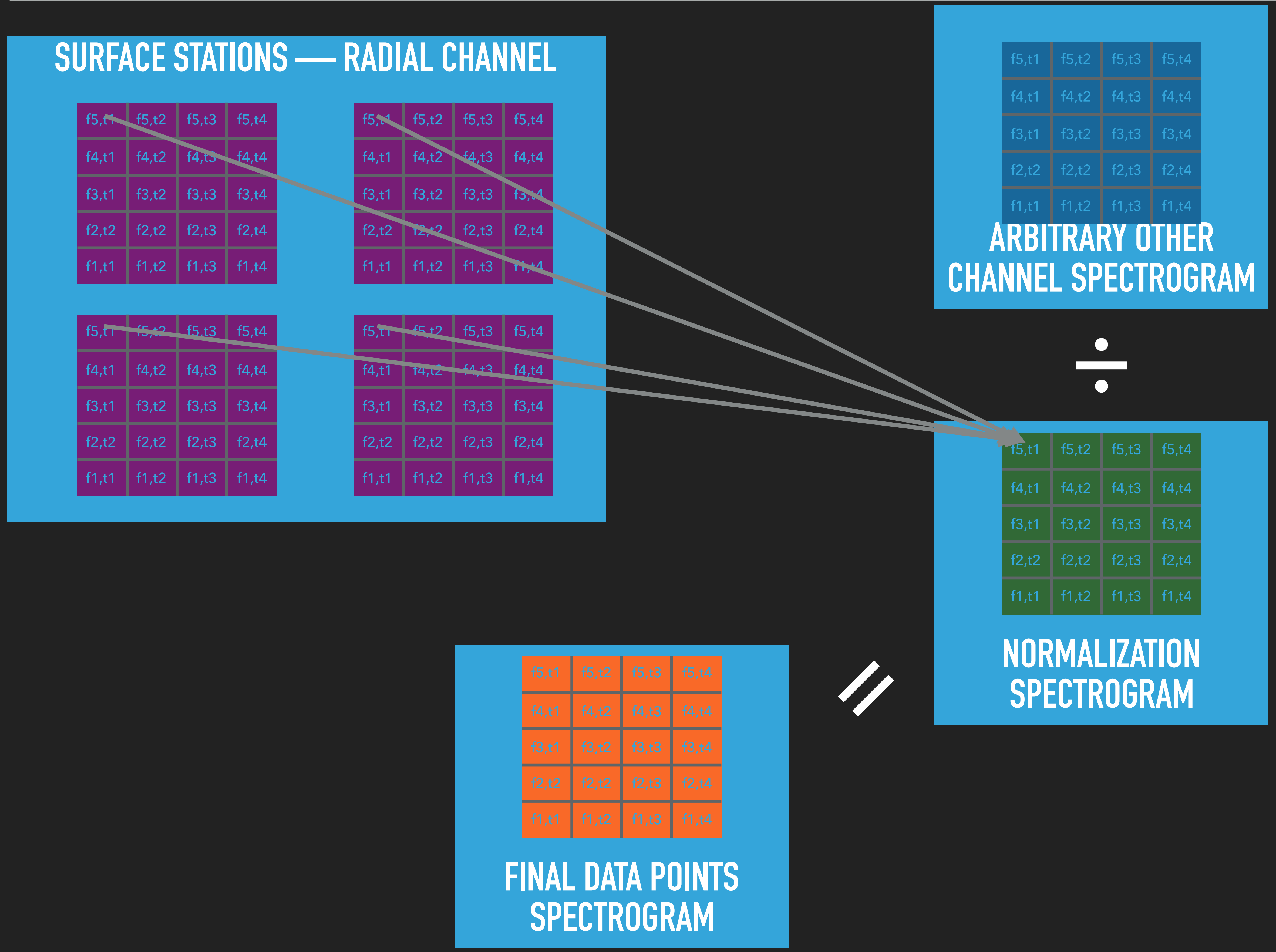
EVENTS

- 28 events
- Typically 50-80 s or so from each event
- If you want times and more info, let me know. I have a full table in my thesis…

ANALYSIS PARAMETERS

- Frequencies: **0.2, 0.3, 0.4 …, 1.2 Hz**
- FFT segment durations: **10 s**
- Everything is done in the frequency domain using spectrograms
  - Initially, each pixel in spectrogram is a **data point**

f5,t1	f5,t2	f5,t3	f5,t4
f4,t1	f4,t2	f4,t3	f4,t4
f3,t1	f3,t2	f3,t3	f3,t4
f2,t1	f2,t2	f2,t3	f2,t4
f1,t1	f1,t2	f1,t3	f1,t4


SURFACE STATIONS — RADIAL CHANNEL
ARBITRARY OTHER CHANNEL SPECTROGRAM
NORMALIZATION SPECTROGRAM
FINAL DATA POINTS SPECTROGRAM

PHASE MEASUREMENTS

- For each pixel measure radial to vertical phase
    - **Retrograde:** assign **(+)** sign to radial measurement
    - **Prograde:** assign **(-)** sign to radial measurement
- All vertical measurements are negative
- This is to be consistent with convention in *Haney/Tsai paper* **[0]**

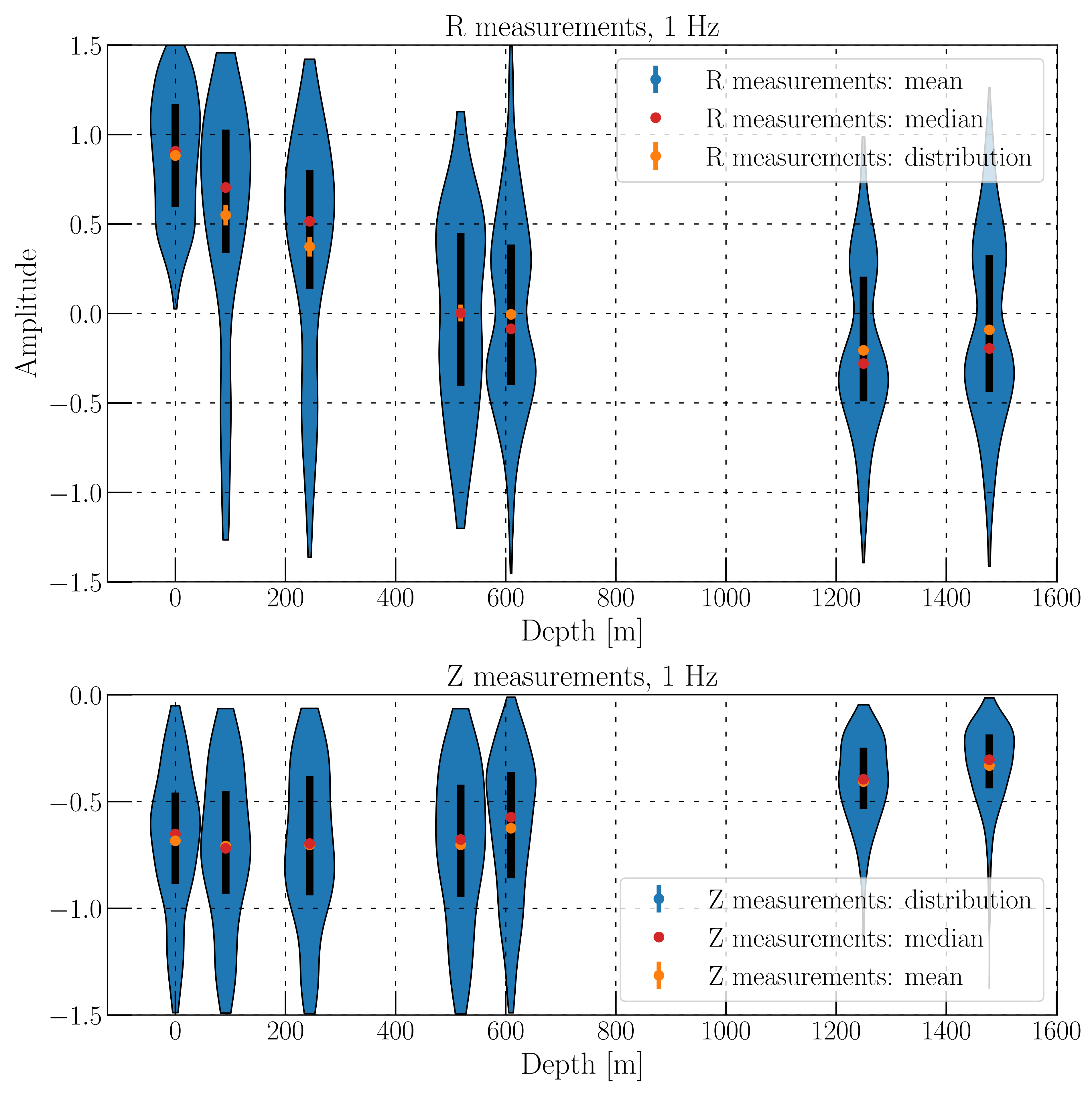
$$\phi = \arctan\left(\frac{\tilde{Z}(f)}{\tilde{R}(f)}\right)$$

[0] Haney, M. M., & Tsai, V. C. (2015). Nonperturbational surface-wave inversion: A Dix-type relation for surface waves. *Geophysics*, *80*(6), EN167–EN177. http://doi.org/10.1190/geo2014-0612.1

FINAL STEPS

- Remove data points with **| normalized amplitude | > 1.5**
- Take mean and standard deviation at each depth and each frequency
- For radial measurements this naturally reduces amplitude in cases R-waves don't dominate
  - **question**: Should we also reduce Z-amplitude by similar factor (i.e. mean(ampR) / mean(| ampR |)
- Convert standard deviation -> standard deviation of the mean (divide by sqrt(N_points))

- Violin plots — Distribution of points at that depth and frequency
- Black bars are 68% intervals



BIEXPONENTIAL MODEL

- Sample over 9 parameters
- **c_2, a_1, a_2, c_3, a_3, a_4, N_{vh}, $v_{intercept}$, v_{slope}**
- Use all frequencies at once as data

$$r_H(f,z) = (e^{-2\pi a_1 fz/v} + c_2 e^{-2\pi a_2 fz/v}) \times \frac{1}{1+c_2}$$

$$r_Z(f,z) = (e^{-2\pi a_3 fz/v} + c_4 e^{-2\pi a_4 fz/v}) \times \frac{N_{vh}}{1+c_4}$$

BIEXPONENTIAL MODEL — A FEW EXPLANATIONS

- “**v**” on the bottom is from dispersion curve
  - Linear for right now (have also also tried power law)
  - Would like to add something different — need something that probably levels off at low frequencies.
- “$\mathbf{N_{vh}}$” is vertical-to-horizontal ratio at the surface

$$v(f) = v_{\text{intercept}} + f \times v_{\text{slope}}$$

LOG LIKELIHOOD

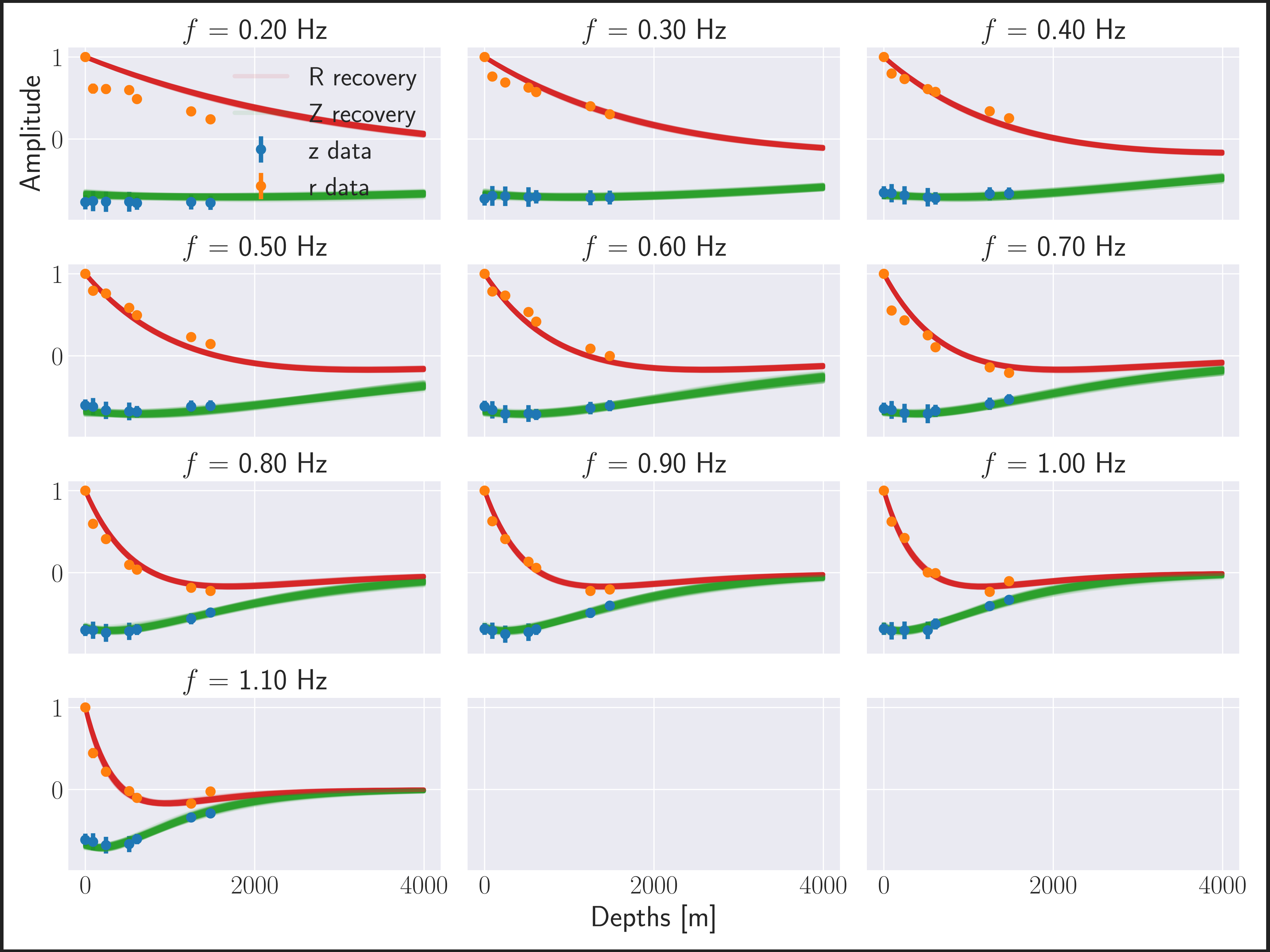
- Sum over frequency **and** depth now.
- (In the past we re-ran sampling for each frequency)

$$\ln\, p(\{\hat{h}, \hat{v}\}|\vec{\theta}) = -\frac{1}{2}\sum_f \sum_z \left(\frac{\left[\hat{h}(f,z) - r_H(f,z;\vec{\theta})\right]^2}{\sigma_H^2(f,z)} + \frac{\left[\hat{v}(f,z) - r_V(f,z;\vec{\theta})\right]^2}{\sigma_V^2(f,z)} \right)$$

PRIOR DISTRIBUTIONS

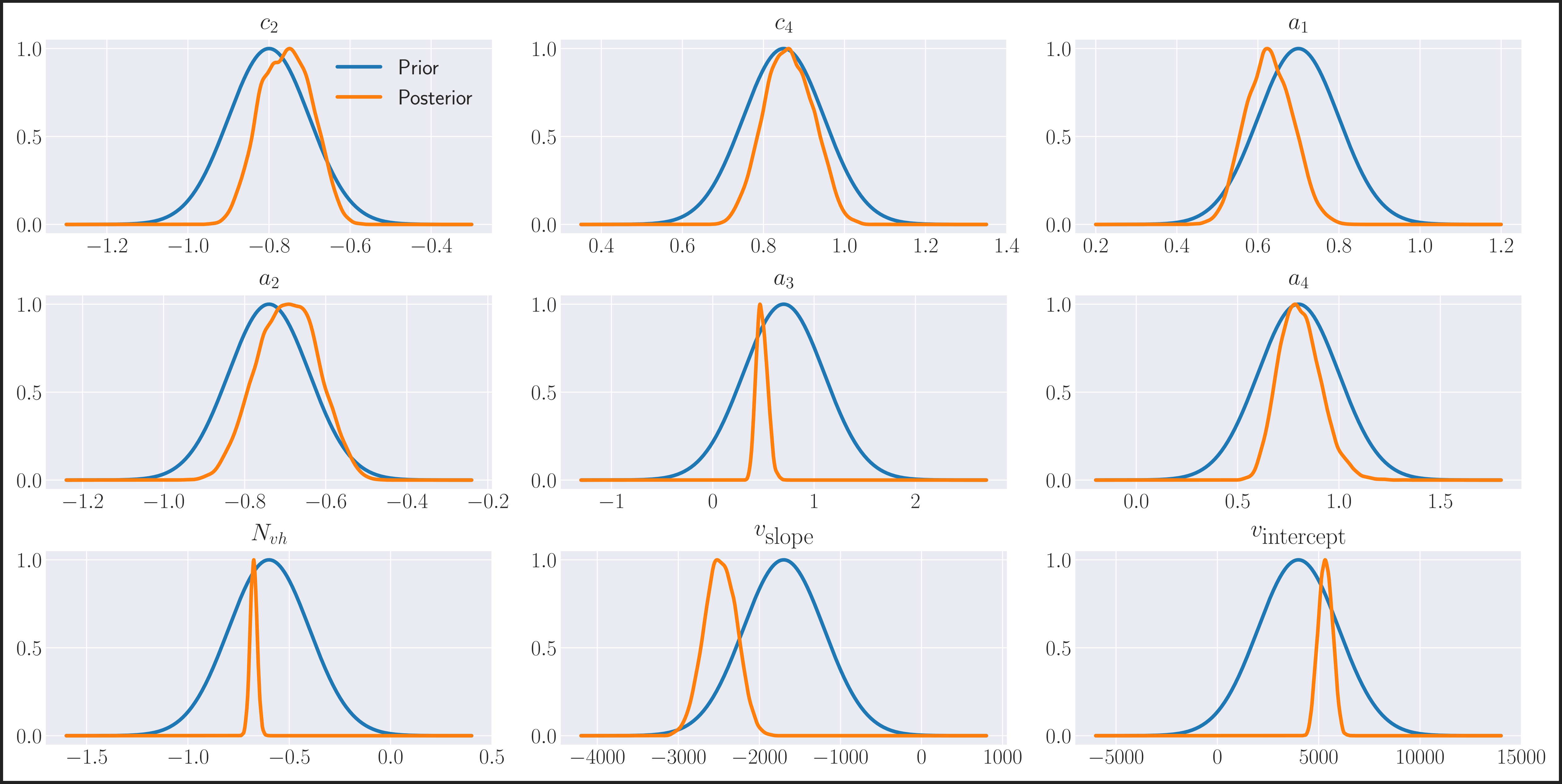
- Use Gaussian priors on each parameter informed by:
  - Victor's paper ("**a**" and "**c**")
  - our data ("N_{vh}")
  - other measurements (velocity dispersion)
- (actual values shown in results in a few slides)

FIT RESULTS



f = 0.20 Hz
f = 0.30 Hz
f = 0.40 Hz
f = 0.50 Hz
f = 0.60 Hz
f = 0.70 Hz
f = 0.80 Hz
f = 0.90 Hz
f = 1.00 Hz
f = 1.10 Hz
R recovery
Z recovery
z data
r data
Amplitude
1
0
0
2000
4000
Depths [m]

PRIOR/POSTERIOR COMPARISON



RESULTS

Parameter	Mean	Error	Prior Mean	Prior Error
c_2	-0.76	0.06	-0.8	0.1
c_4	0.86	0.06	0.85	0.1
a_1	0.63	0.06	0.7	0.1
a_2	-0.69	0.07	-0.74	0.1
a_3	0.49	0.06	0.7	0.4
a_4	0.81	0.1	0.8	0.2
N_{vh}	-0.68	0.02	-0.6	0.2
v_{slope}	-2500	200	-1700.0	500.0
$v_{\text{intercept}}$	5330	340	4000.0	2000.0

SOME COMMENTS

- Making priors larger results in some bi-modal results
  - Not surprising (Daniel has seen something similar)
- Velocity dispersion curve makes sense
  - Would like to add a better one (maybe 3 parameters?)
- In the end, I think the MCMC is probably overkill
  - Unless there is a physical model associated with these parameters having specific values, I think from an empirical standpoint it's simpler to state that the space is very degenerate and what we quote are simply effective parameters (continued…)

SOME COMMENTS

- If people are interested in testing/including more models, I’d be happy to run the MCMC with that model
- I can also share the final data points if people are interested.

CONCLUSIONS — R-WAVE PE

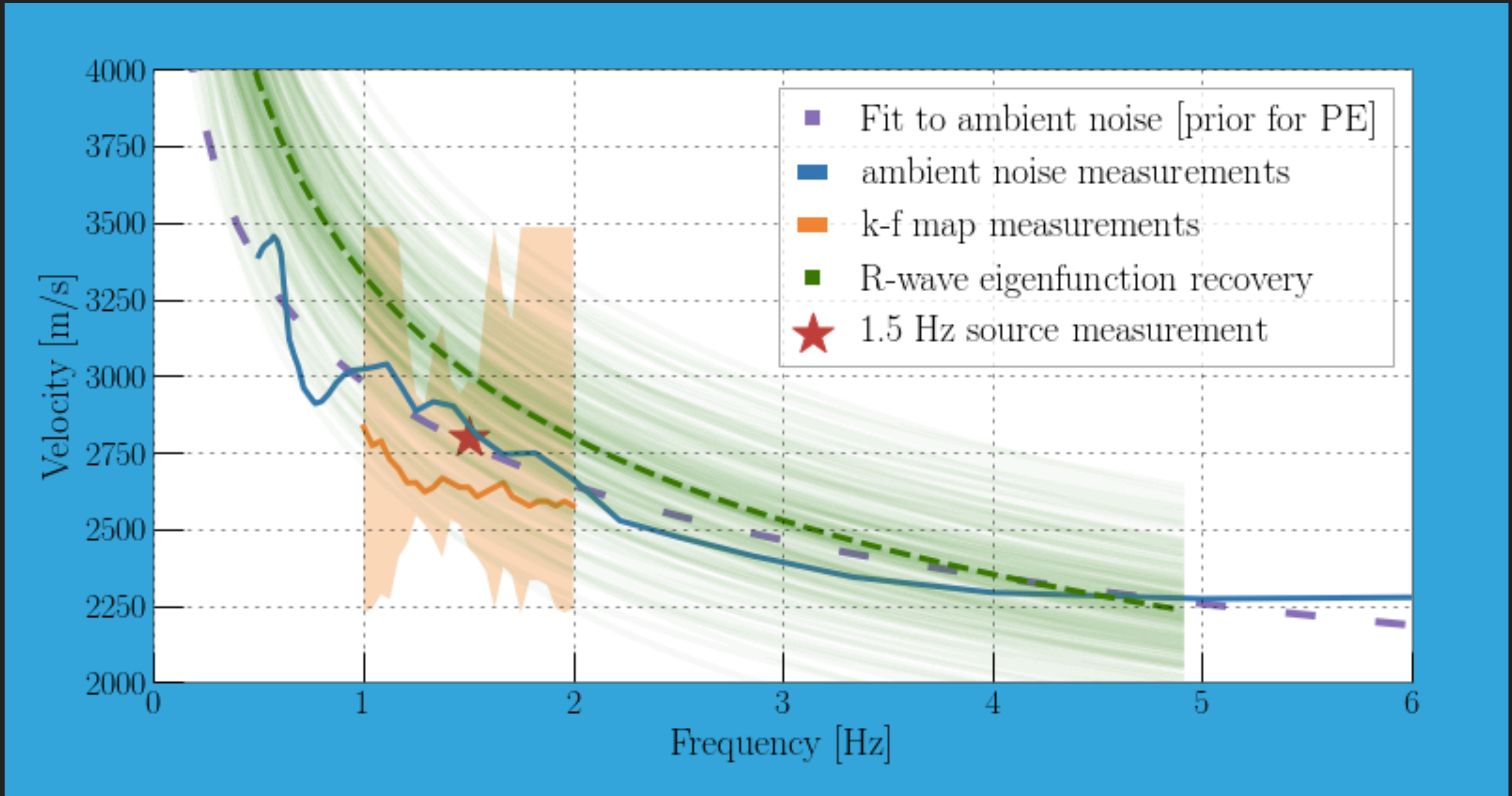
- What do we want the focus of the paper to be?
  - Separate call?
- Do we let measurements speak for themselves and include a set of biexponential fit parameters?
- Would we like to look at some physical models? If so, which ones?
- I can start putting text into paper. Apologies for not doing that sooner.

EXTRAS

- **OTHER DATA METHODS:**
  - could use each data point when sampling
    - takes longer. errors would be from pre-event data. need to propagate error in normalization as well. becomes messy, but infrastructure currently exists.
  - could add model for phase
    - estimate distribution of radial to vertical phase on surface
    - use this to generate a likelihood for R-wave phase at depth

REDOING THIS ANALYSIS WITH A POW

- Comparison of velocities — powerlaw velocity dispersion used instead of linear
  - qualitative fits were similar for eigenfunctions; below are results from Daniel, Michael and myself for the different velocity measurements we've done.

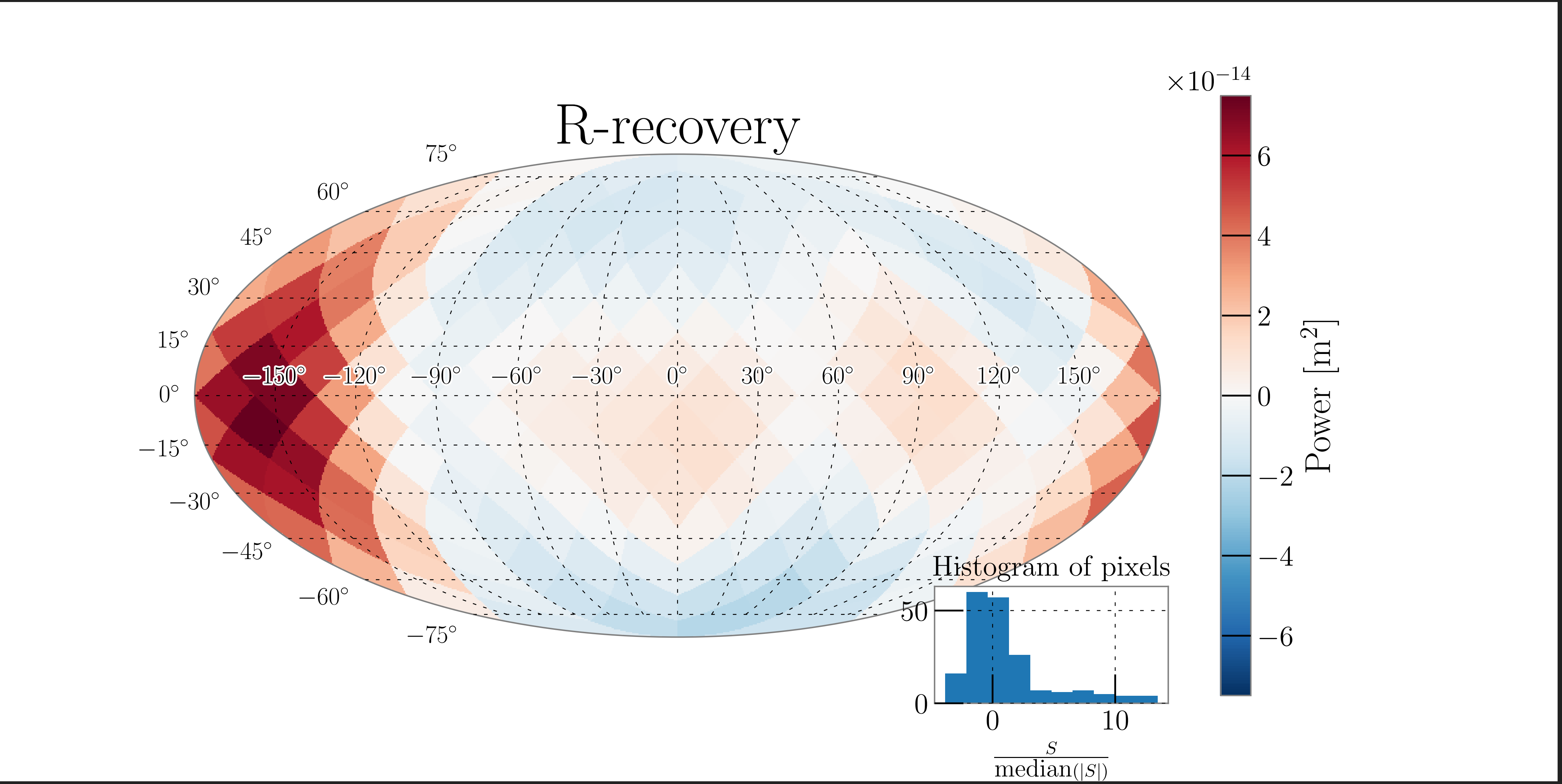


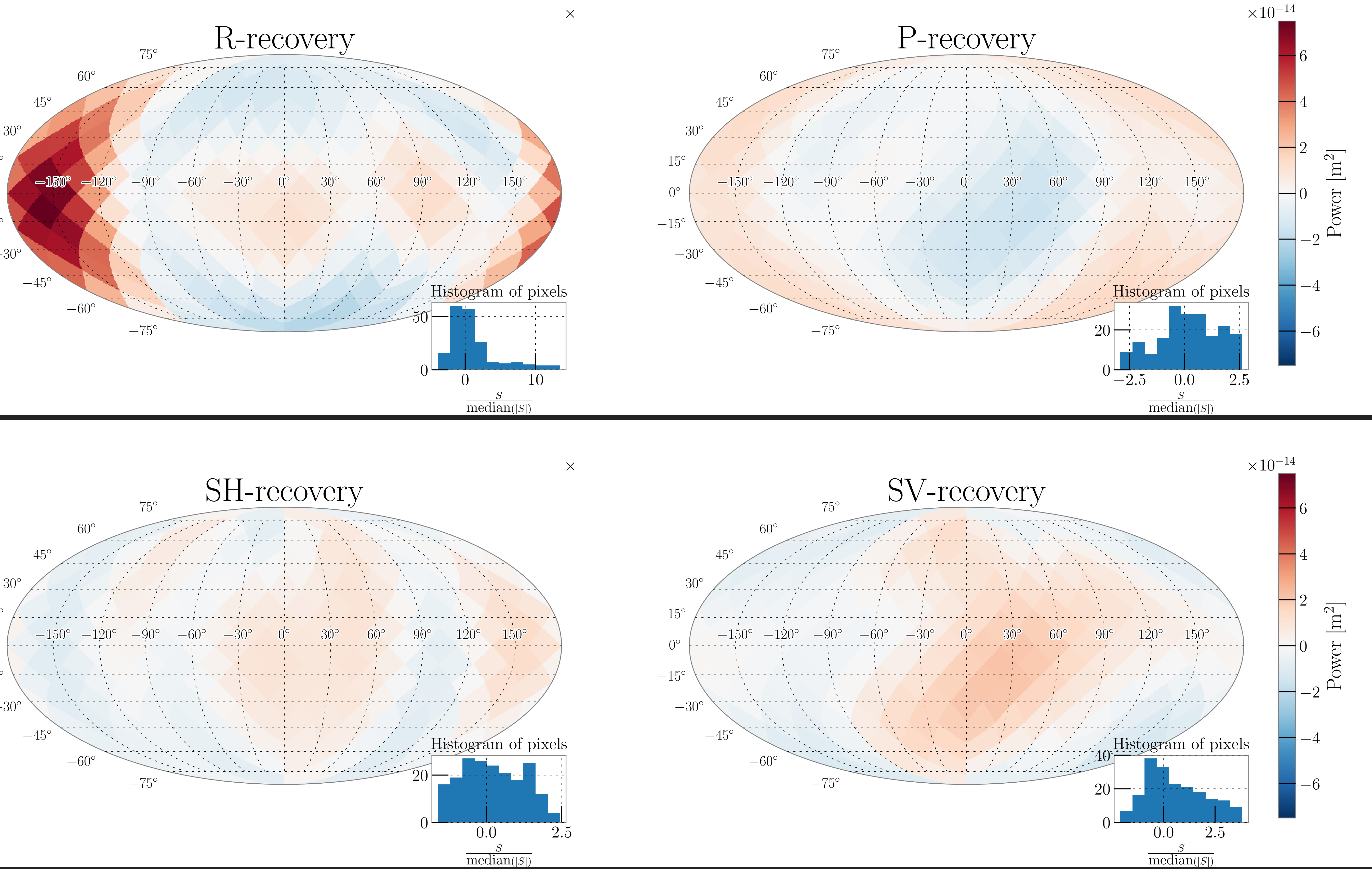
APPLYING EIGENFUNCTION TO SEISMIC RADIOMETER

- We can apply this to the radiometer — all maps **show propagation direction**
  - Microseism measurements from Michael/Jan [I used ~12 hours instead of full day to speed things up]
    - “Noisy day” — consistent with R-waves
    - “Quiet day” — mixed-wave content
  - 1.5 Hz source
  - **Future** — average coherences for mine blasts from roughly same direction
- Using injections, I’ve found that amplitude recoveries can be suspect, but direction information (even for multiple injected sources) is generally reliable.
- I’m still checking the units. There’s a chance I’m missing a (1/(2 * pi * f))^2 in all of these!

MICROSEISM (NOISY DAY)

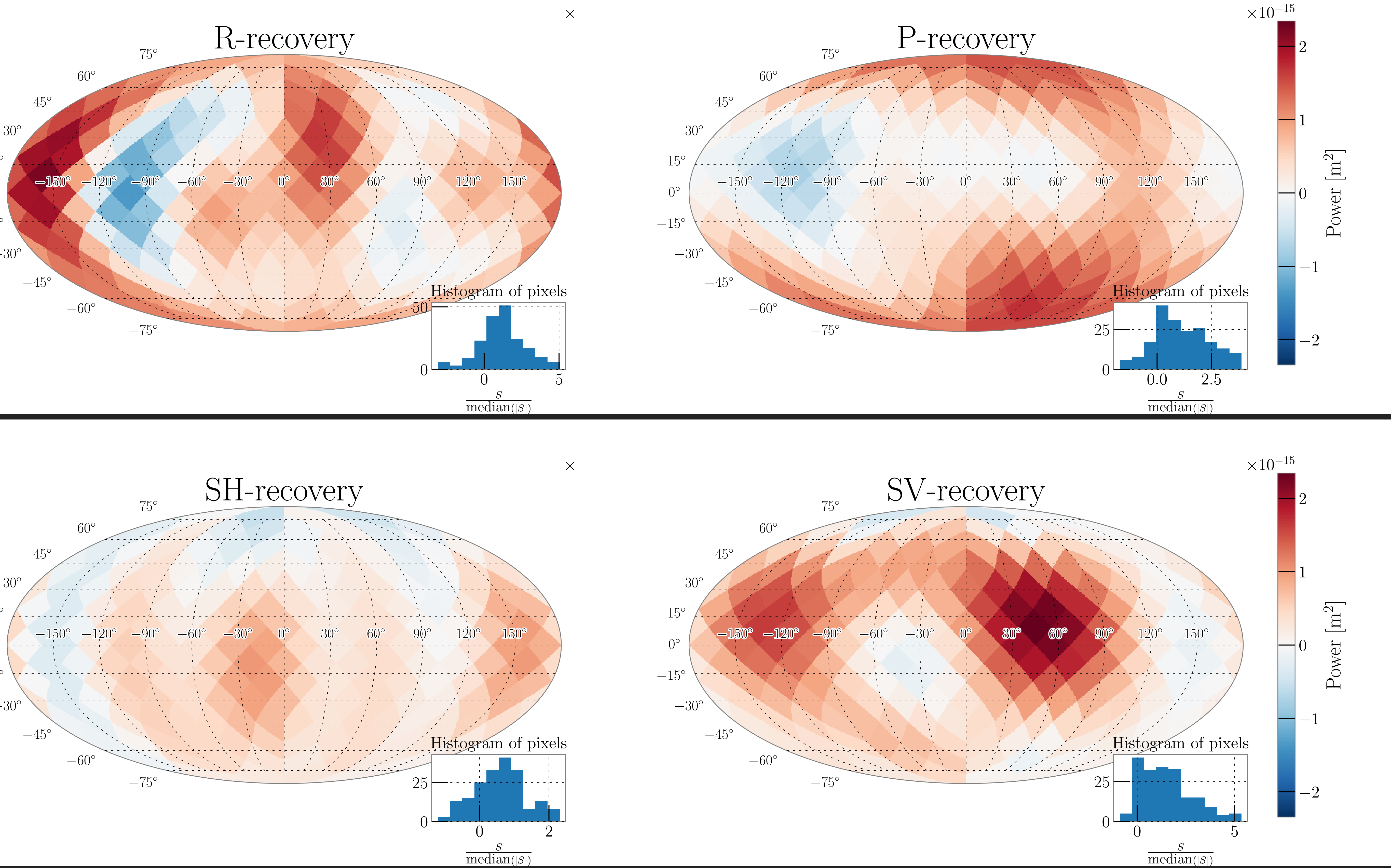
- Indicates wave propagating in **westward** direction





P/S/R -WAVES NOISY DAY

v_R=3.5 km/s, v_P = 7km/s, v_S=5km/s



P/S/R -WAVES QUIET DAY

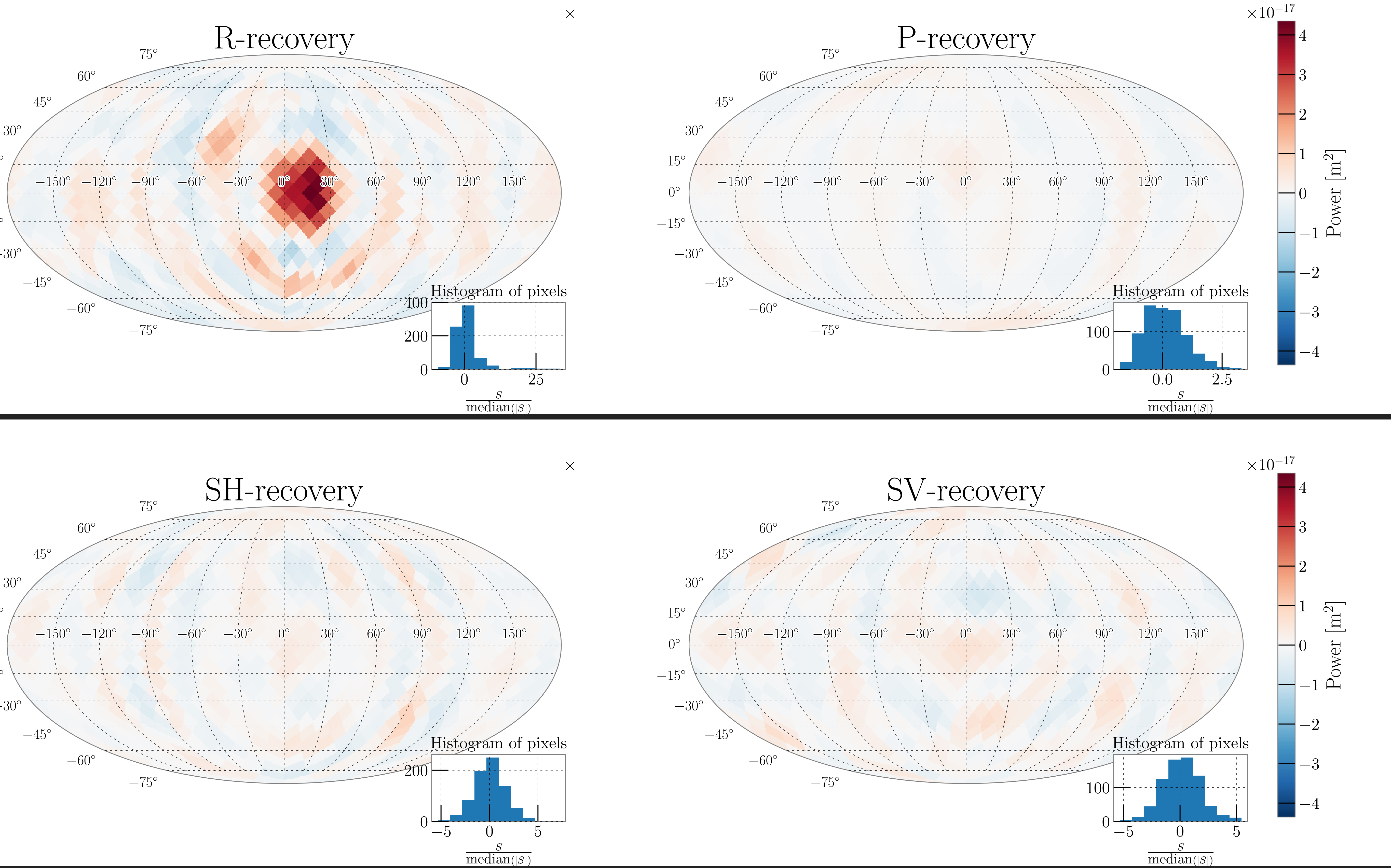
v_R=3.5 km/s, v_P = 7km/s, v_S=5km/s

NOISY DAY

Wave type	Total map power [m^2]
R	1.19×10^{-12}
P	1.59×10^{-13}
S_h	2.24×10^{-13}
S_v	6.33×10^{-13}
Total Body	1.02×10^{-12}

QUIET DAY

Wave type	Total map power [m^2]
R	9.93×10^{-14}
P	9.32×10^{-14}
S_h	5.03×10^{-14}
S_v	1.26×10^{-13}
Body waves	2.7×10^{-13}



1.5 HZ SOURCE

v_R = 2.8 km/s, v_P = 6 km/s, v_S = 3.5km/s

1.5 HZ SOURCE

Wave type	Total map power $[10^{-15}\mathrm{m}^2]$
R	1.17
P	0.16
S_h	0.08
S_v	0.15
Total Body	0.39

CONCLUSIONS

- Hoping to run on a few more interesting frequencies
- Maybe convert these into NN measurements
- Try MCMC sampling, which could naturally give uncertainties.