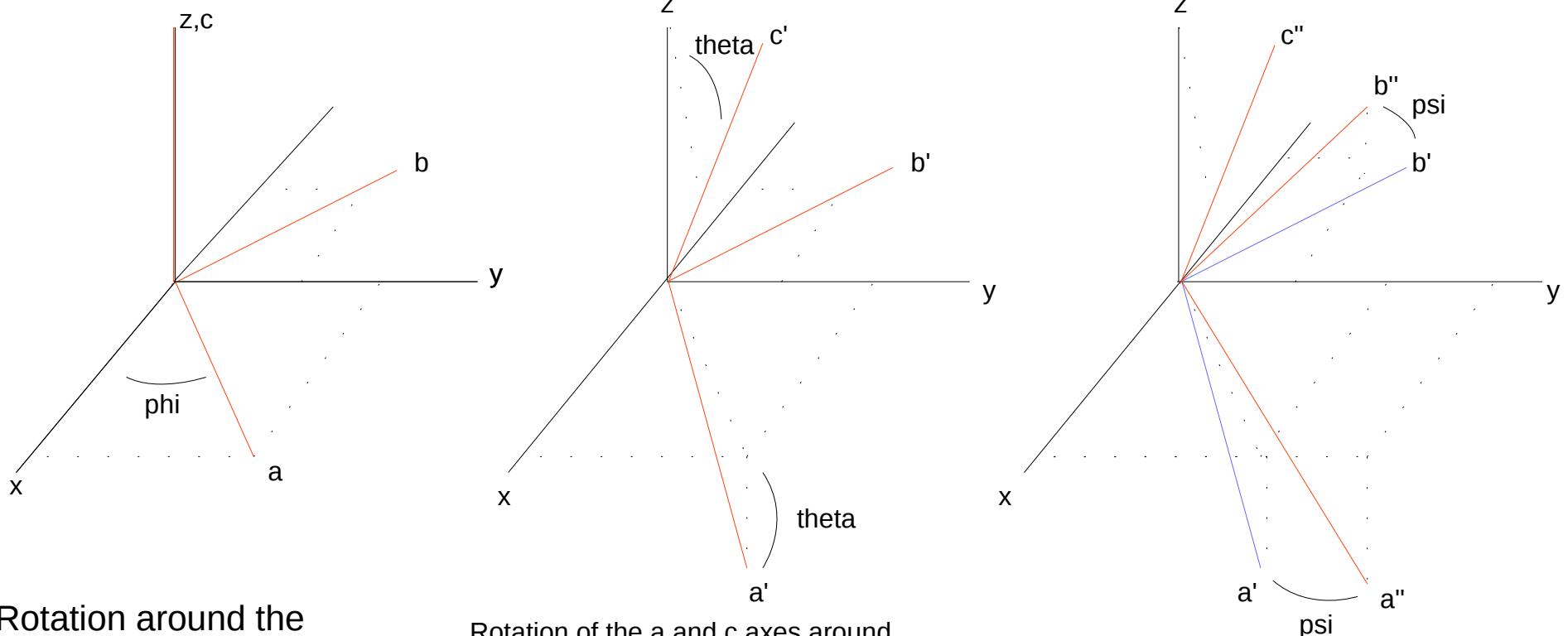


Swave Tests 2.0

Noah Bittermann

Polarization Convention

The following diagrams depict how Euler Angles are used to construct and characterize S-waves. Let x,y,z be set axes of the coordinate system in which the wave, passing through the origin, is observed.



Rotation around the
z axis by phi to
form the a,b, and c
axes

Rotation of the a and c axes around
the b axis to form the a', b', and c'
axes. Note the deviation from the
“usual” euler characterization, which
instead contains a rotation by theta
around the a axis. The purpose of this
deviation is so that (theta, phi) signifies
the propagation direction of the wave
in geographical coordinates, which are
used throughout

Rotation of the a'
and b' axes around
the c' axis to form
the a'', b'', and c''
axes.

Polarization Convention Cont.

Hence using these transformations, c'' is the propagation direction of the S-wave, and is characterized by (θ, ϕ) as it would be in geographical coordinates. The accumulation of all of the preceding transformations is below:

$$\begin{pmatrix} \cos(\psi)\cos(\phi+90)-\cos(\theta)\sin(\phi+90)\sin(\psi) \\ \cos(\psi)\sin(\phi+90)+\cos(\theta)\cos(\phi+90)\sin(\psi) \\ \sin(\theta)\sin(\phi+90) \end{pmatrix}$$

Ψ is the polarization angle; it characterizes in which direction the amplitude of the wave oscillates. It is implicitly defined so that when $\psi = 0$, the amplitude oscillates in the b'' direction. This was determined by substituting $\theta = 0$, $\phi = 0$, and $\psi = 0$ into the above transformation, so that a'' , b'' , c'' coincided with x, y, z . The calculation gave $\text{Transpose}([0, 1, 0])$.

Scale Testing

- The following parameters are kept constant:

number of detectors = 8

Phi = 90 degrees

Theta = 30 degrees

Psi = 45 degrees

monochromatic source of freq = 1 Hz

Amplitude = 10 m

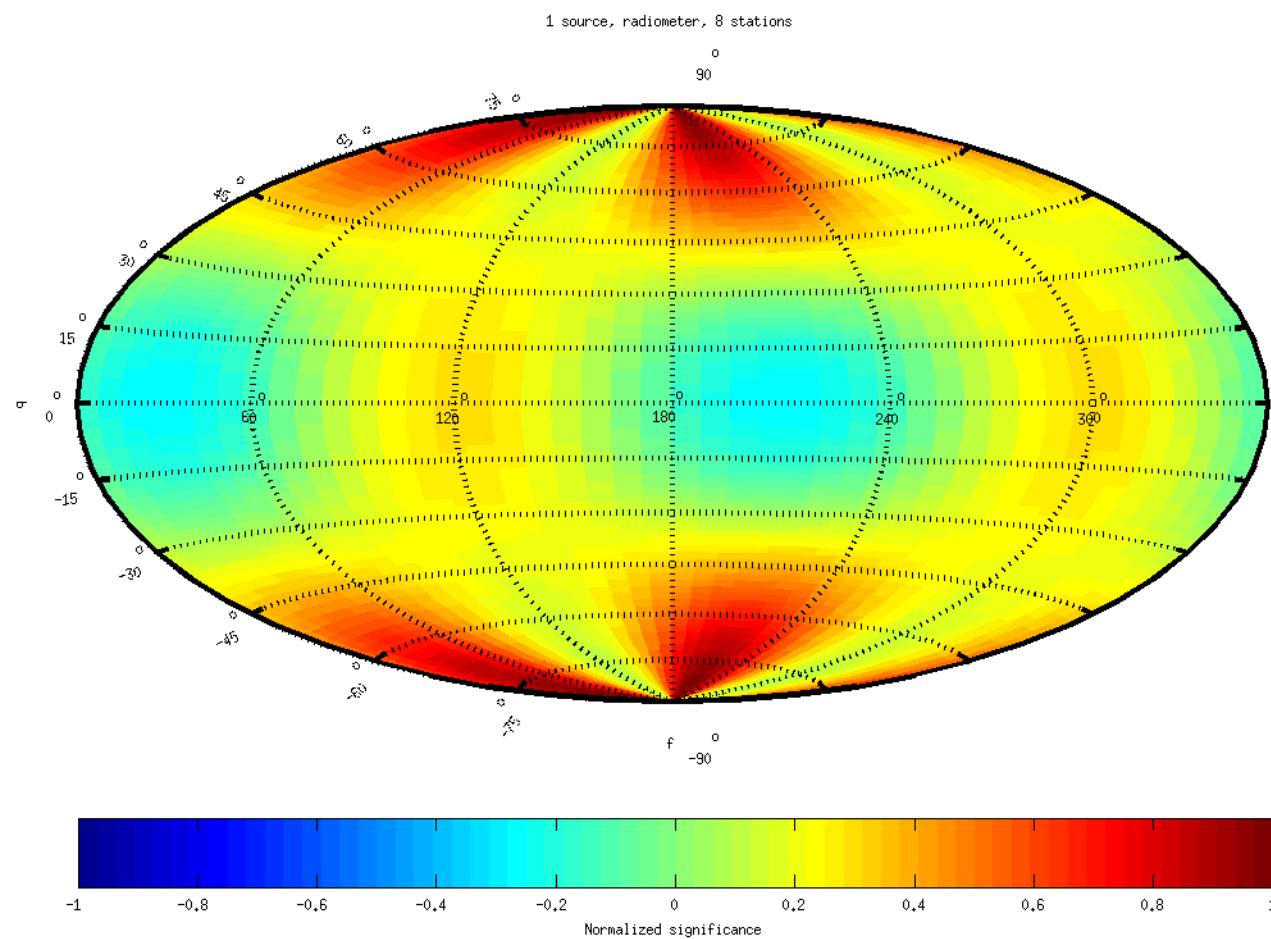
Speed = 2100 m/s

Detector Location is varied thusly:

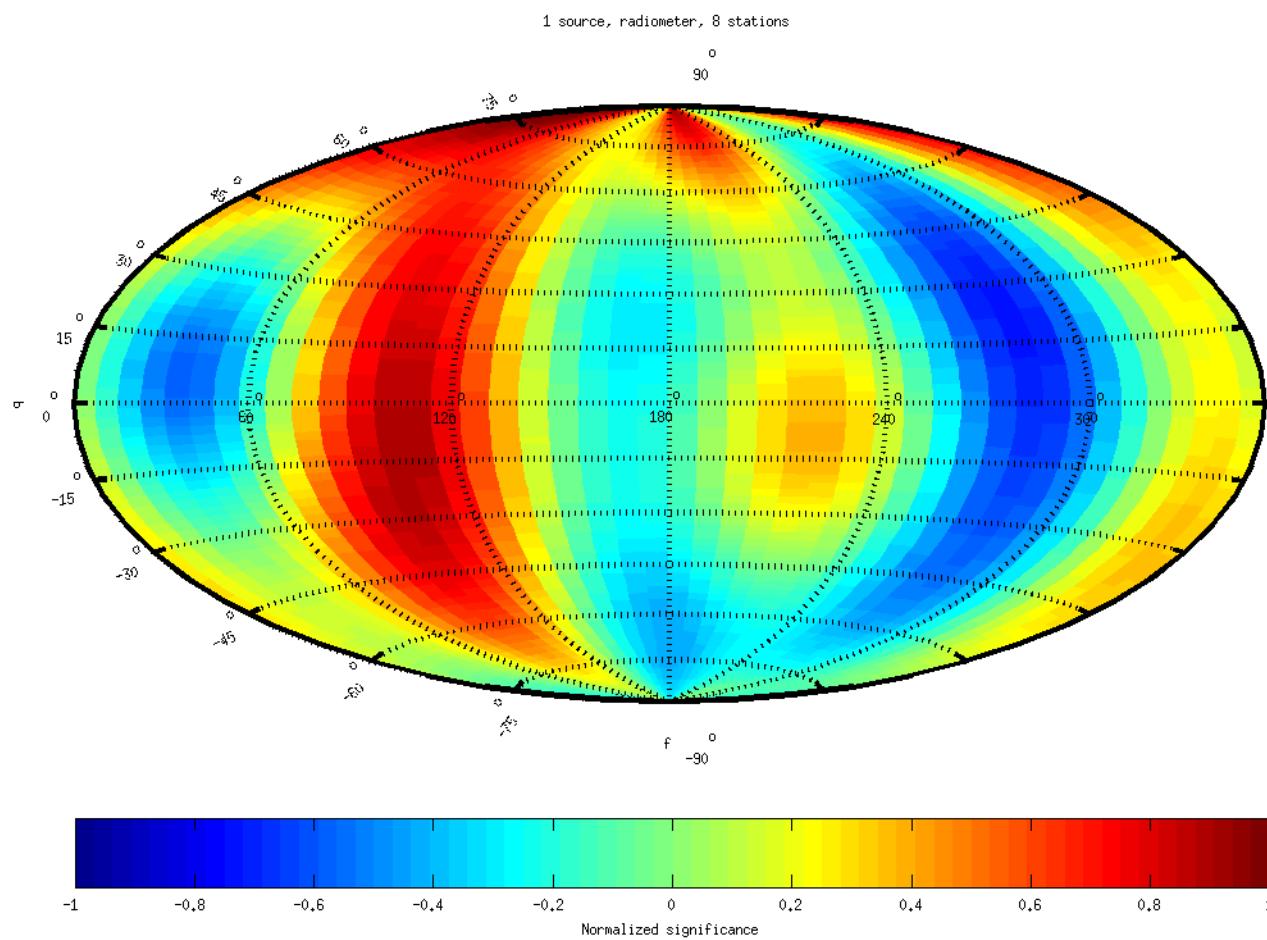
detloc = scale*

0.2356	0.2256	0.2556
0.2257	0.2978	0.1350
0.5375	0.9833	0.4396
0.9891	0.0892	0.1755
0.8970	0.7286	0.9501
0.8163	0.8914	0.2313
0.1514	0.5209	0.7084
0.1264	0.5037	0.8124

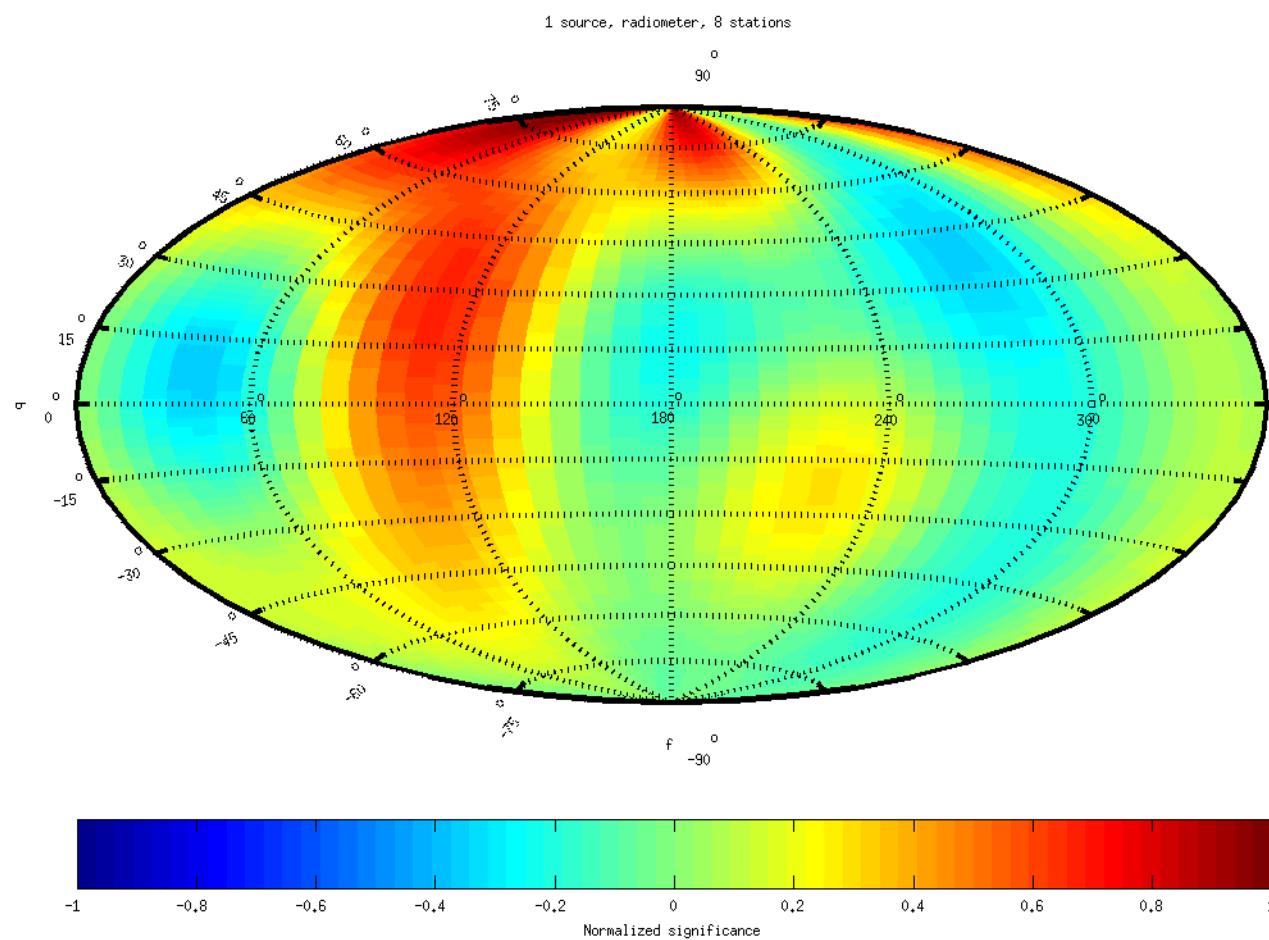
Scale = 1 m nstest1



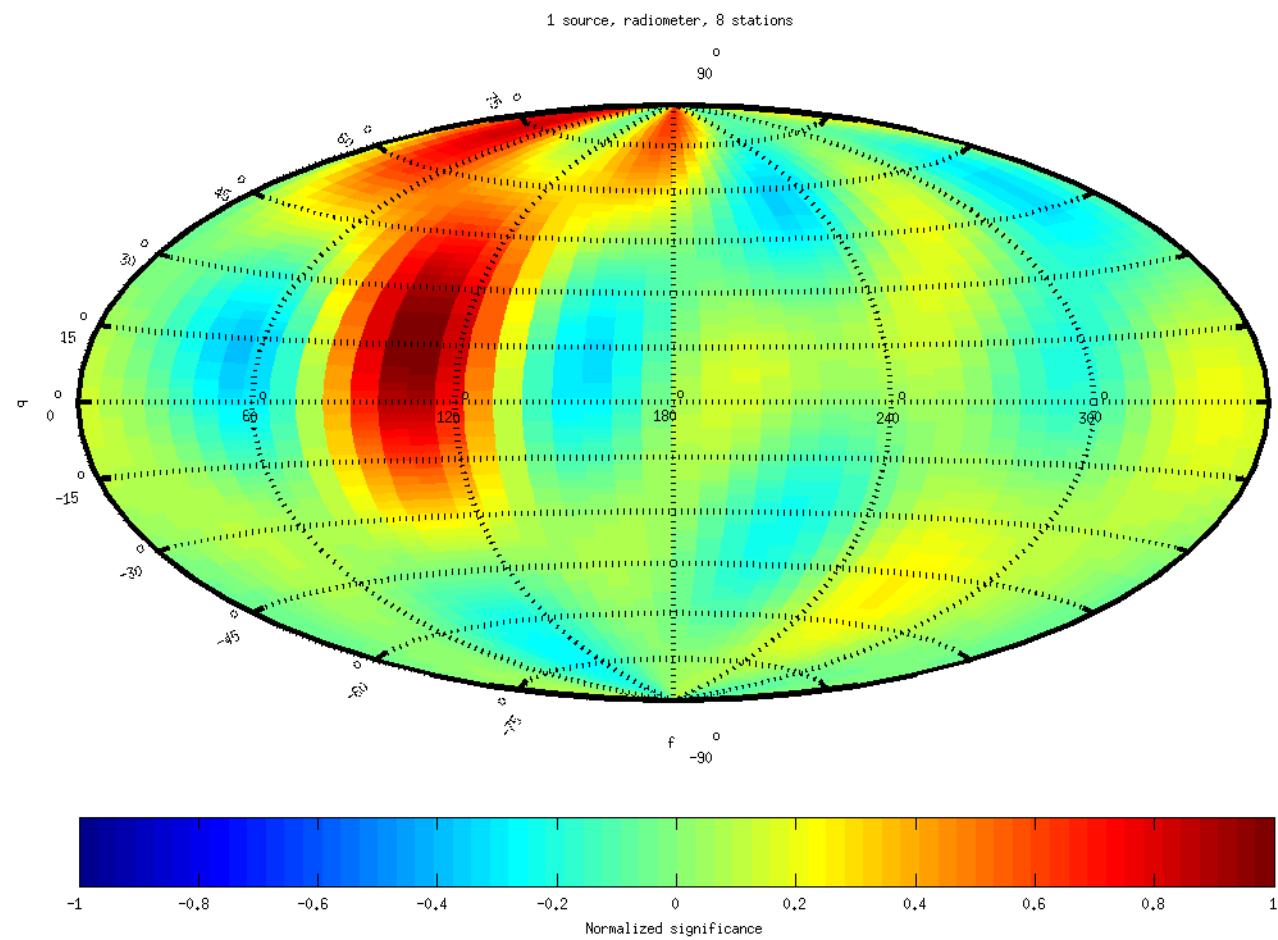
Scale = 10 m nstest2



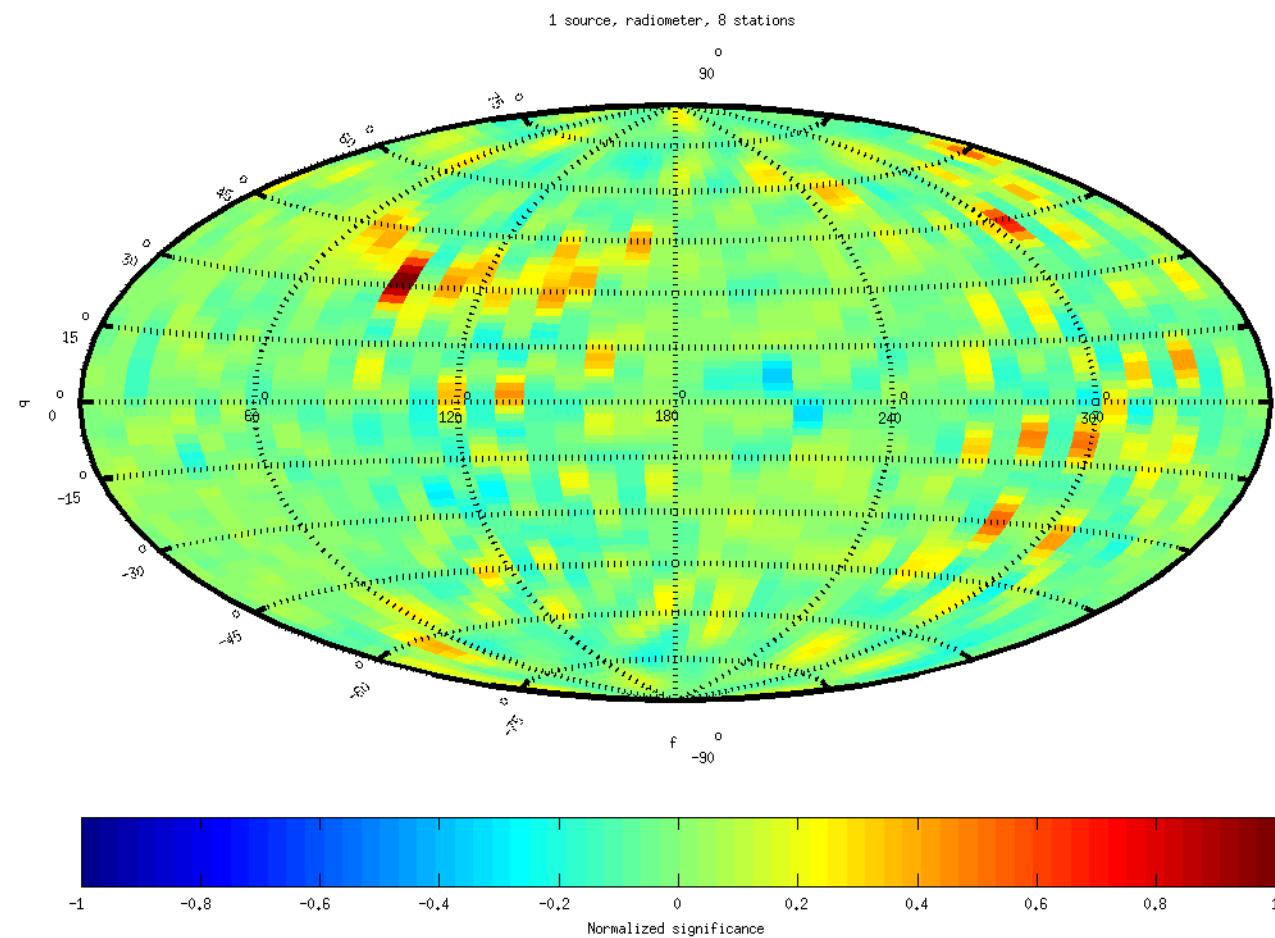
Scale = 100 m nstest3



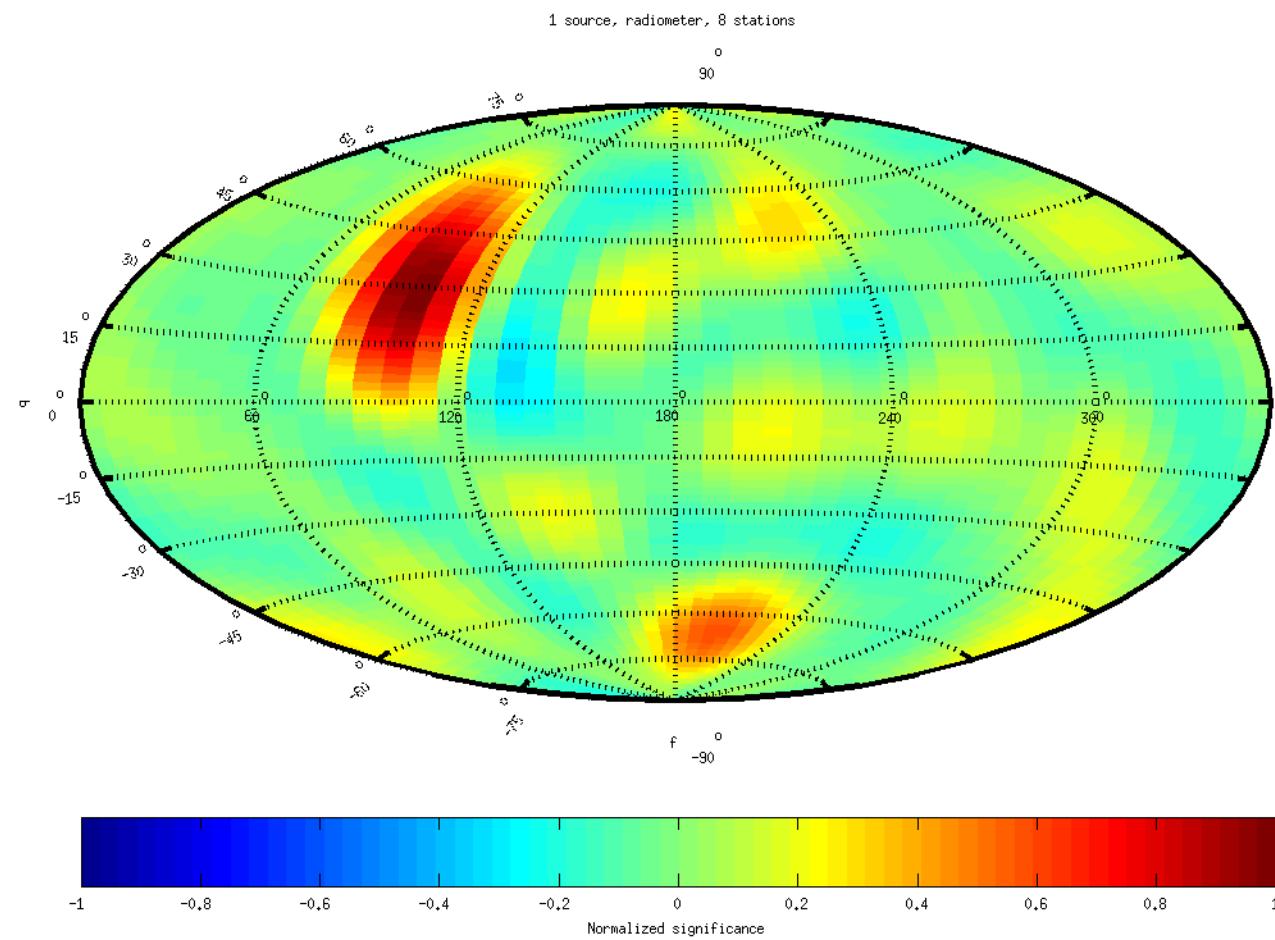
Scale = 1000 m nstest4



Scale = 10000 m nstest5



Scale = 2000 m nstest6

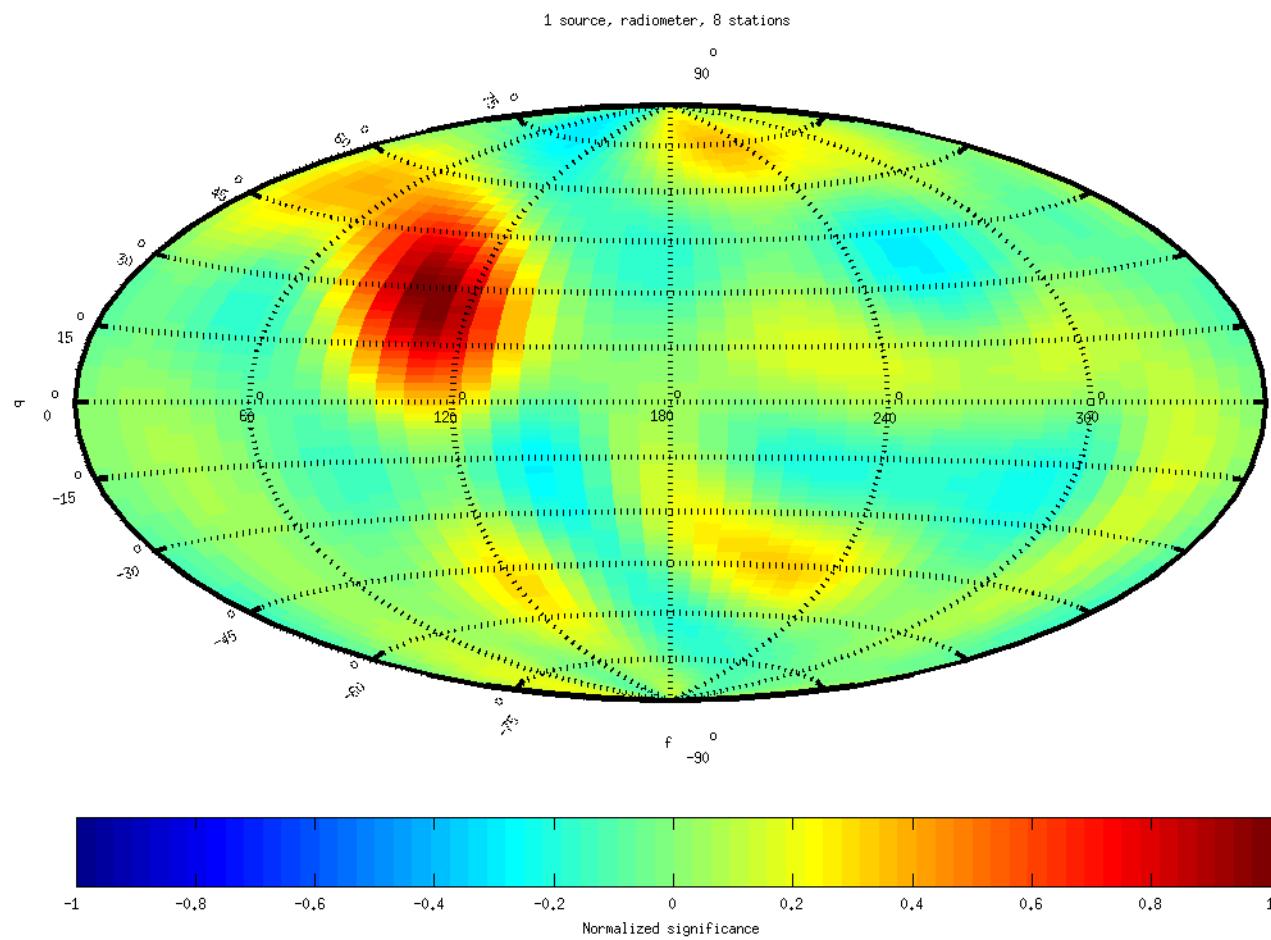


Detector Location Testing

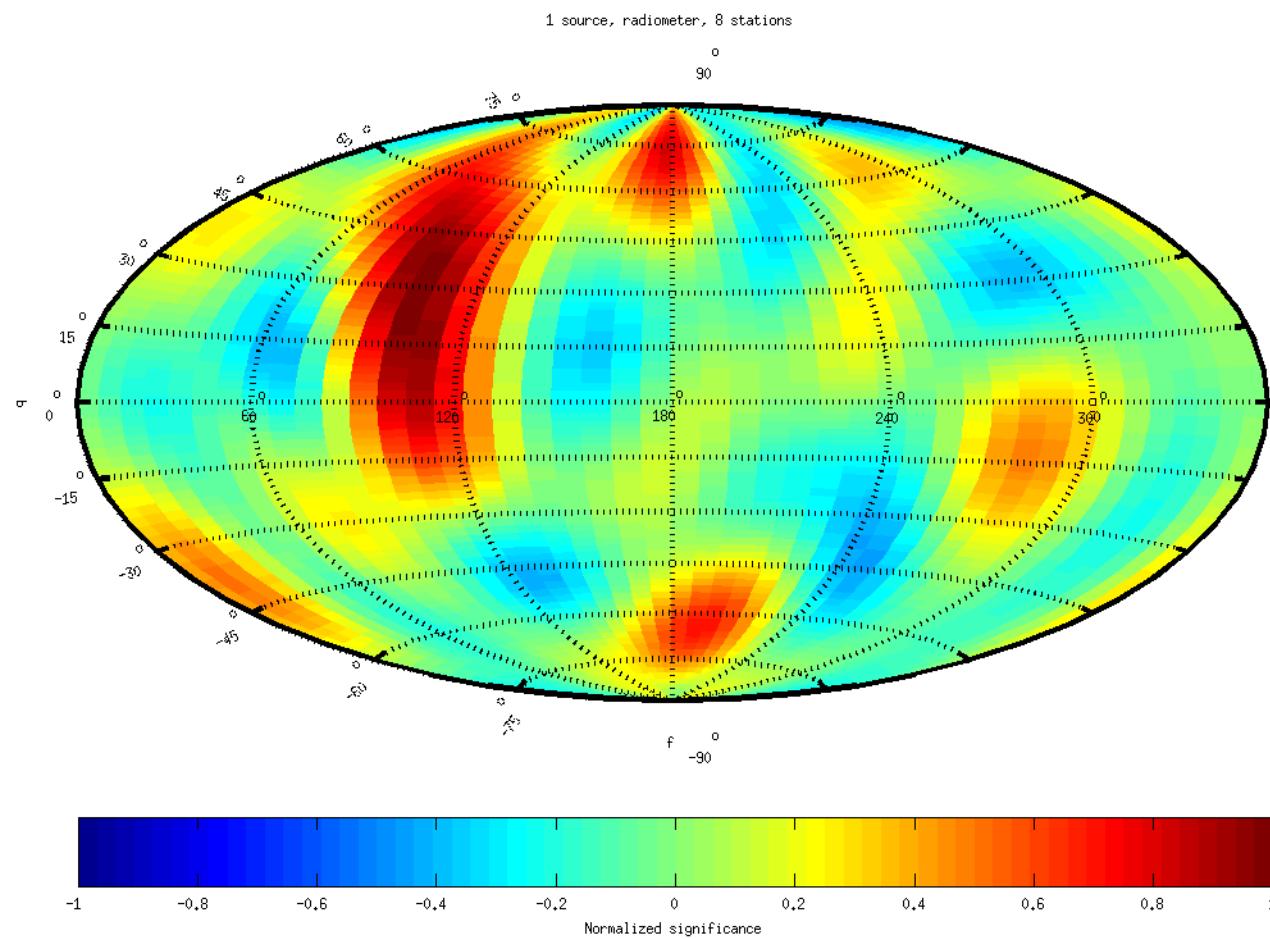
- In this series of test, random detectors are chosen and placed in a box of size 2000 m. All arrays below should be multiplied by 1000 m. All other parameters are the same as before.

nstest7	nstest8	nstest9
0.1029	0.2454	0.8232
0.8982	0.8457	1.9393
1.4311	0.8961	0.6799
1.2009	0.0914	0.3934
1.0414	0.5484	1.6074
0.8644	1.5812	0.5884
1.8625	1.5193	0.1478
0.7602	0.2315	0.9579
nstest10	nstest11	
0.7531	1.8556	1.0045
0.8954	1.9566	1.0640
1.2440	1.8548	1.9017
0.9623	1.9656	0.7943
1.1583	1.9147	0.2922
1.0722	0.7694	0.6055
0.6859	0.2447	0.5166
1.5639	1.4368	0.8301

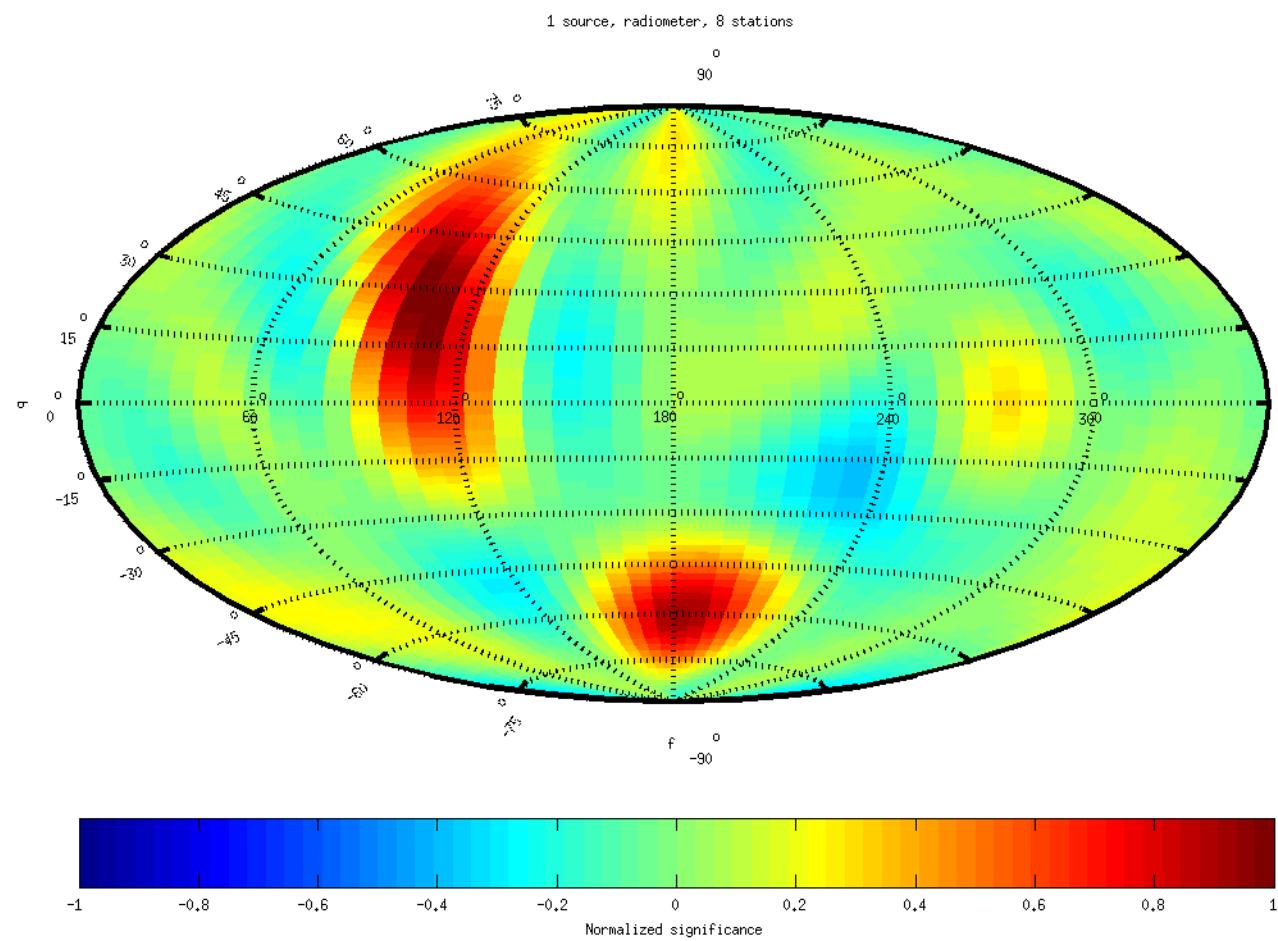
nstest7



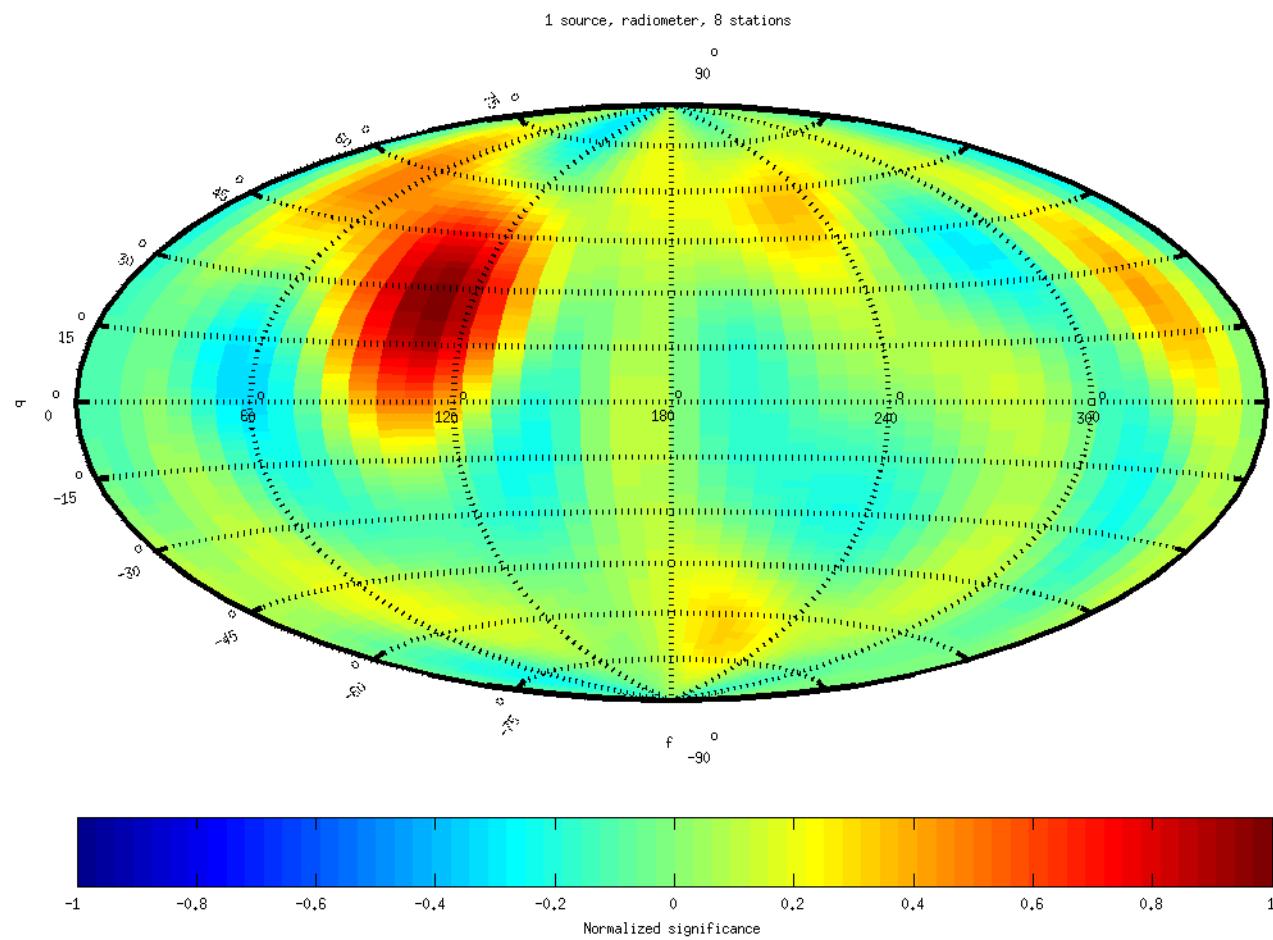
nstest8



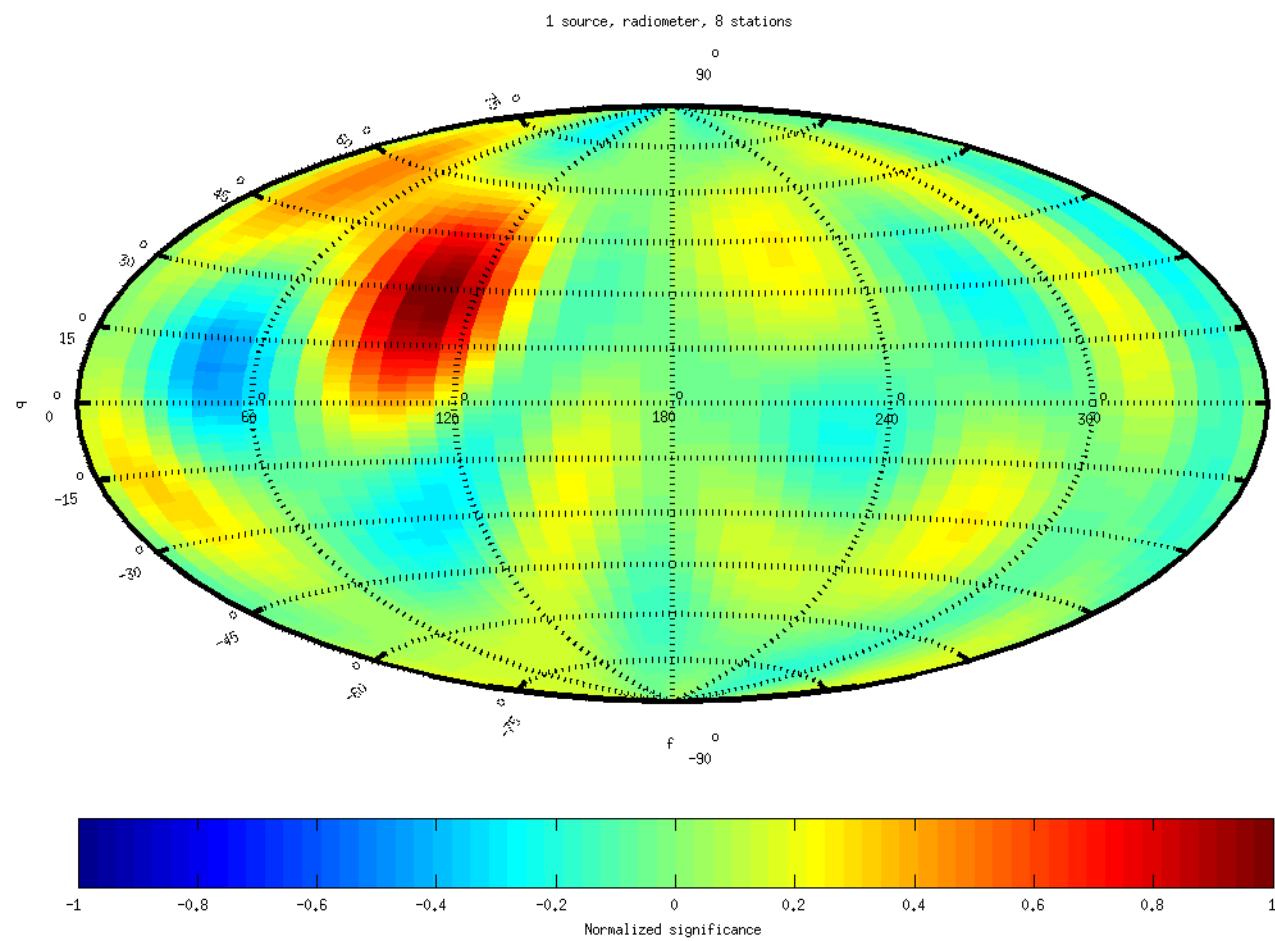
nstest9



nstest10



nstest11



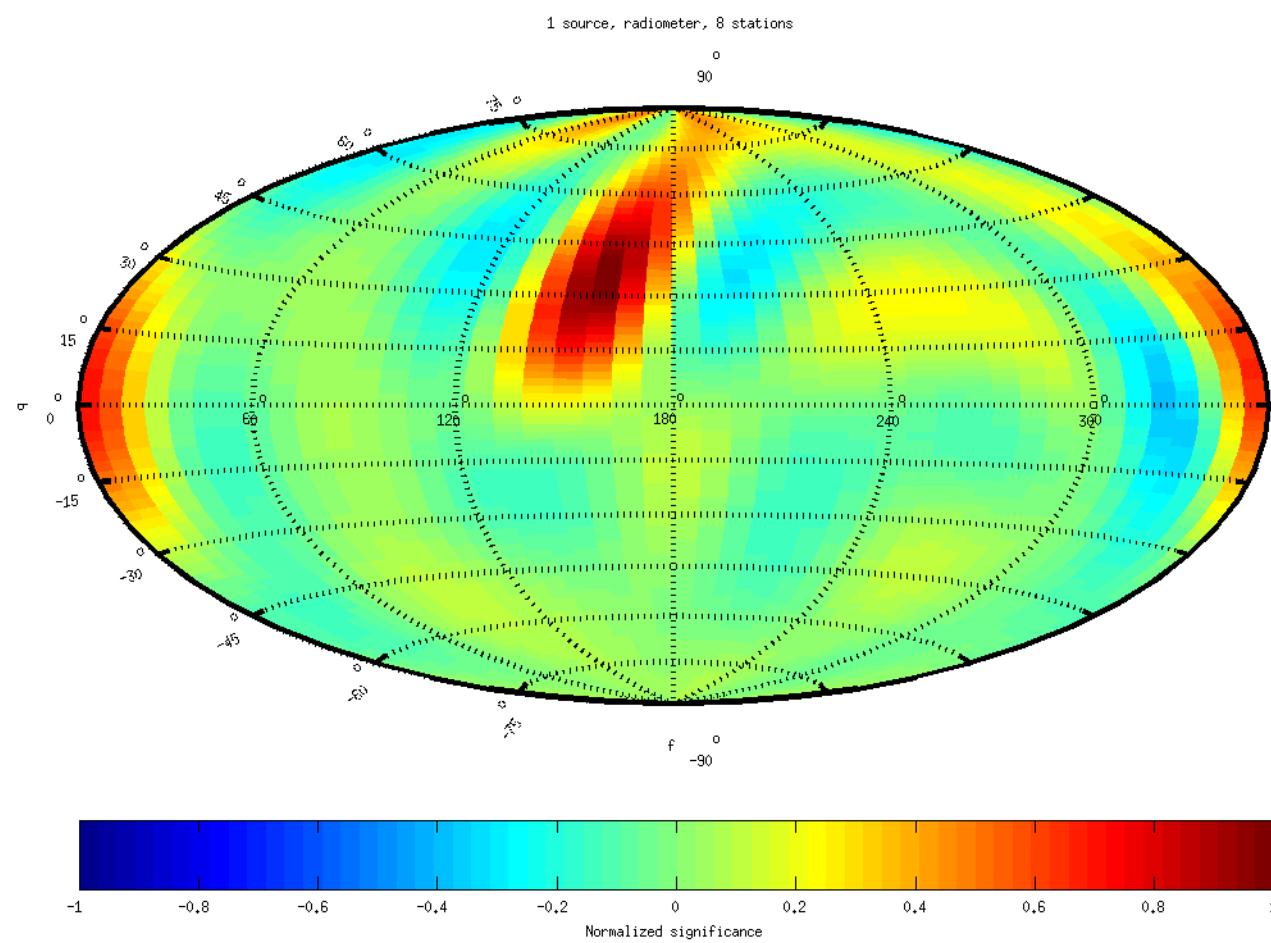
Source Location Testing

The following detector array is used(from nstest11):

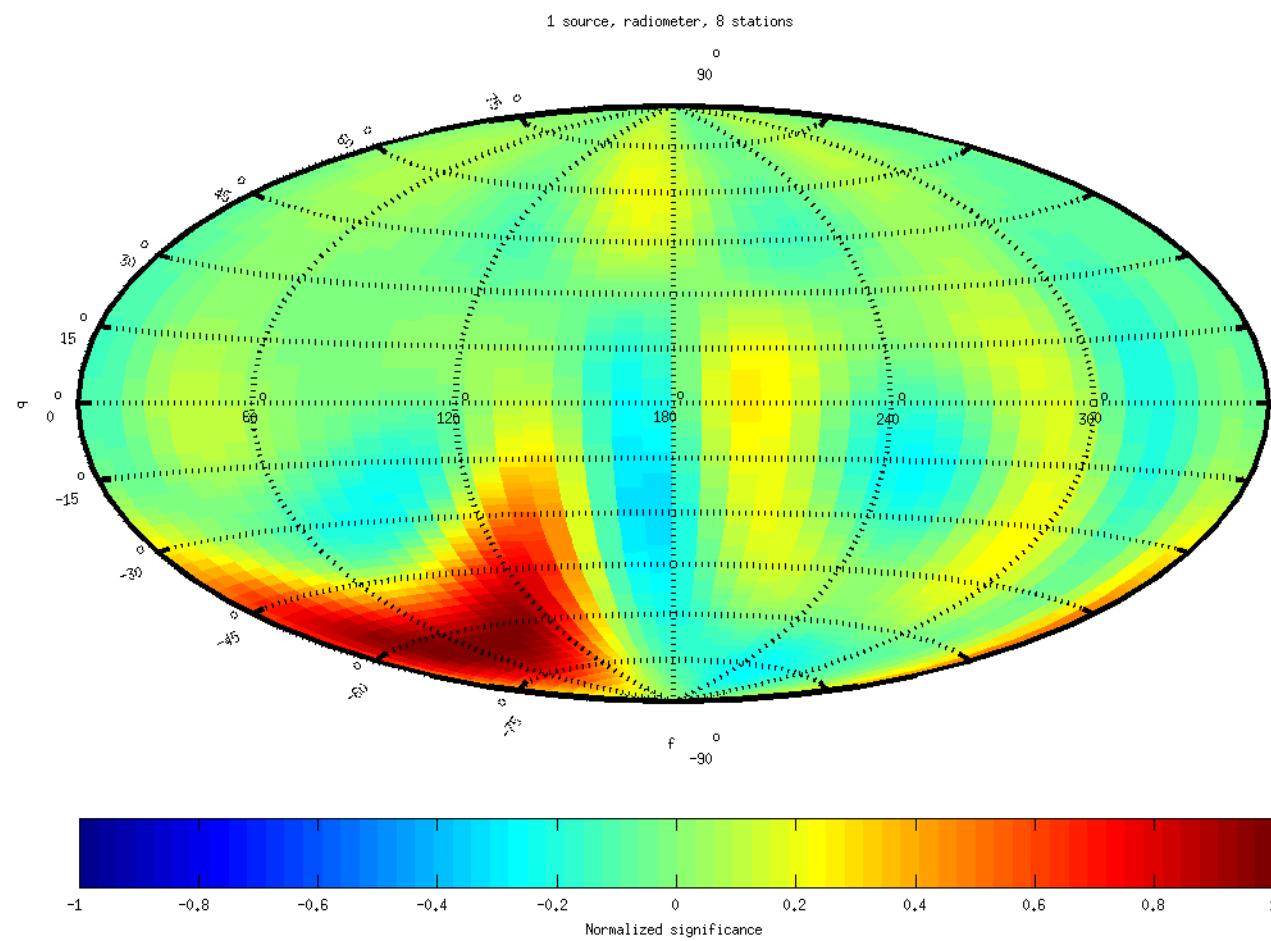
```
detloc = 1.0e+03 m *
1.0049  0.8696  1.5024
0.9540  1.4846  0.5536
1.7282  1.7792  1.8847
0.8537  0.4295  1.0085
0.7172  0.3935  1.1054
1.4601  1.3522  1.1130
0.8472  0.0663  1.3949
0.8863  0.5722  0.4442
```

The source location is chosen randomly. All other parameters are the same as before.

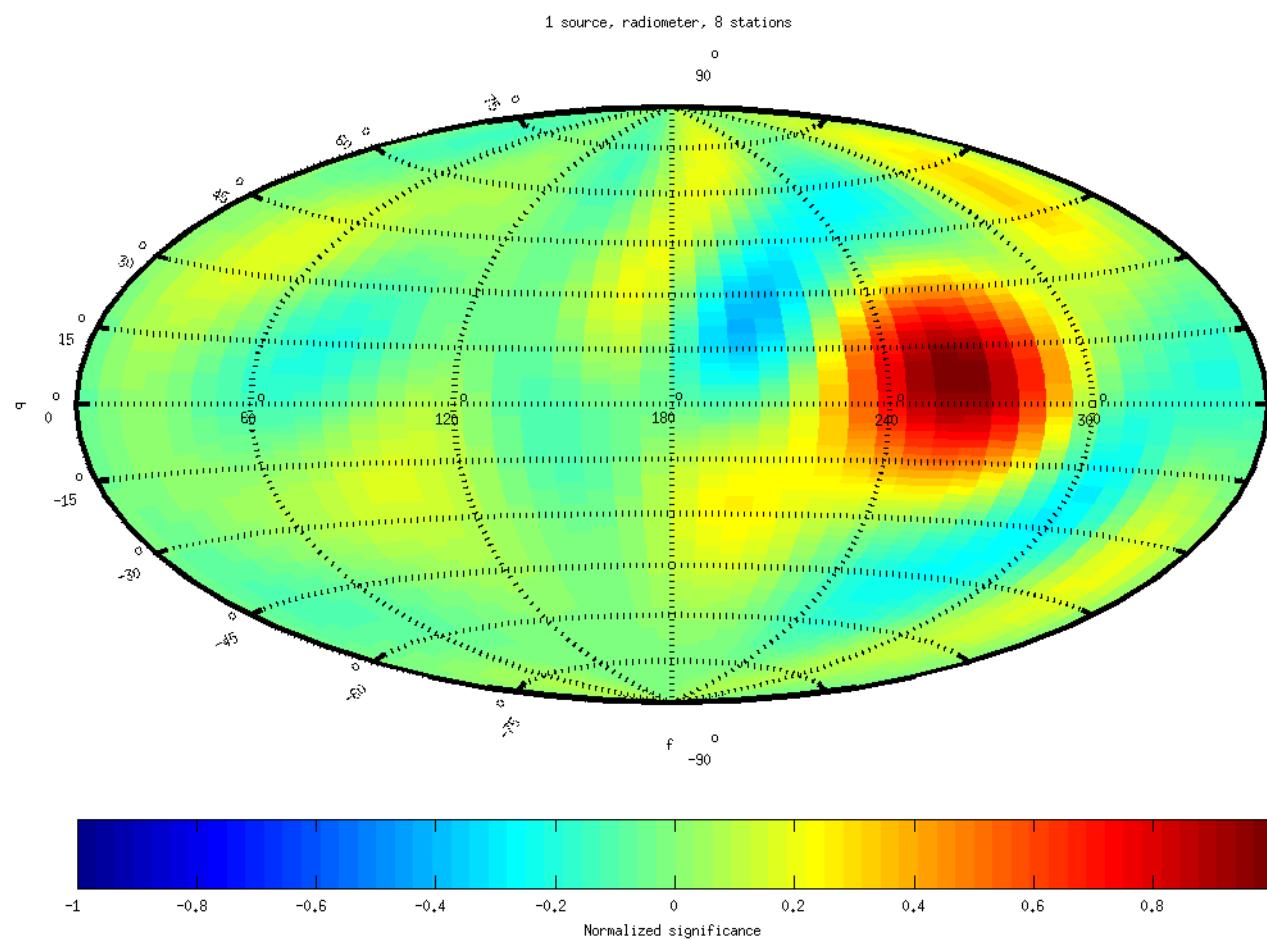
(theta, phi) = (39.8411, 138.4983) nstest12



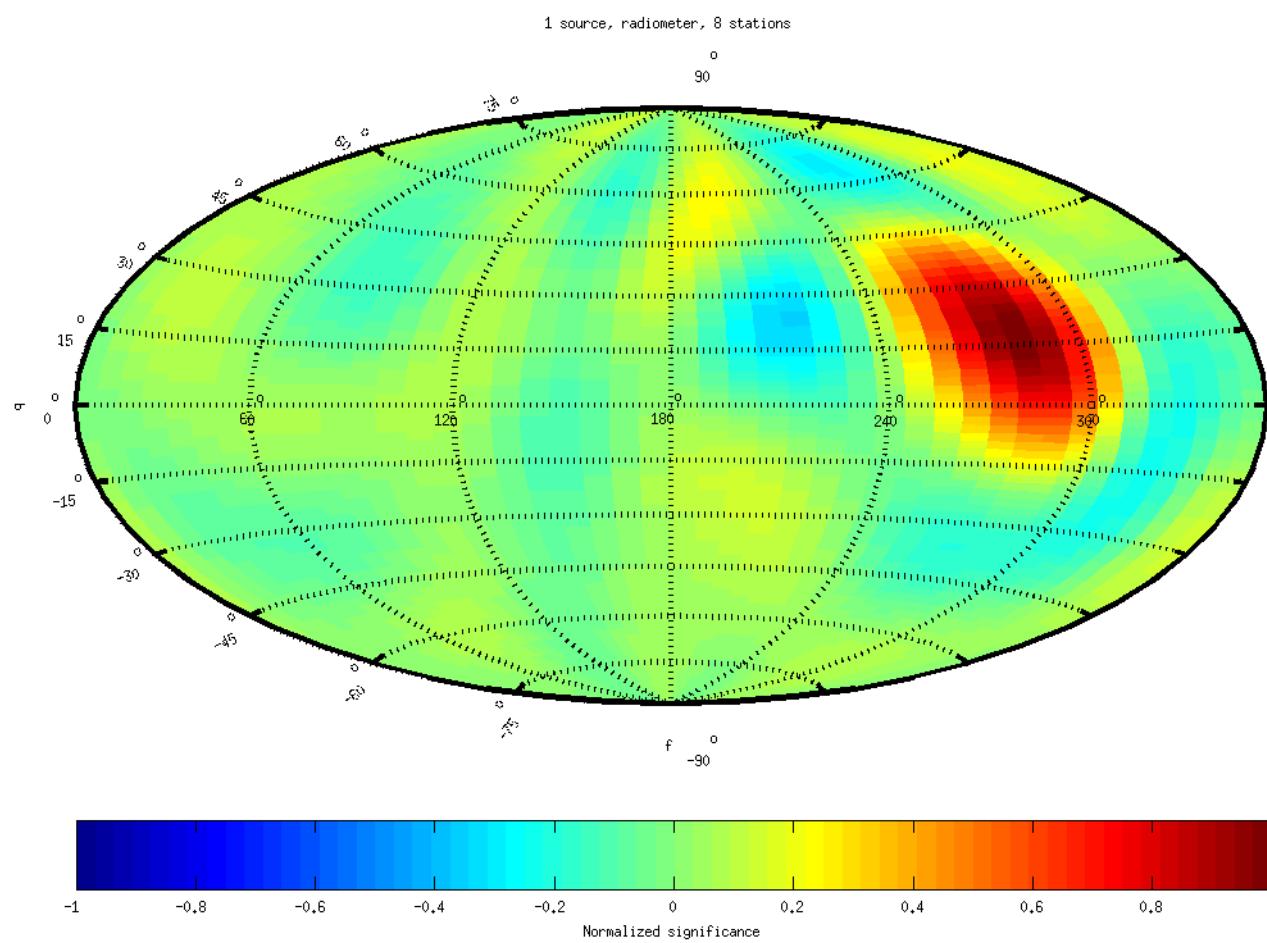
(theta, phi) = (-63.203634.6356) nstest13



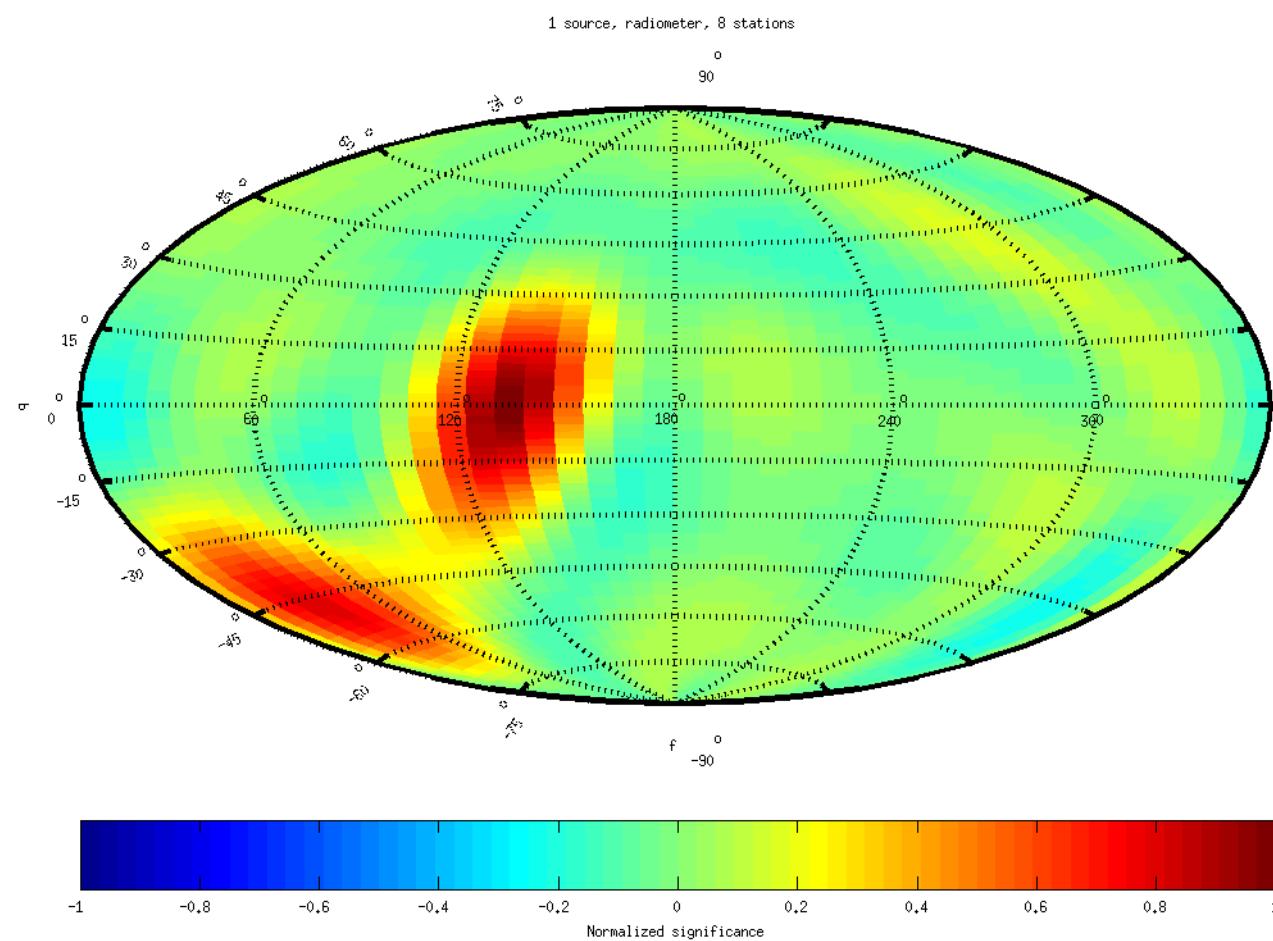
(theta, phi) = (8.1062, 243.8633) nstest 14



(theta, phi) = (27.3395, 249.6201) nstest15



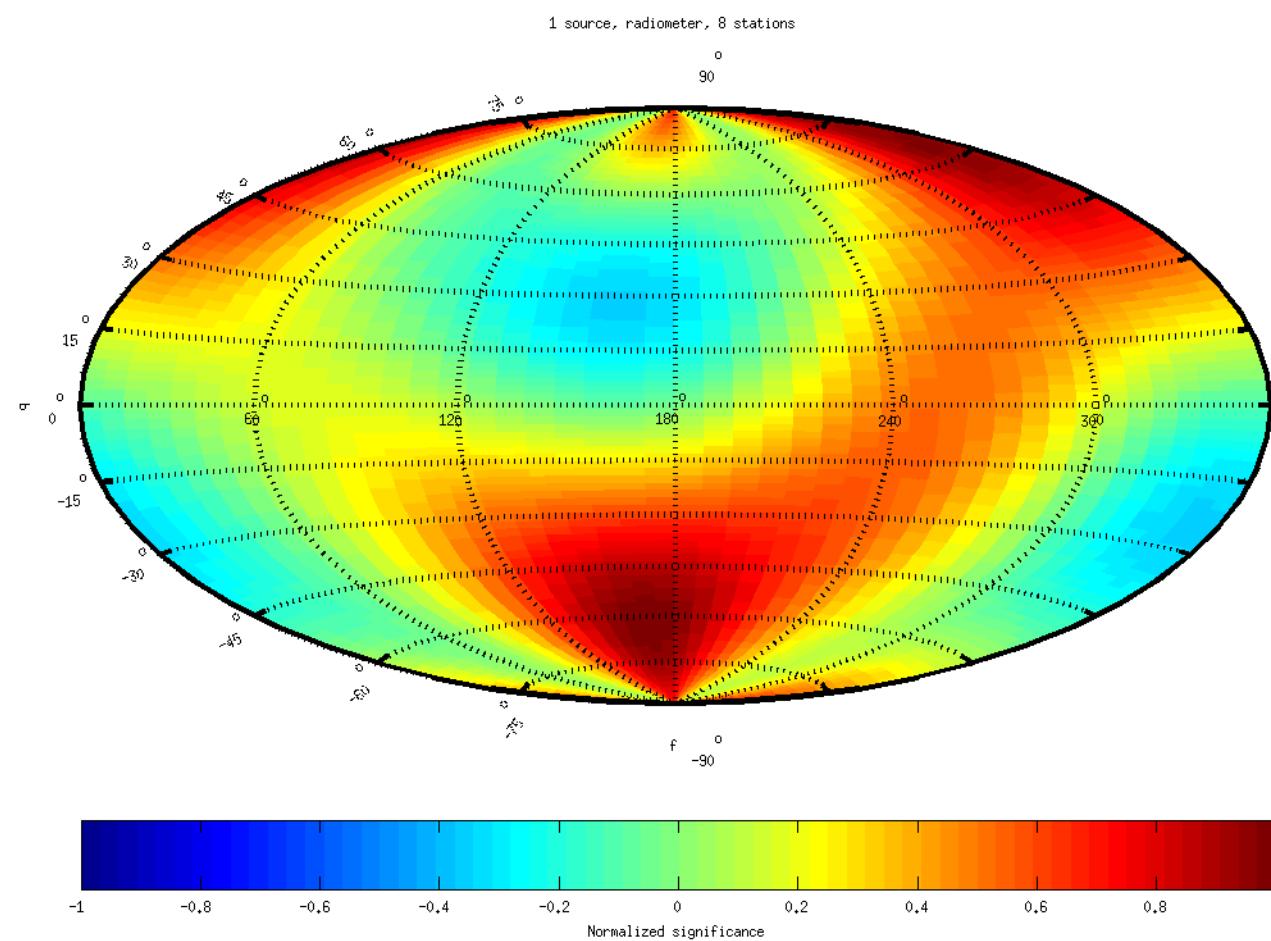
(theta, phi) = (-11.5677,127.6468) nstest16



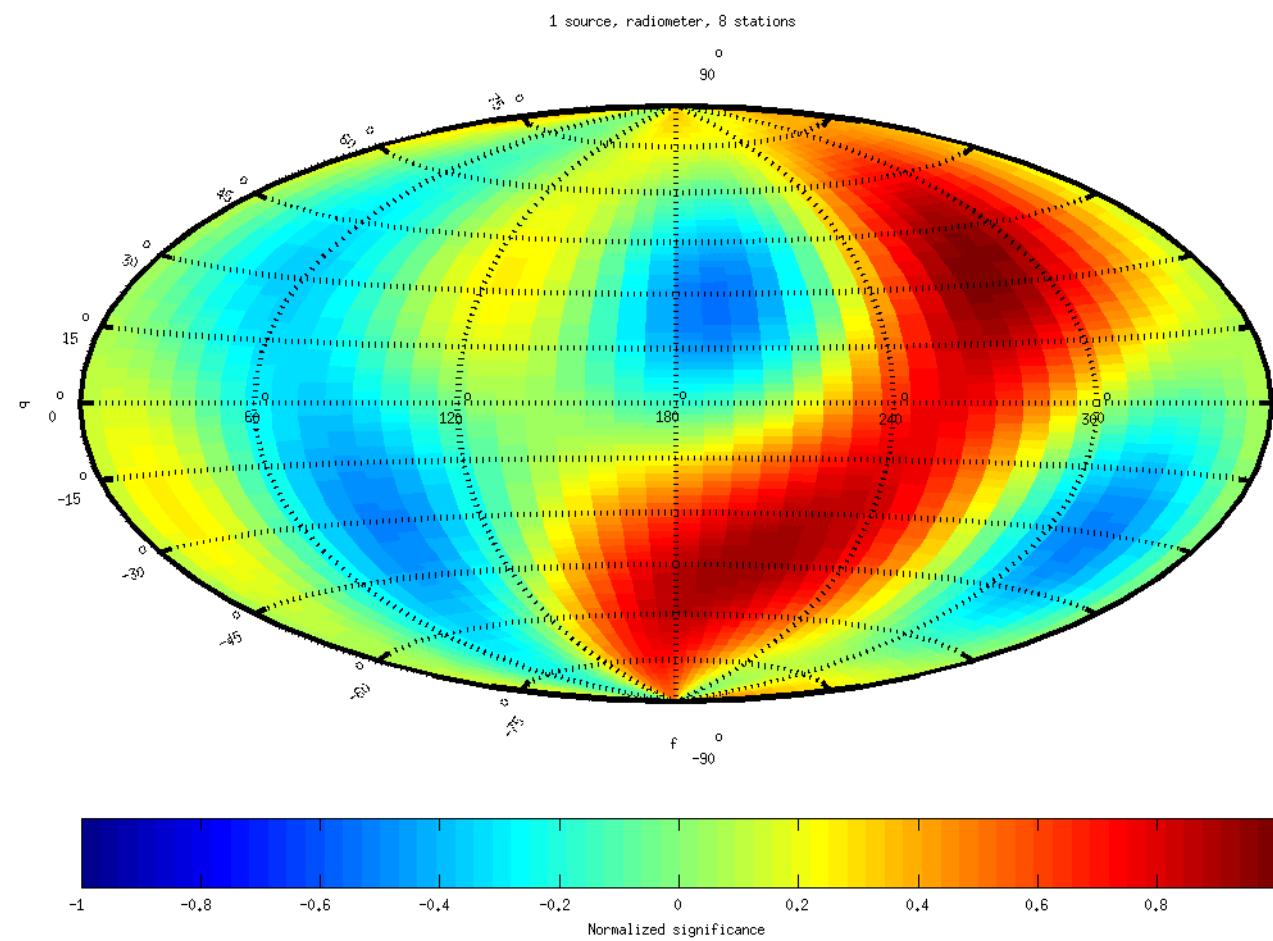
Frequency Dependence Testing

- In the following tests a source location of (theta, phi) = (8.1062, 243.8633) (from nstest14). The frequency of the injected data and of the recovery are varied. All other parameters are kept the same as before. `f_analyse` indicates the frequency of the injected data and of the recovery.

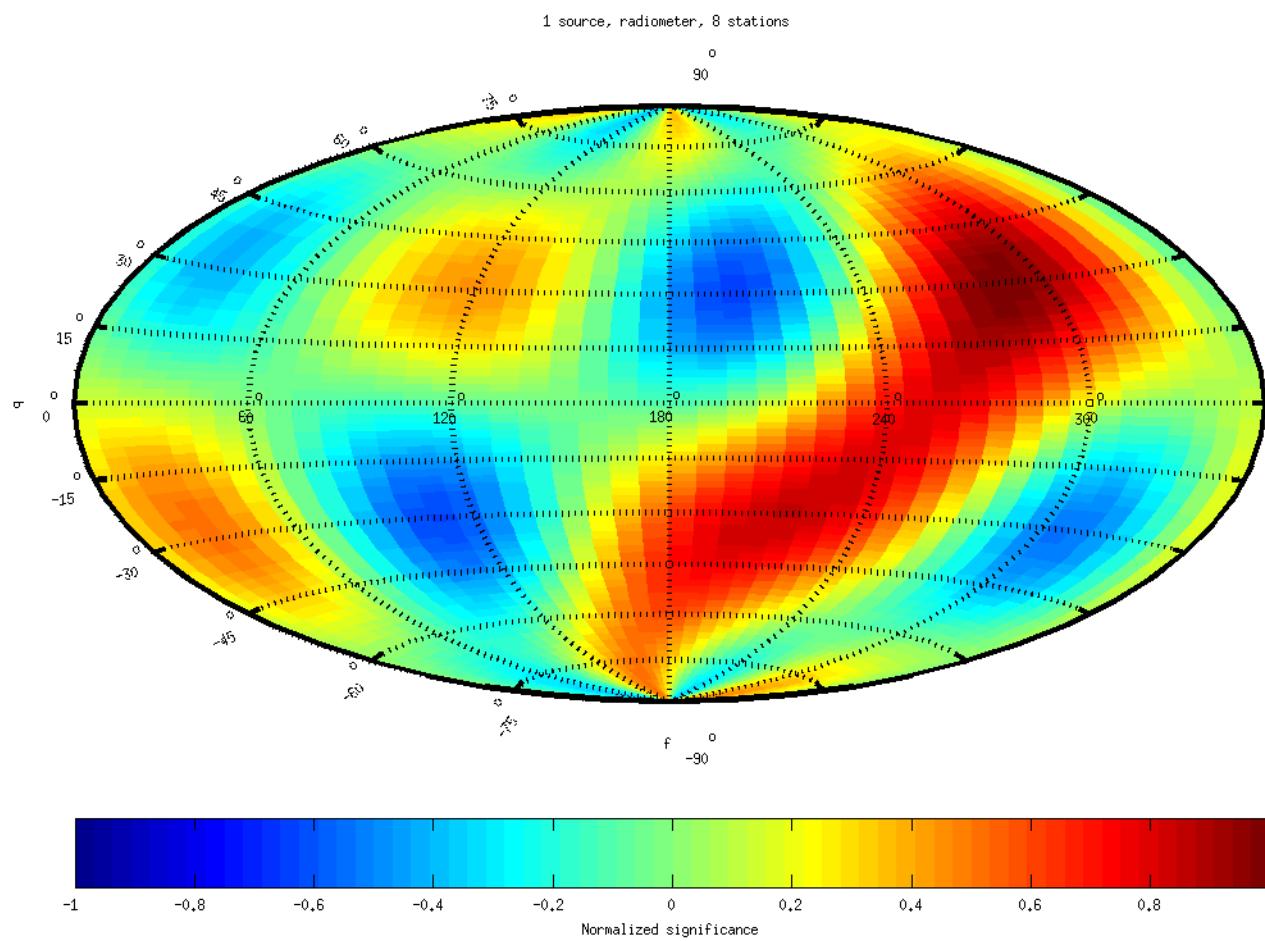
$f_{\text{analyse}} = .001 \text{ Hz}$ nstest17



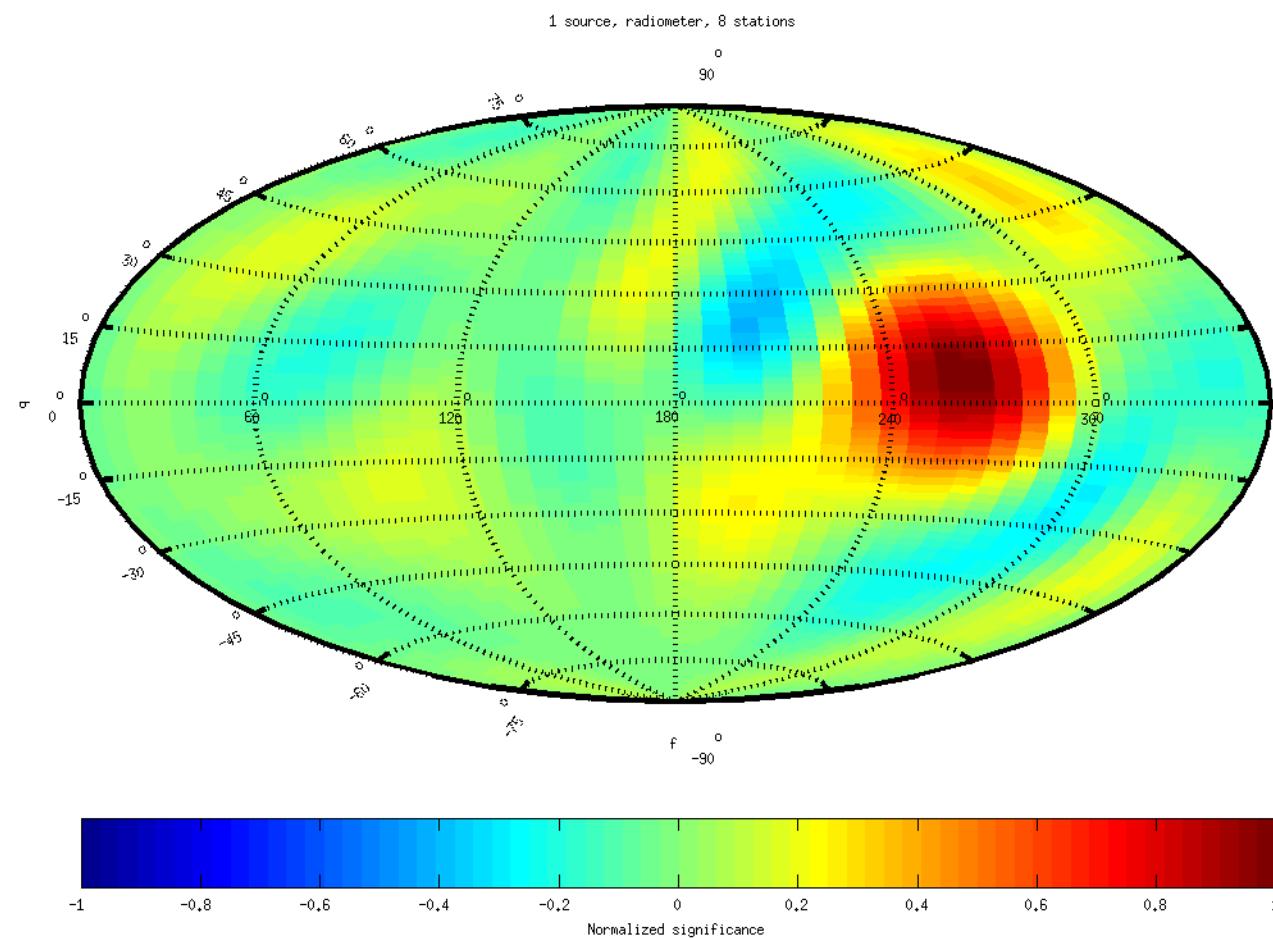
$f_{\text{analyse}} = .01 \text{ Hz}$ nstest18



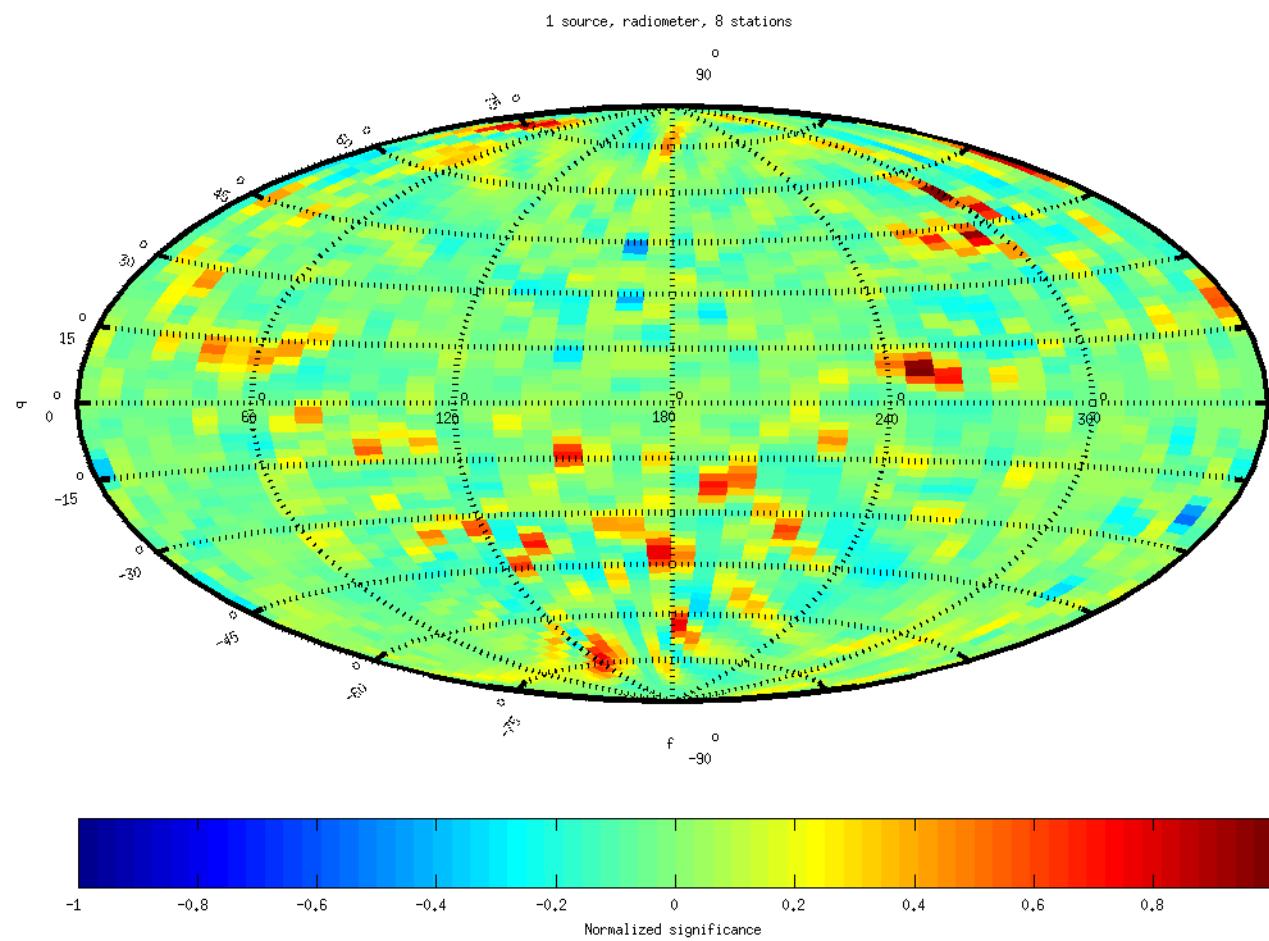
$f_{\text{analyse}} = .1 \text{ Hz}$ nstest19



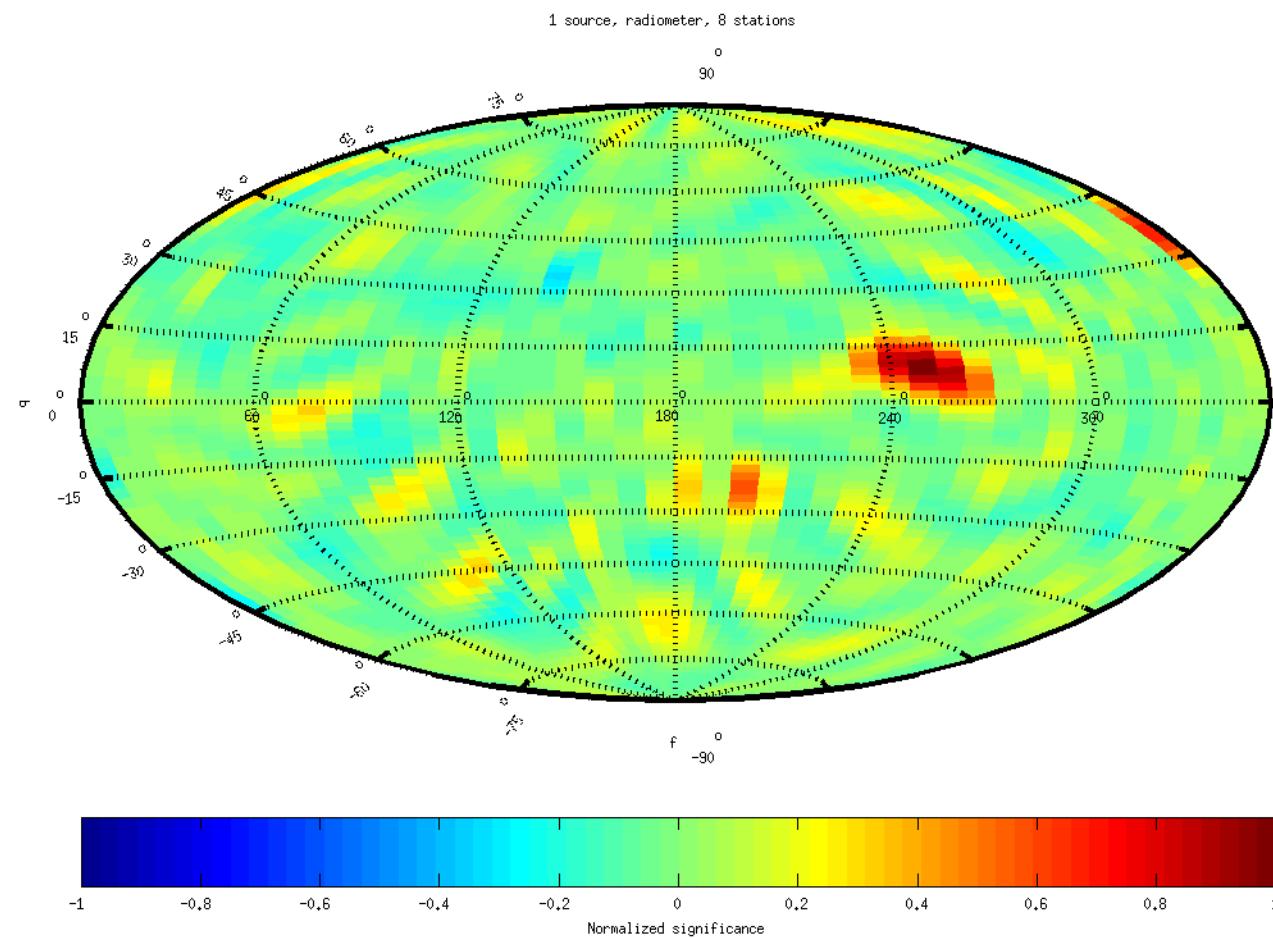
$f_{\text{analyse}} = 1 \text{ Hz}$ nstest20



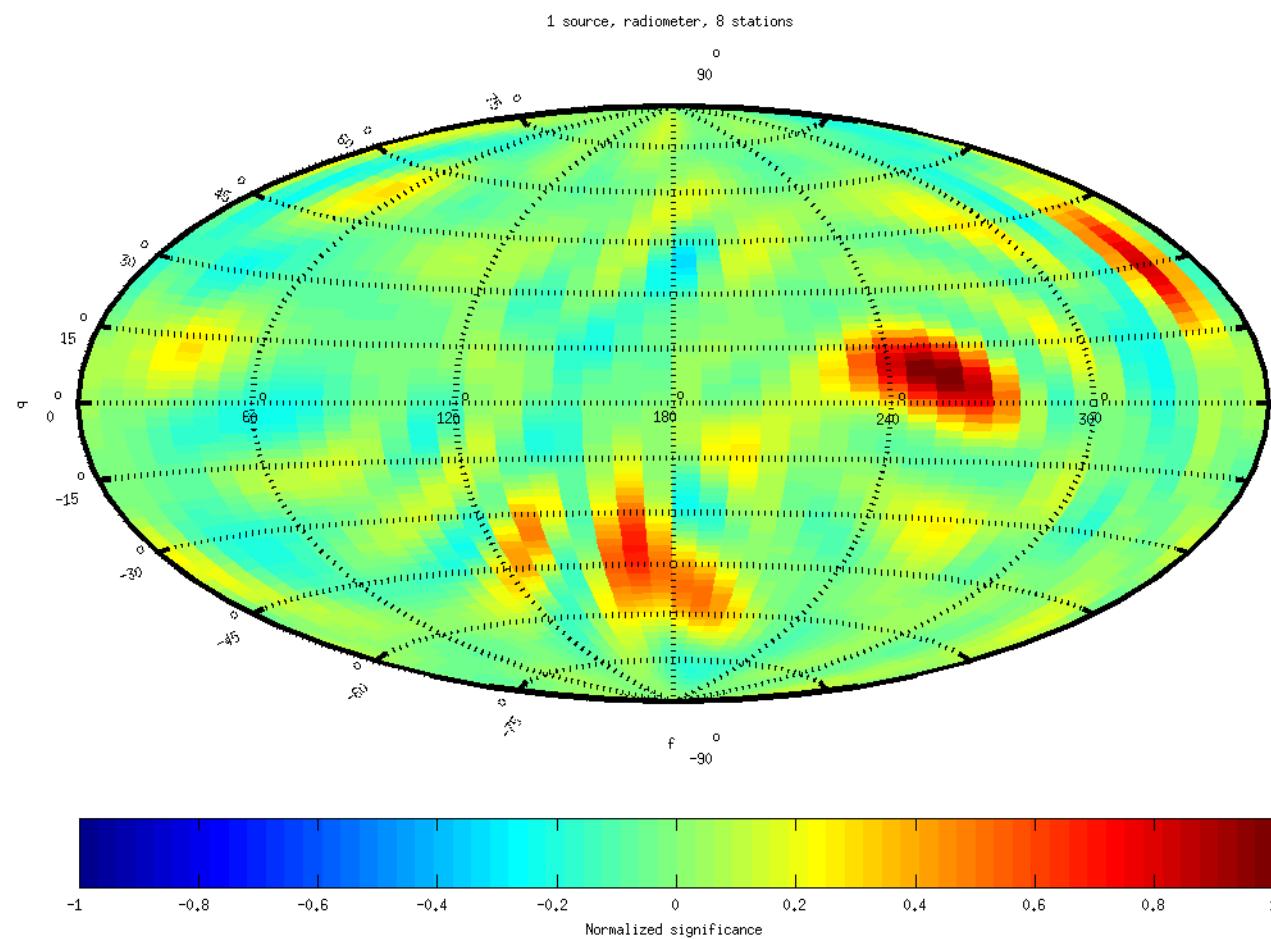
$f_{\text{analyse}} = 10 \text{ Hz}$ nstest21



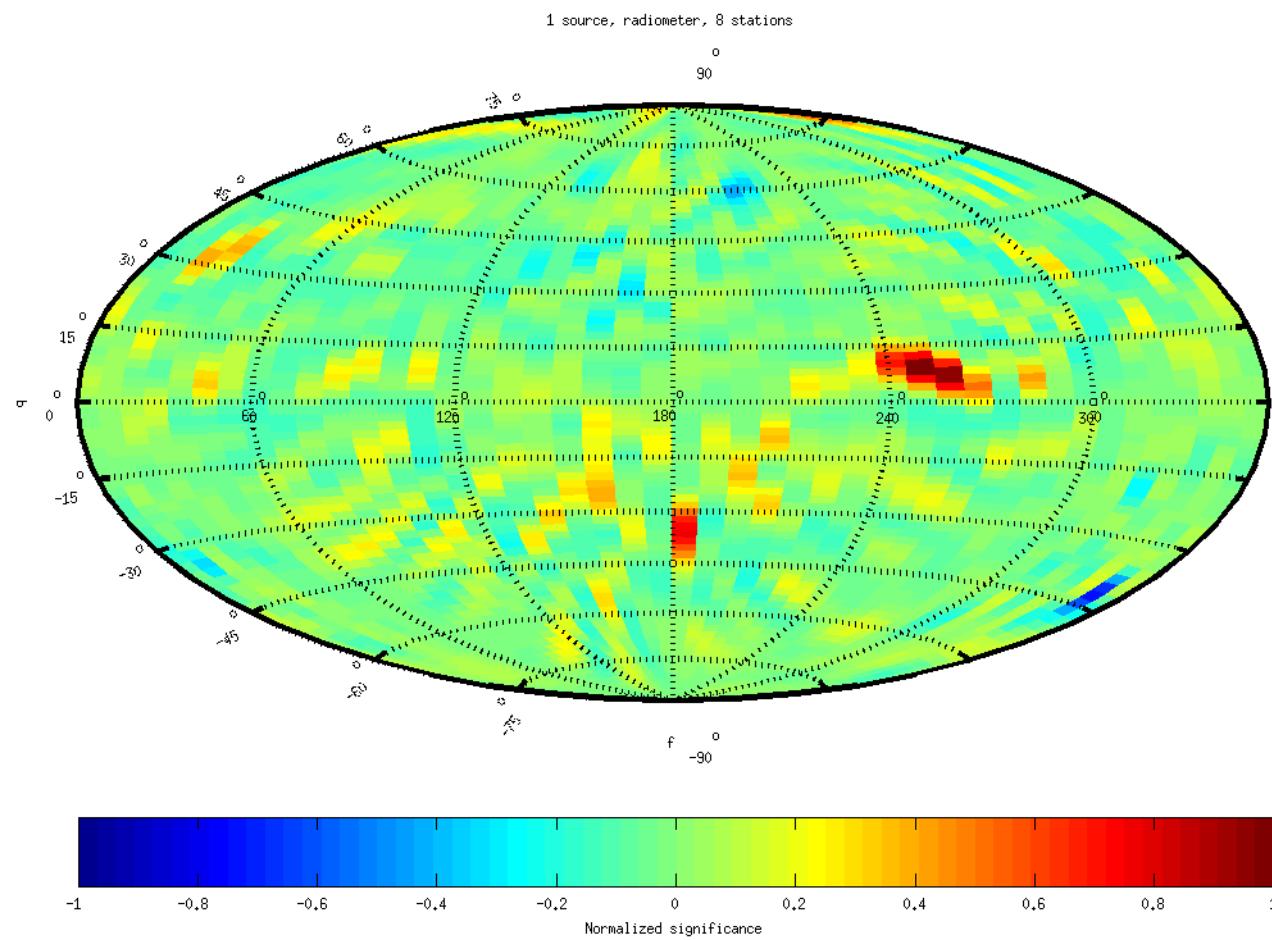
$f_{\text{analyse}} = 5 \text{ Hz}$ nstest22



$f_{\text{analyse}} = 3 \text{ Hz}$ nstest23



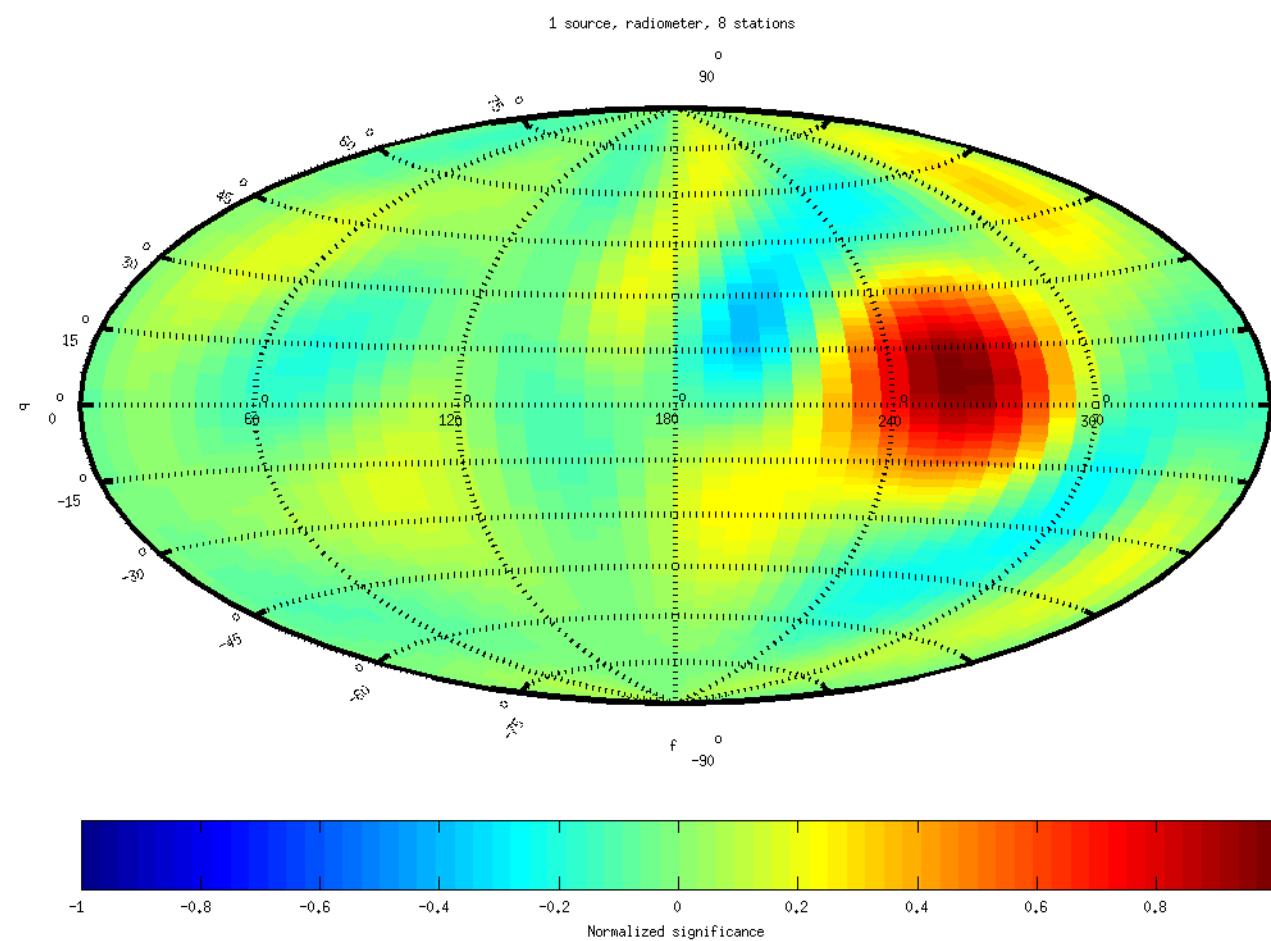
$f_{\text{analyse}} = 7 \text{ Hz}$ nstest24



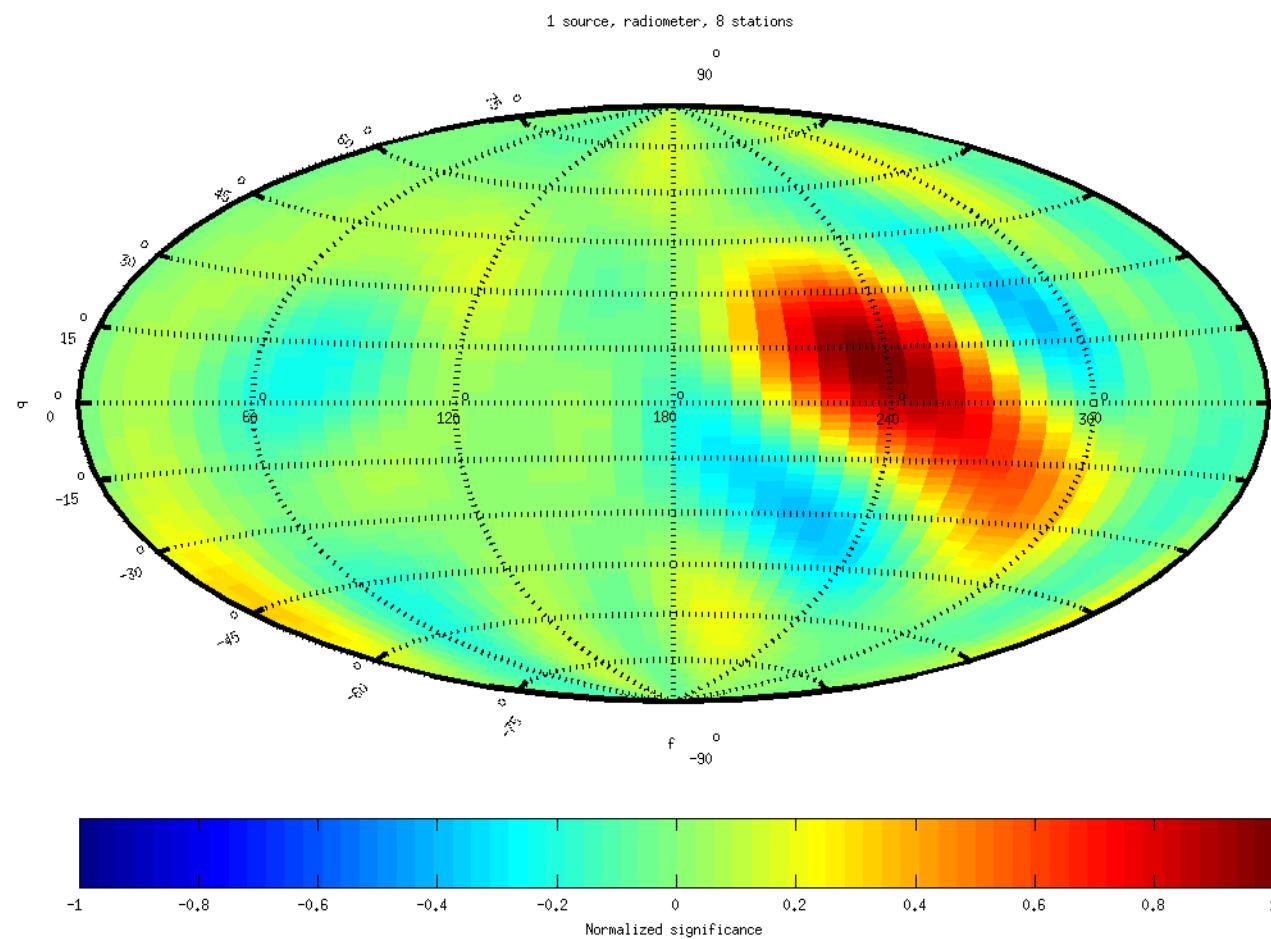
Polarization Testing

- A monochromatic source of either 1 Hz or 5 Hz is used. Random polarizations are used. All other parameters are the same as before.

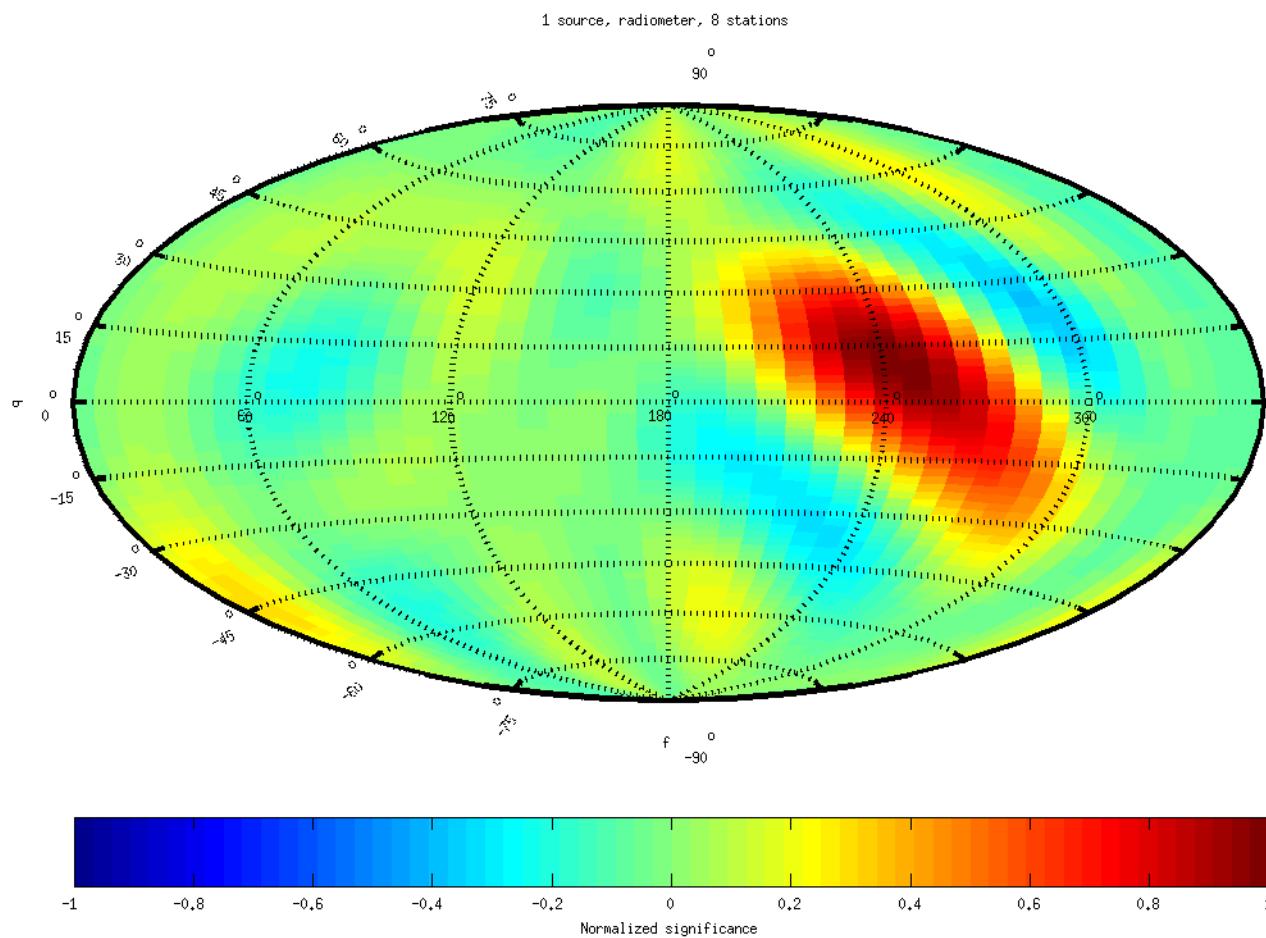
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 40.2956$ nstest25



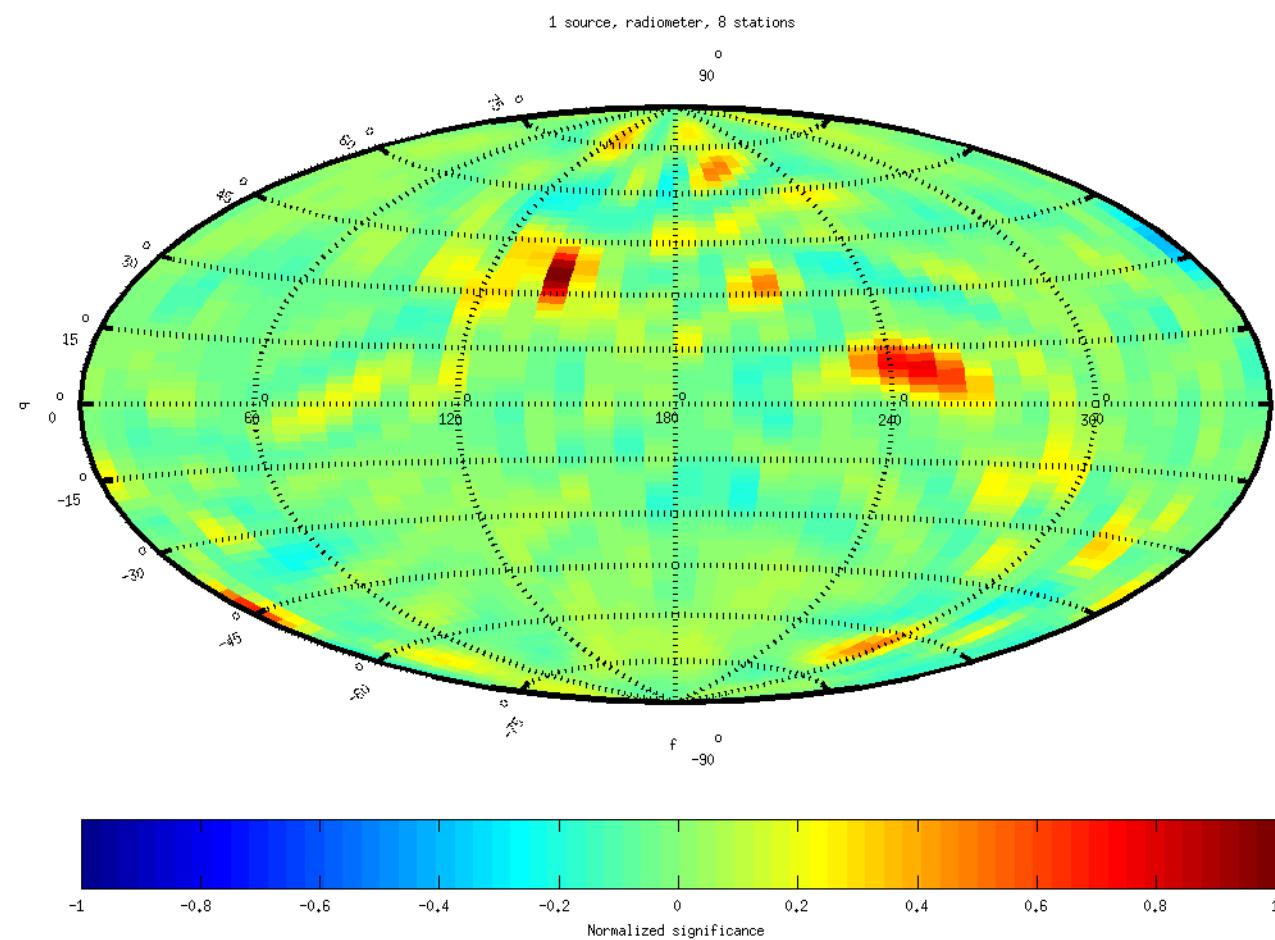
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 138.0781$ nstest26



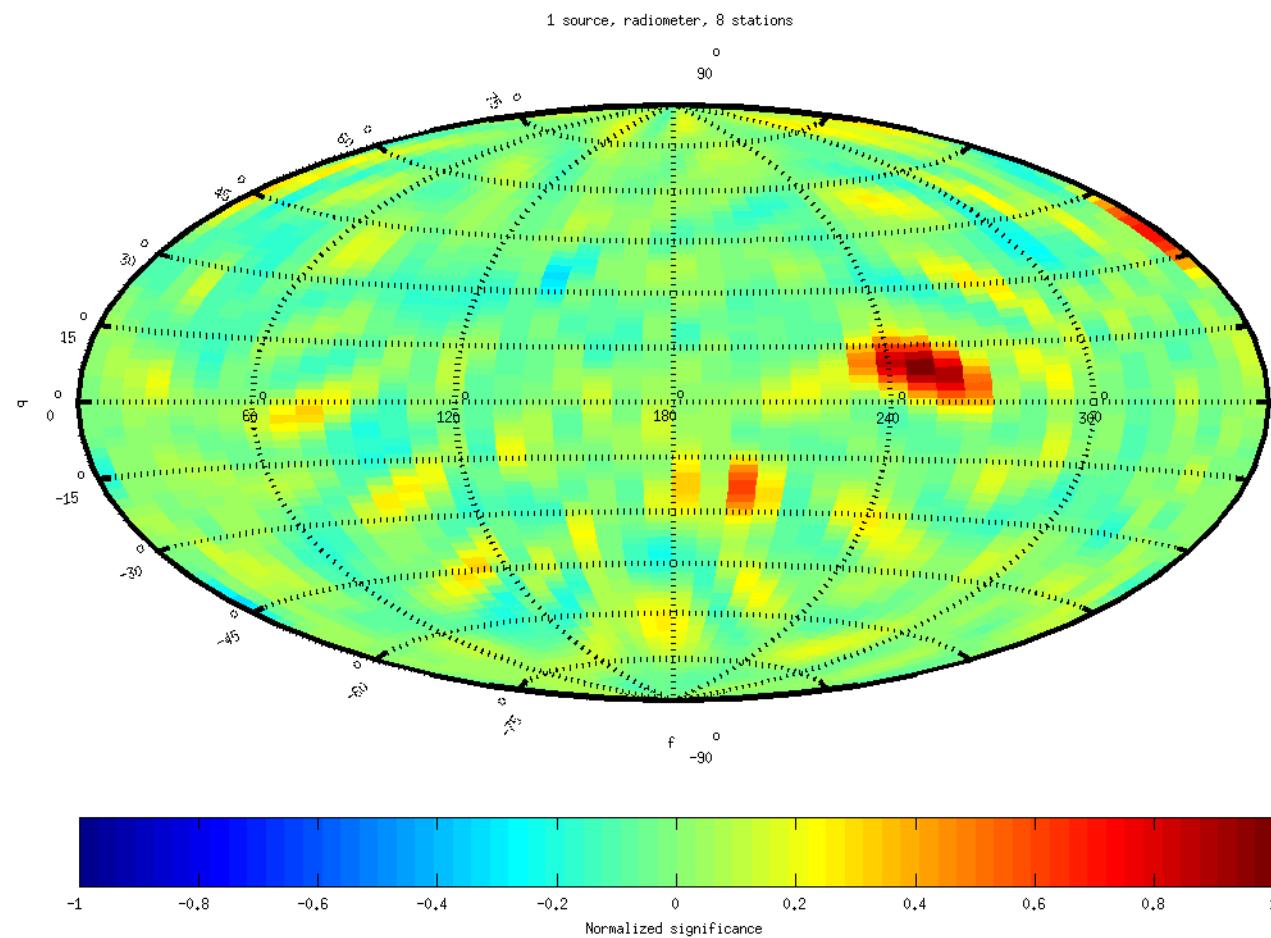
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 190.9751$ nstest27



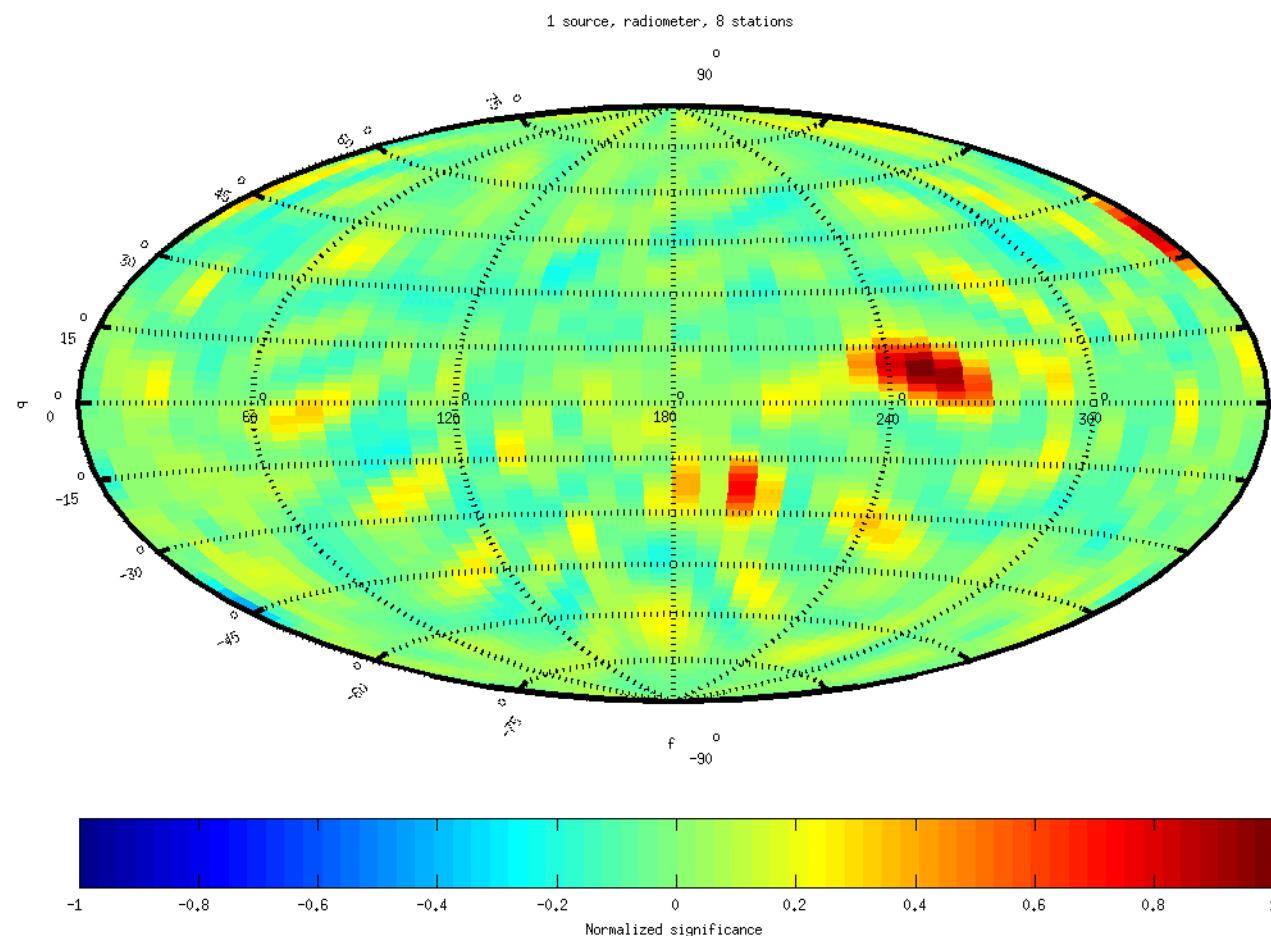
f_analyse = 5 Hz, psi = 220.9382 nstest28



$f_{\text{analyse}} = 5 \text{ Hz}$, $\psi = 50.8770$ nstest29



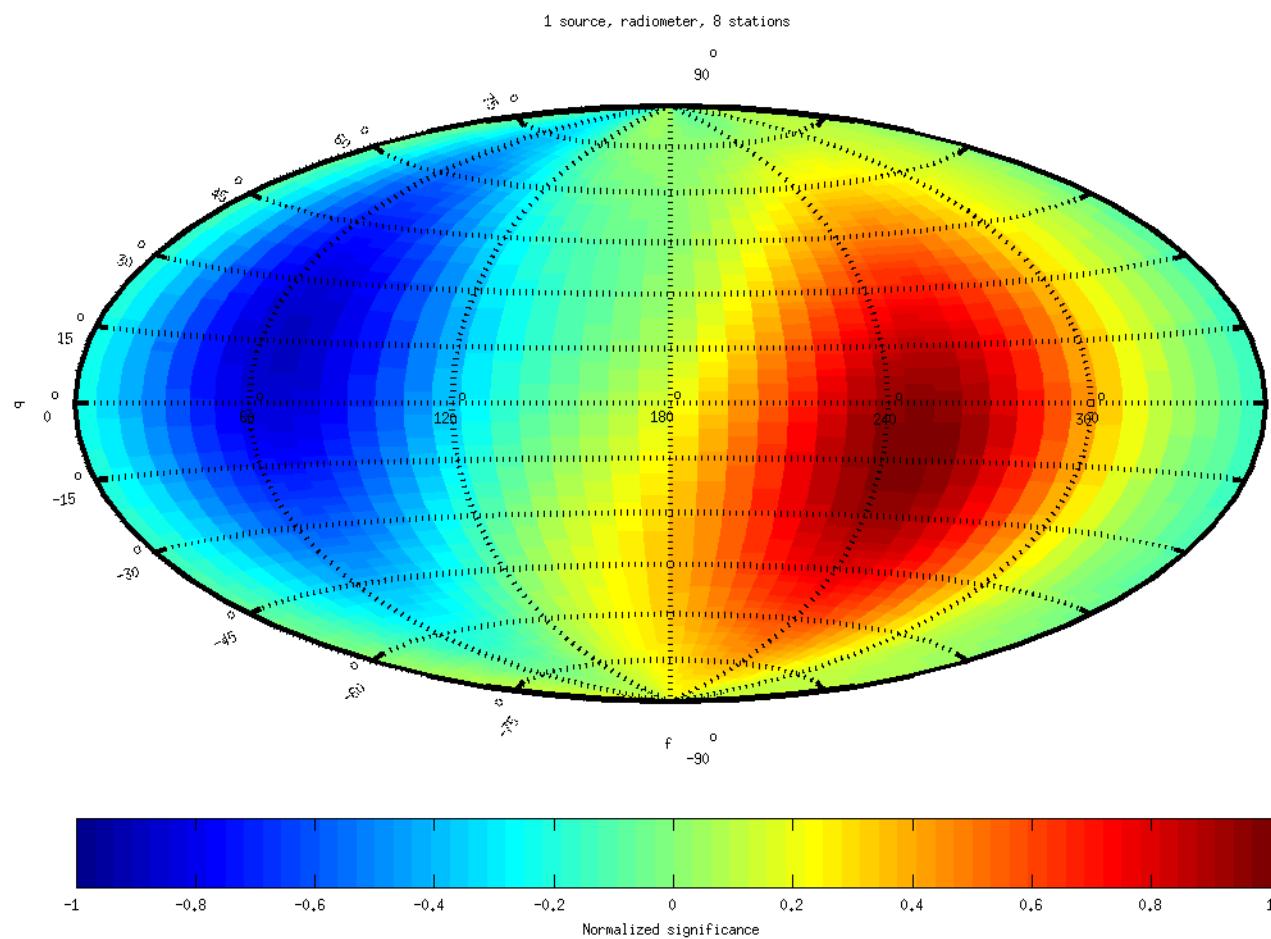
$f_{\text{analyse}} = 5 \text{ Hz}$, $\psi = 64.7787$ nstest30



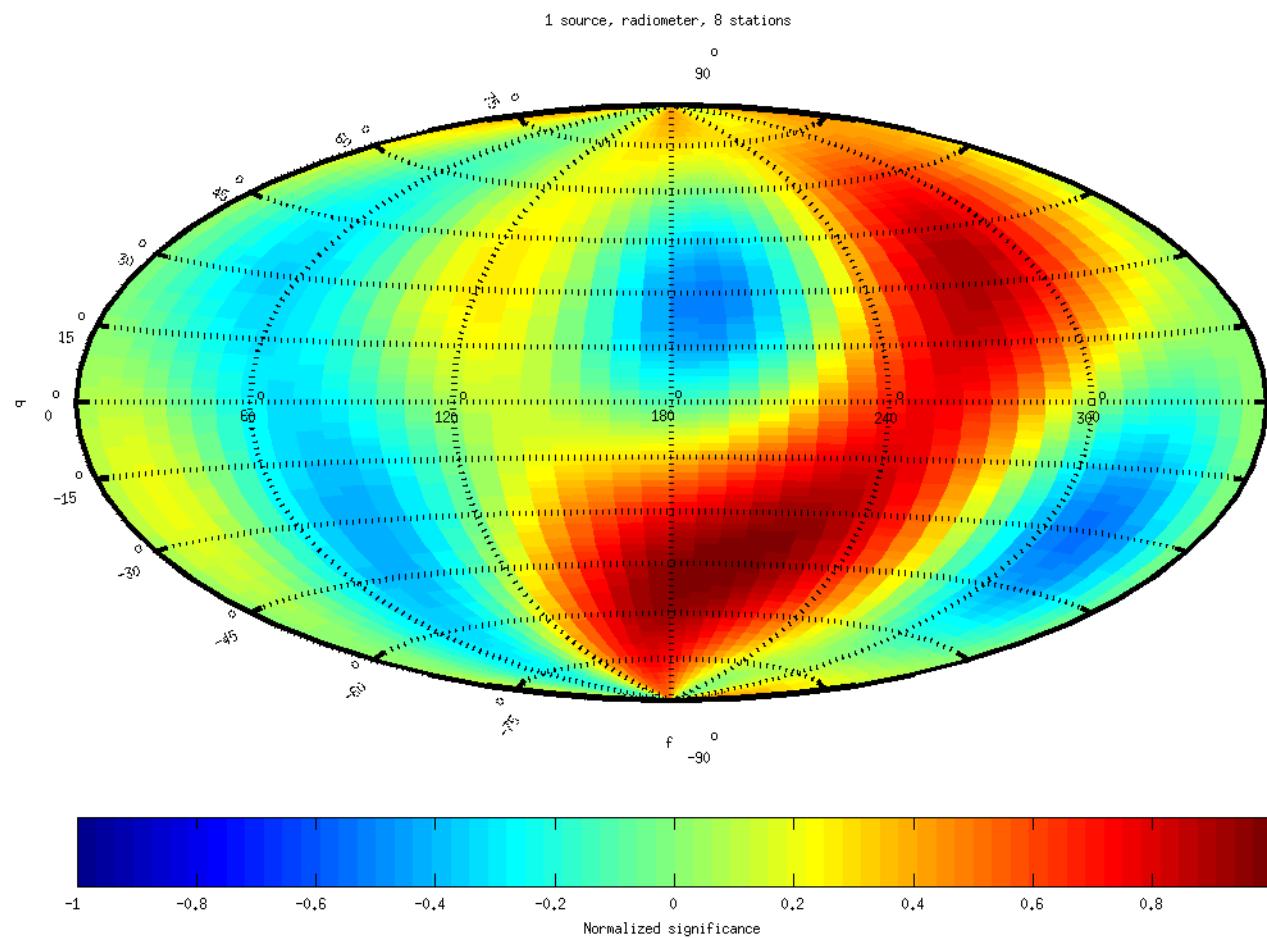
Broadband Signal Testing

- In the next series of tests, a broadband injection signal is used. f_{analyse} refers to the frequency at which the recovery was performed. All other parameters are kept constant, with ψ being kept at 45.

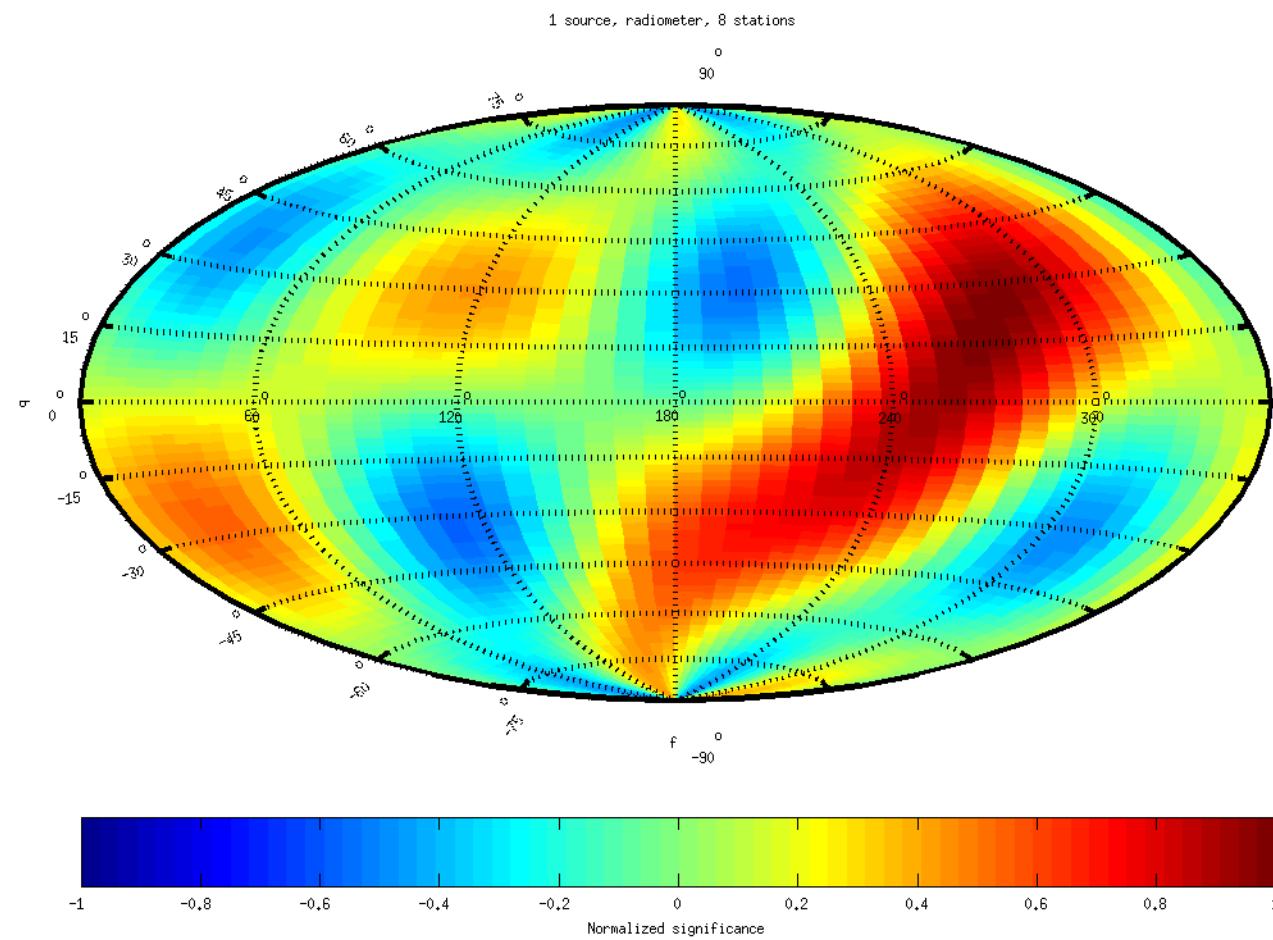
$f_{\text{analyse}} = .001 \text{ Hz}$ nstest31



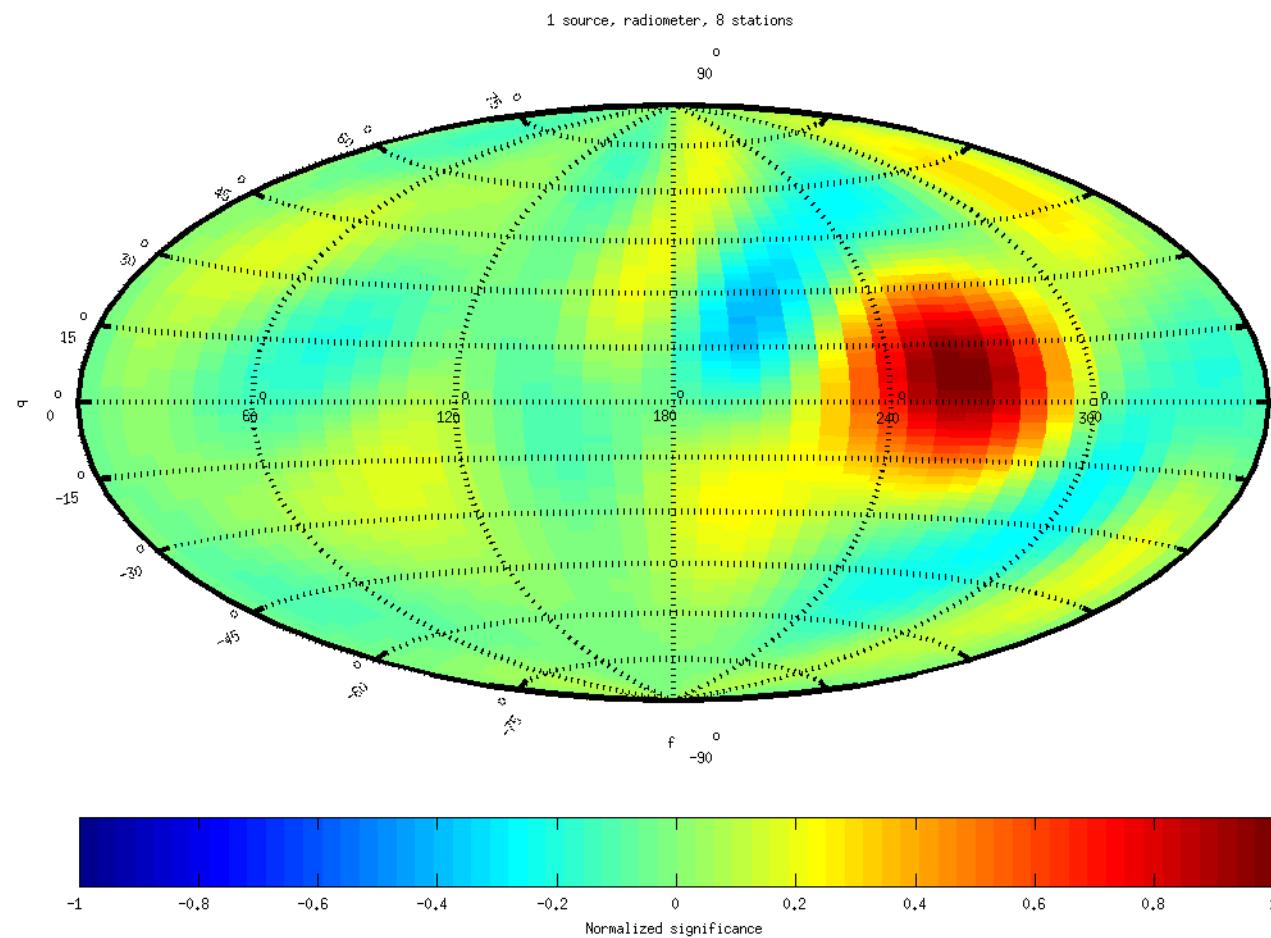
$f_{\text{analyse}} = .01 \text{ Hz}$ nstest32



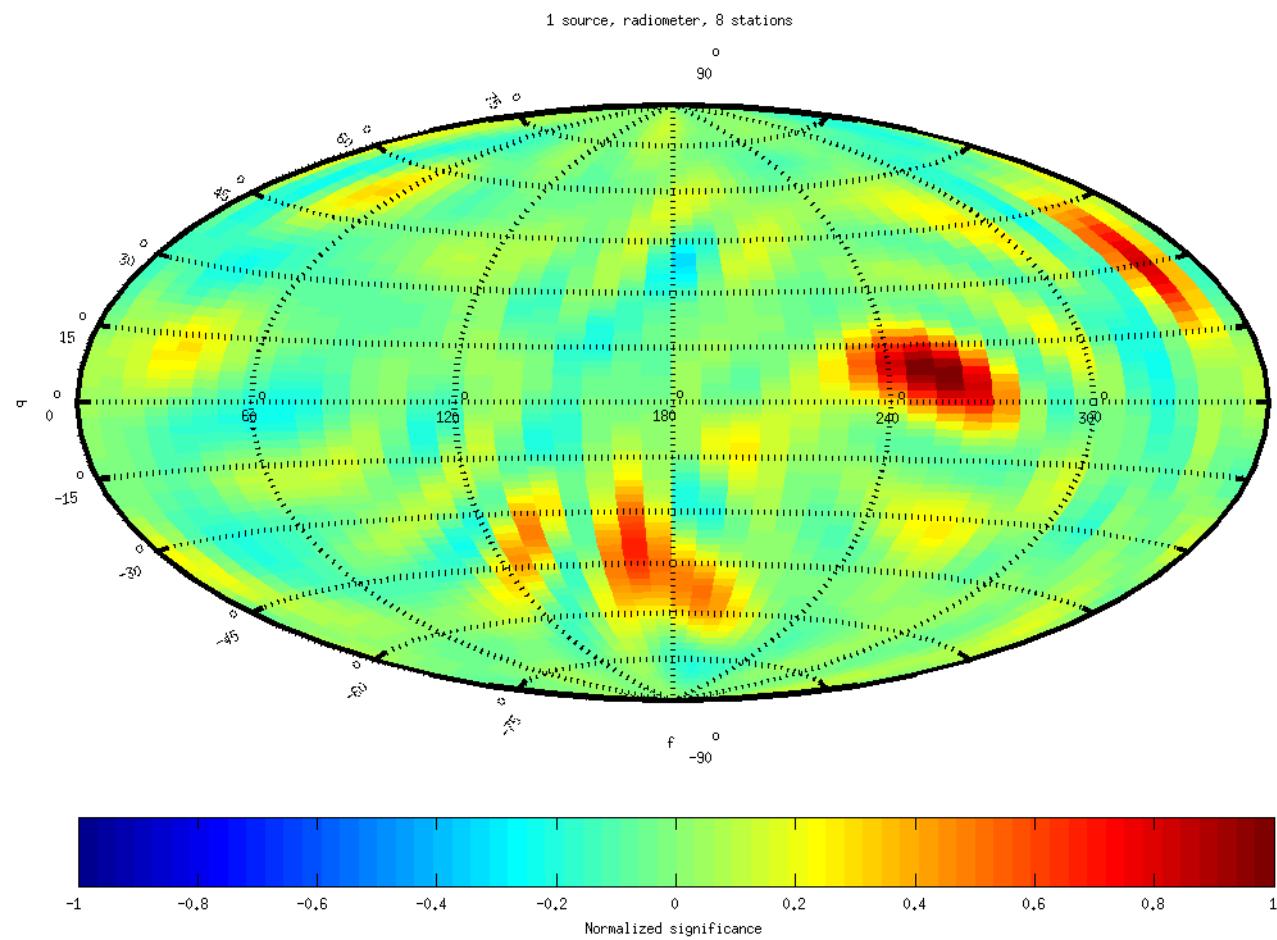
$f_{\text{analyse}} = .1 \text{ Hz}$ nstest33



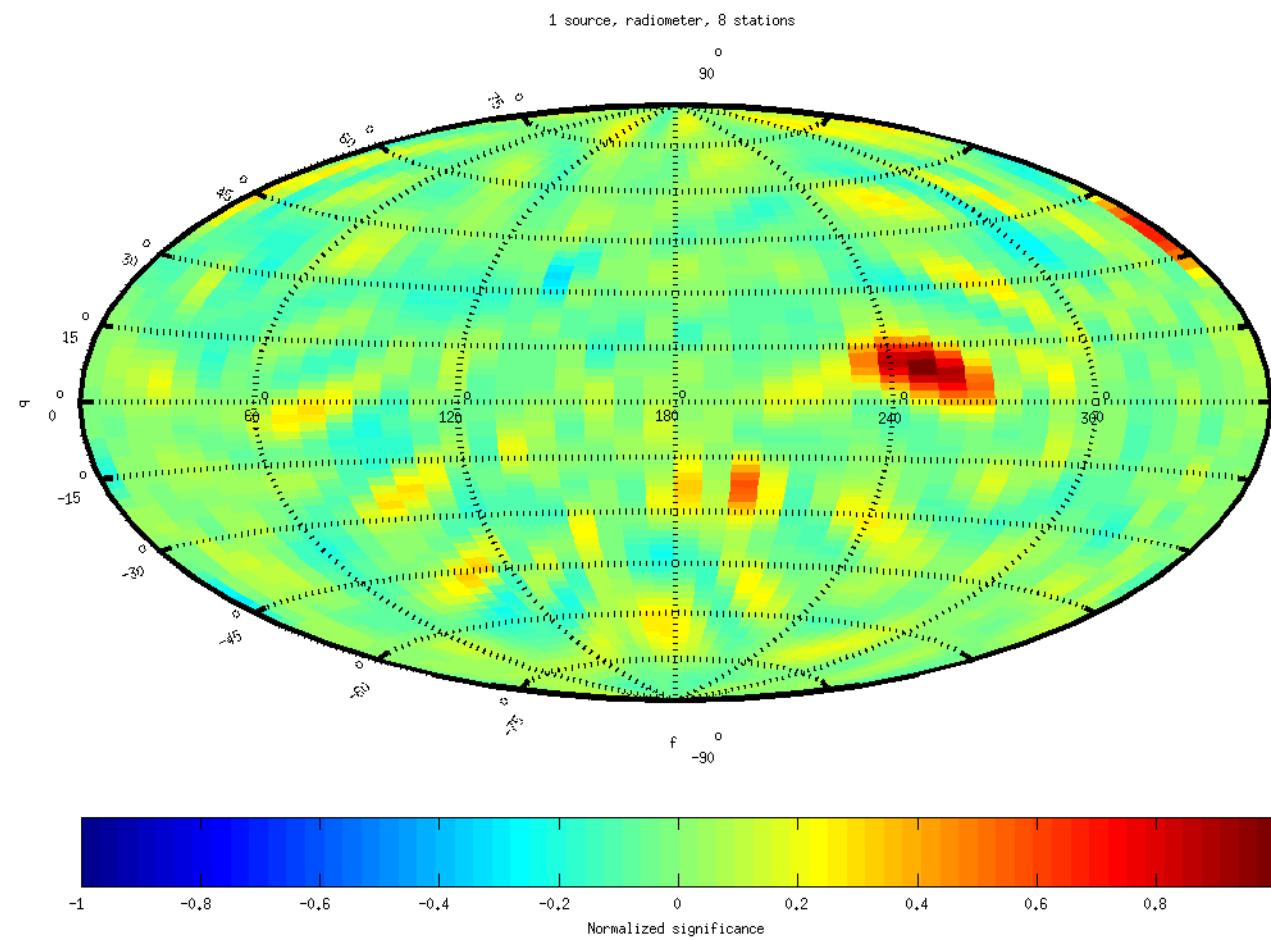
f_analyse = 1 Hz nstest34



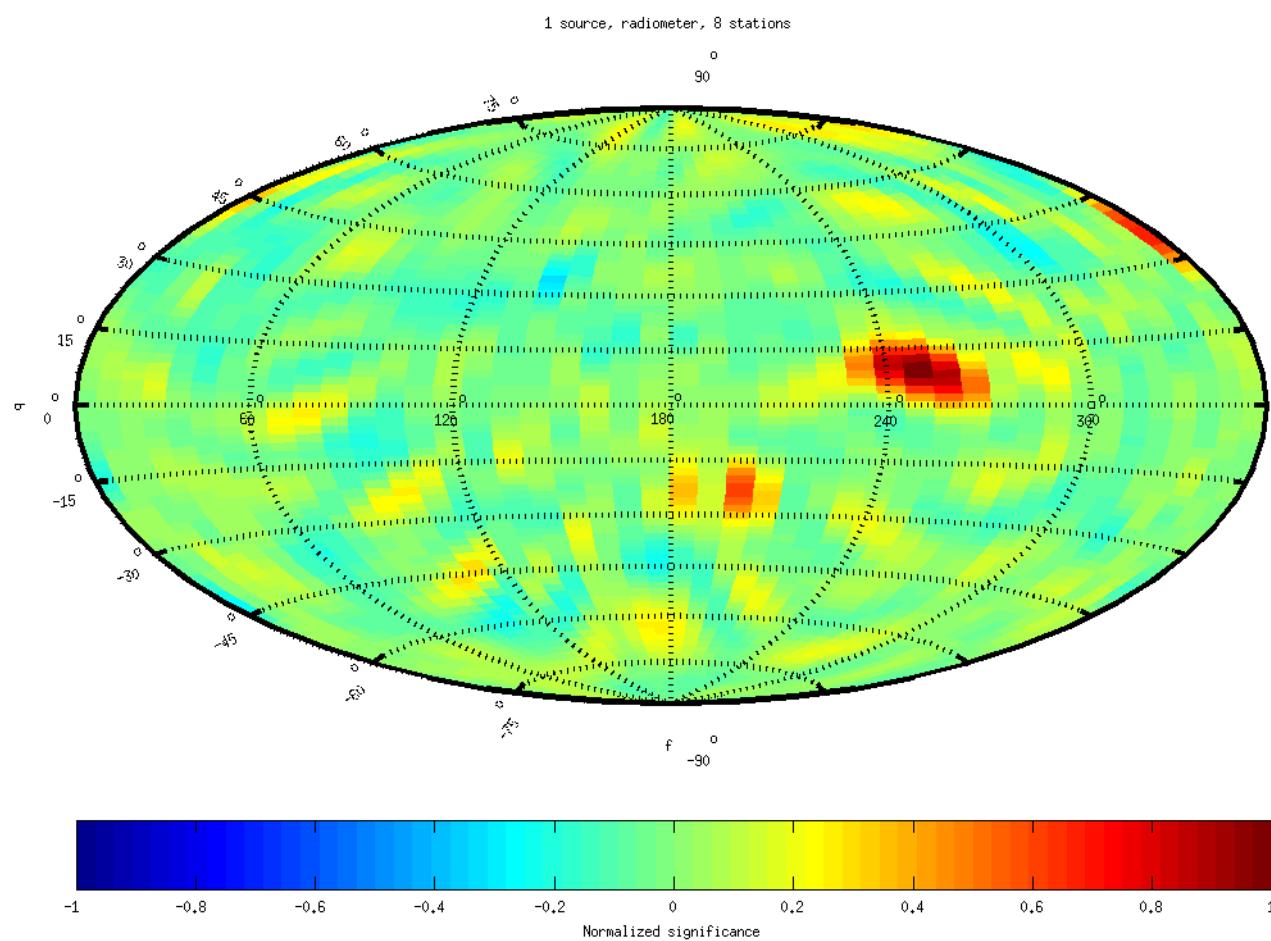
$f_{\text{analyse}} = 3 \text{ Hz}$ nstest35



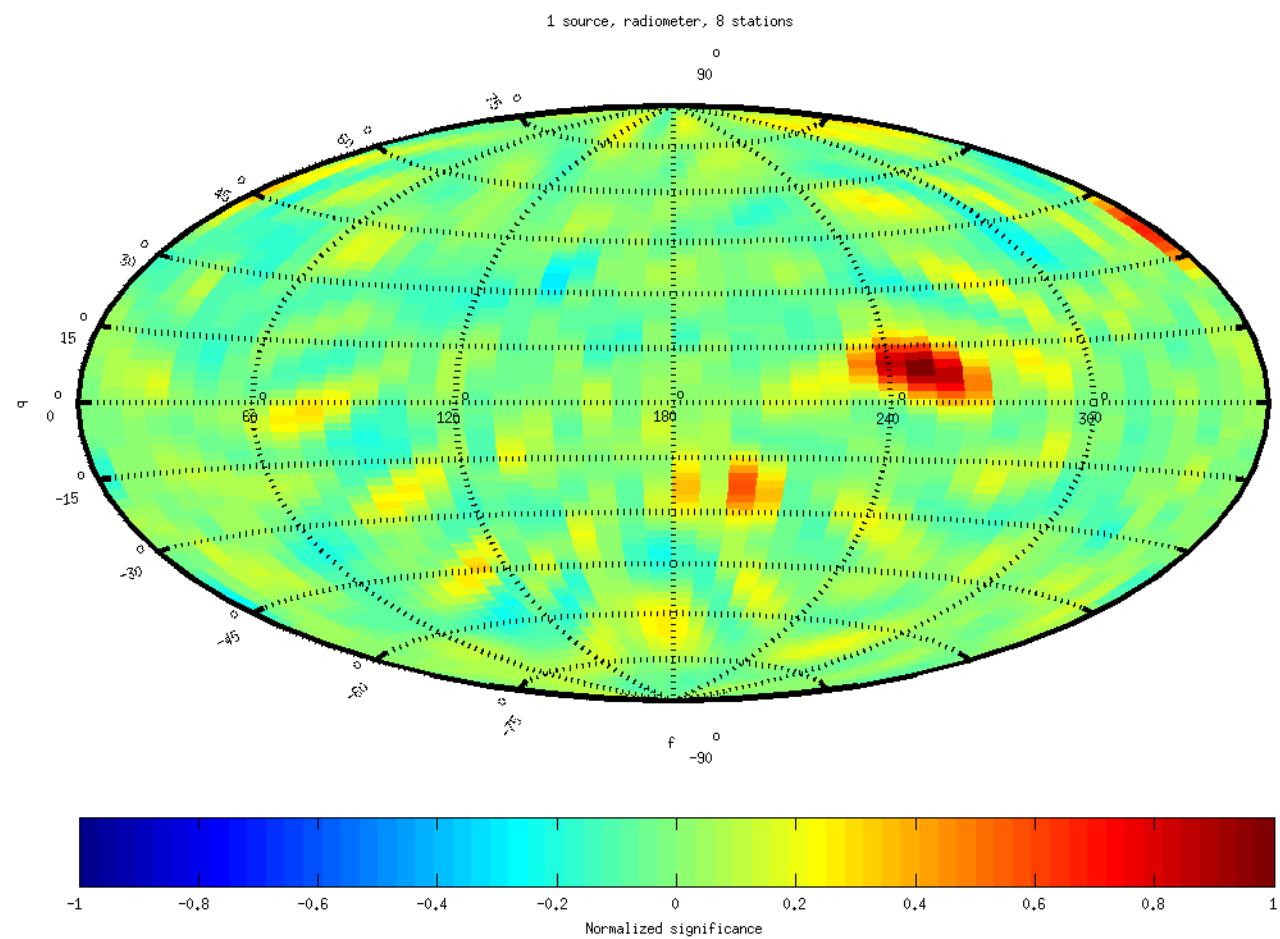
f analyse = 5 Hz nstest36



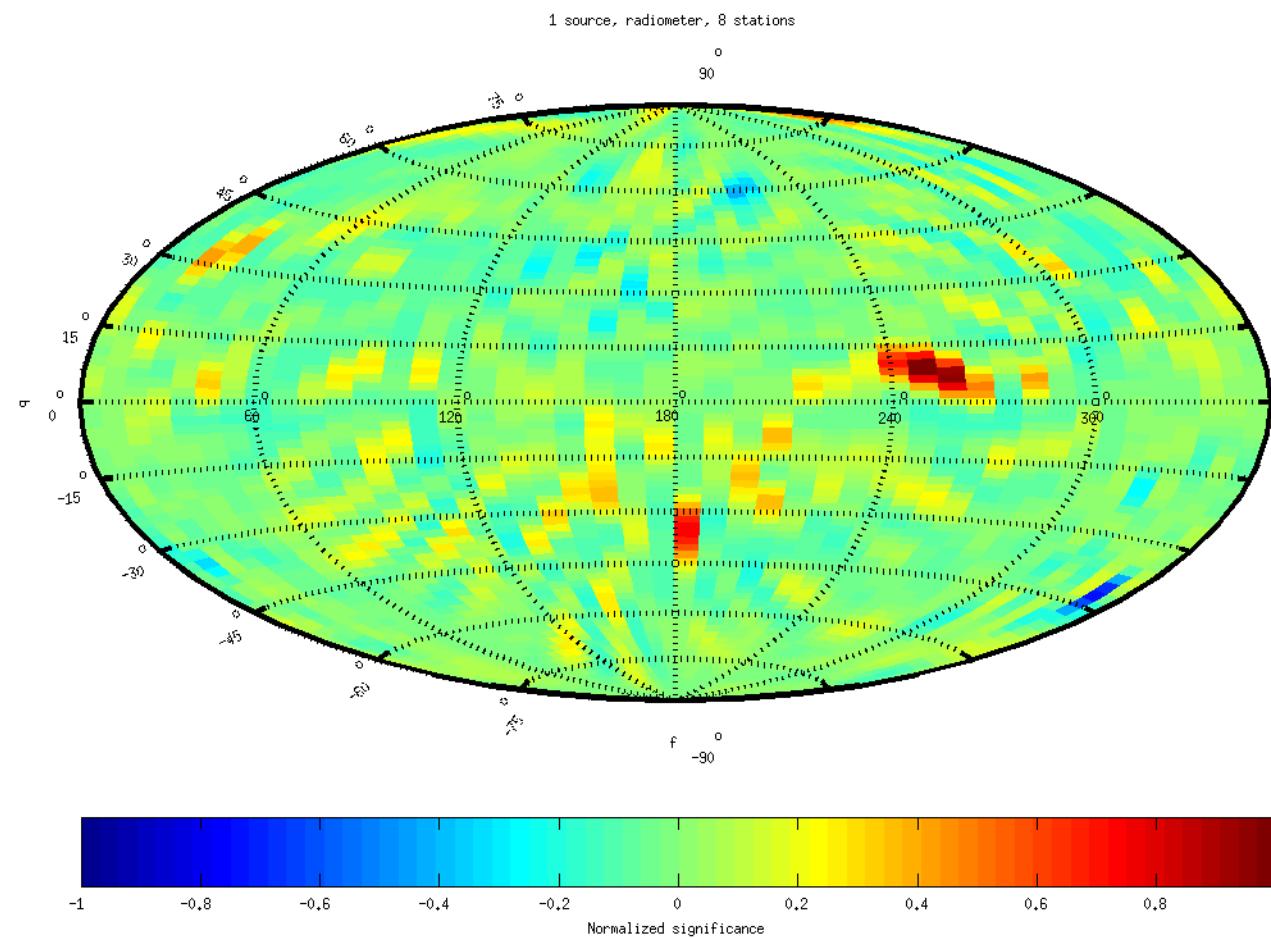
Repeat of nstest36 ***current image is nstest60



Repeat of nstest36 ***current image is nstest61



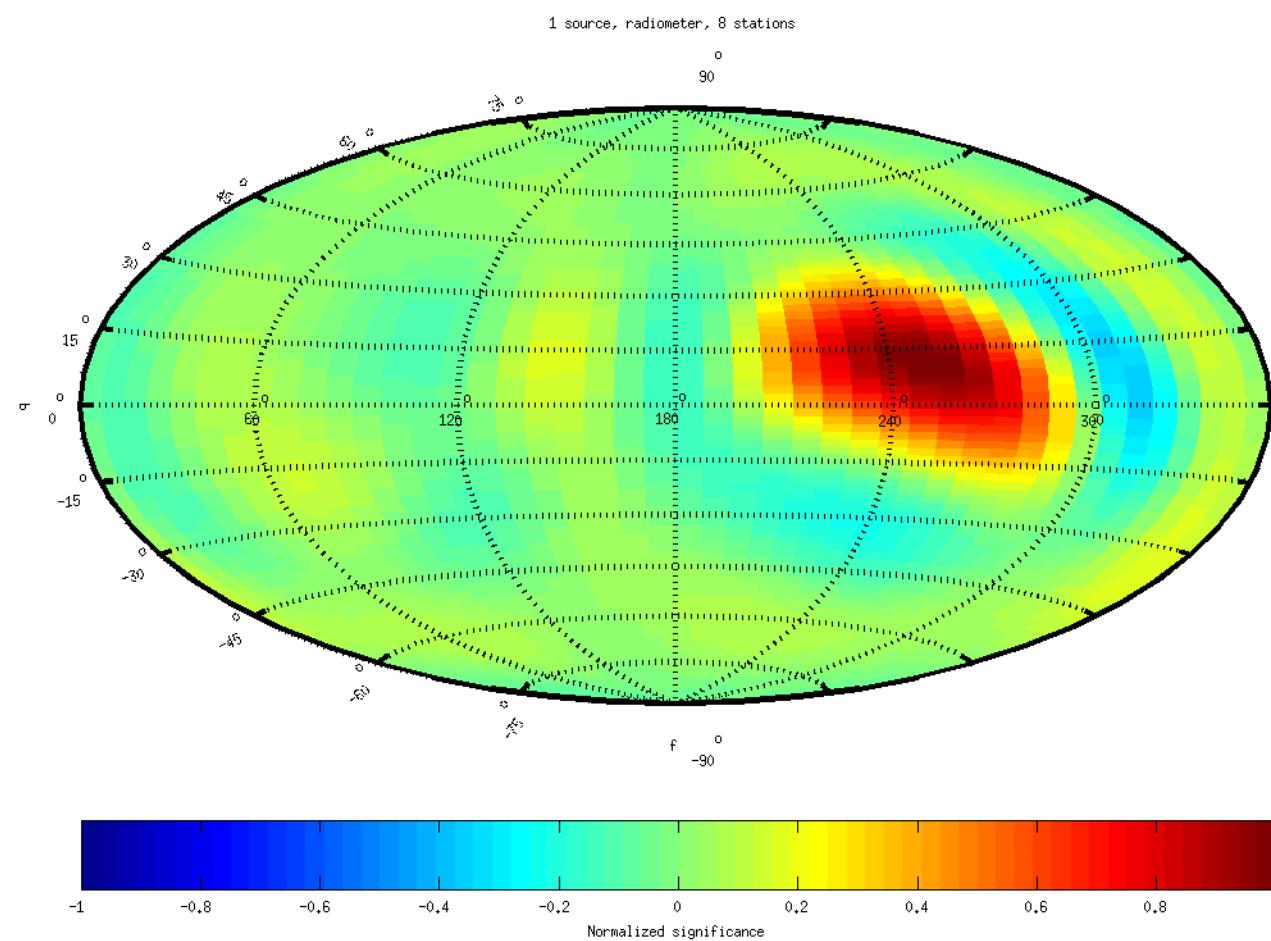
$f_{\text{analyse}} = 7 \text{ Hz}$ nstest37



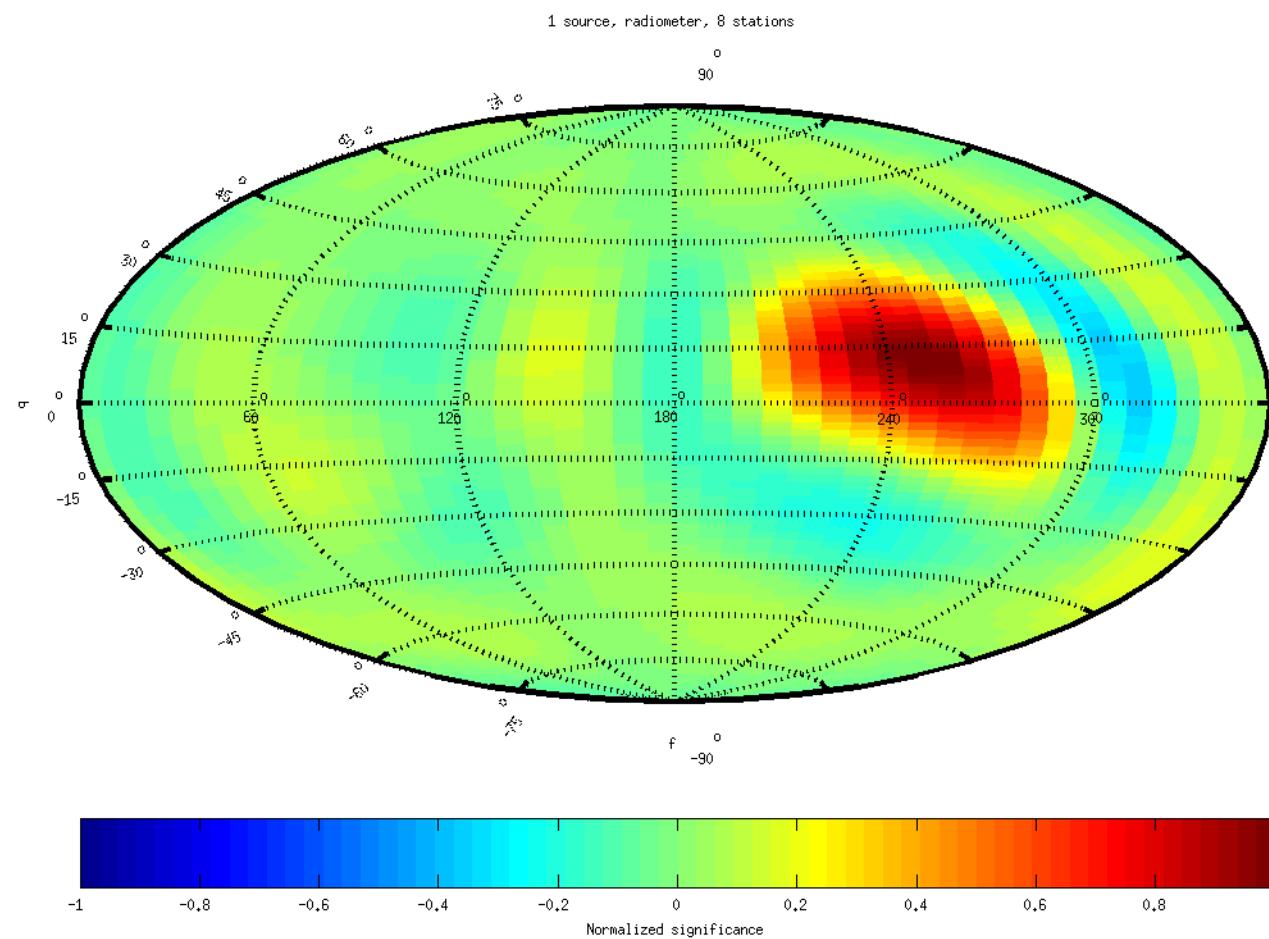
Broadband/Polarization Testing

- In this series of tests, the injection signal is again broadband. With the recovery performed at either 1 Hz or 3 Hz, the signal was equipped with random polarizations. All other parameters are the same as before.

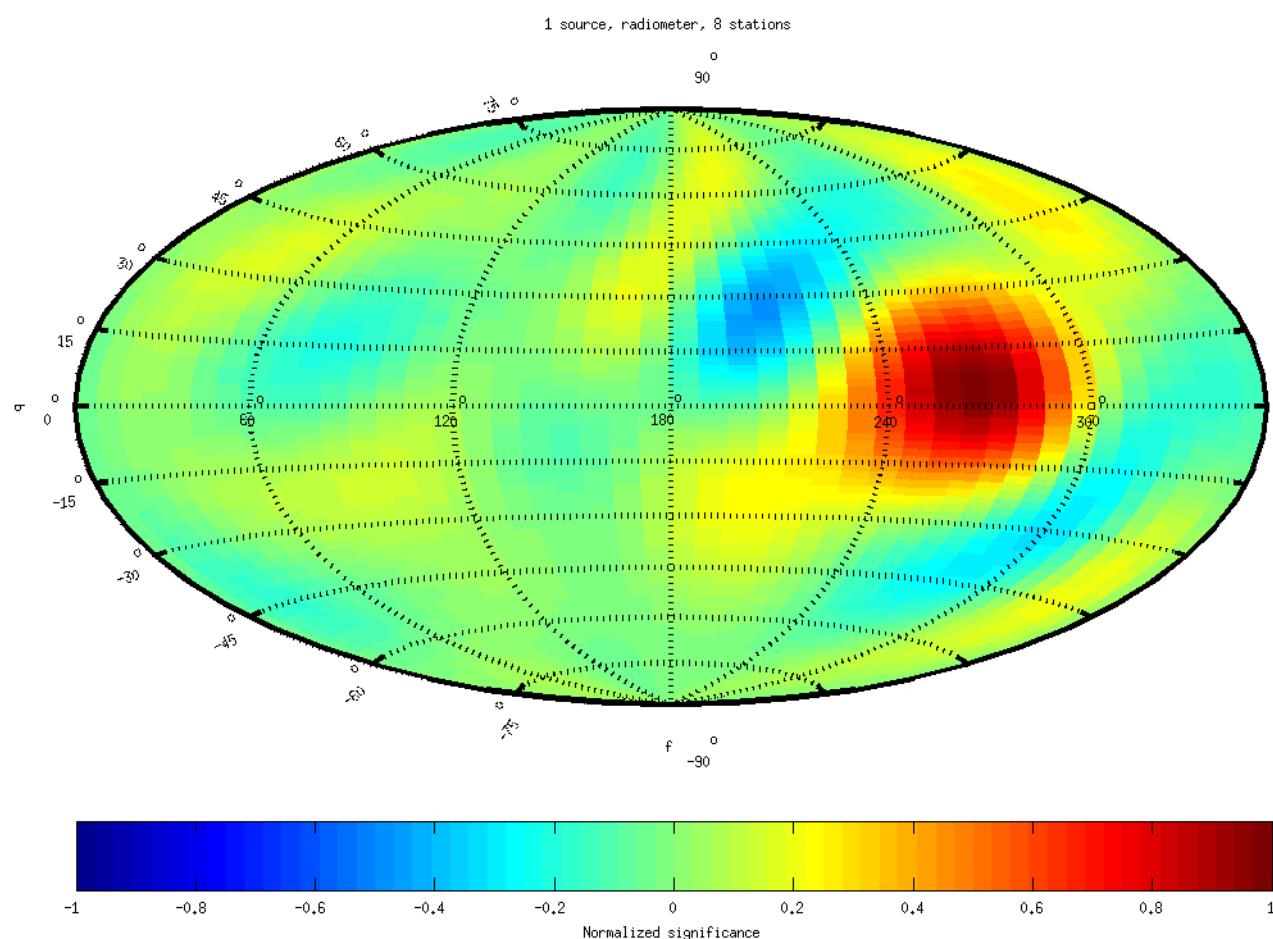
f_analyse = 1 Hz, psi = 264.7032 nstest38



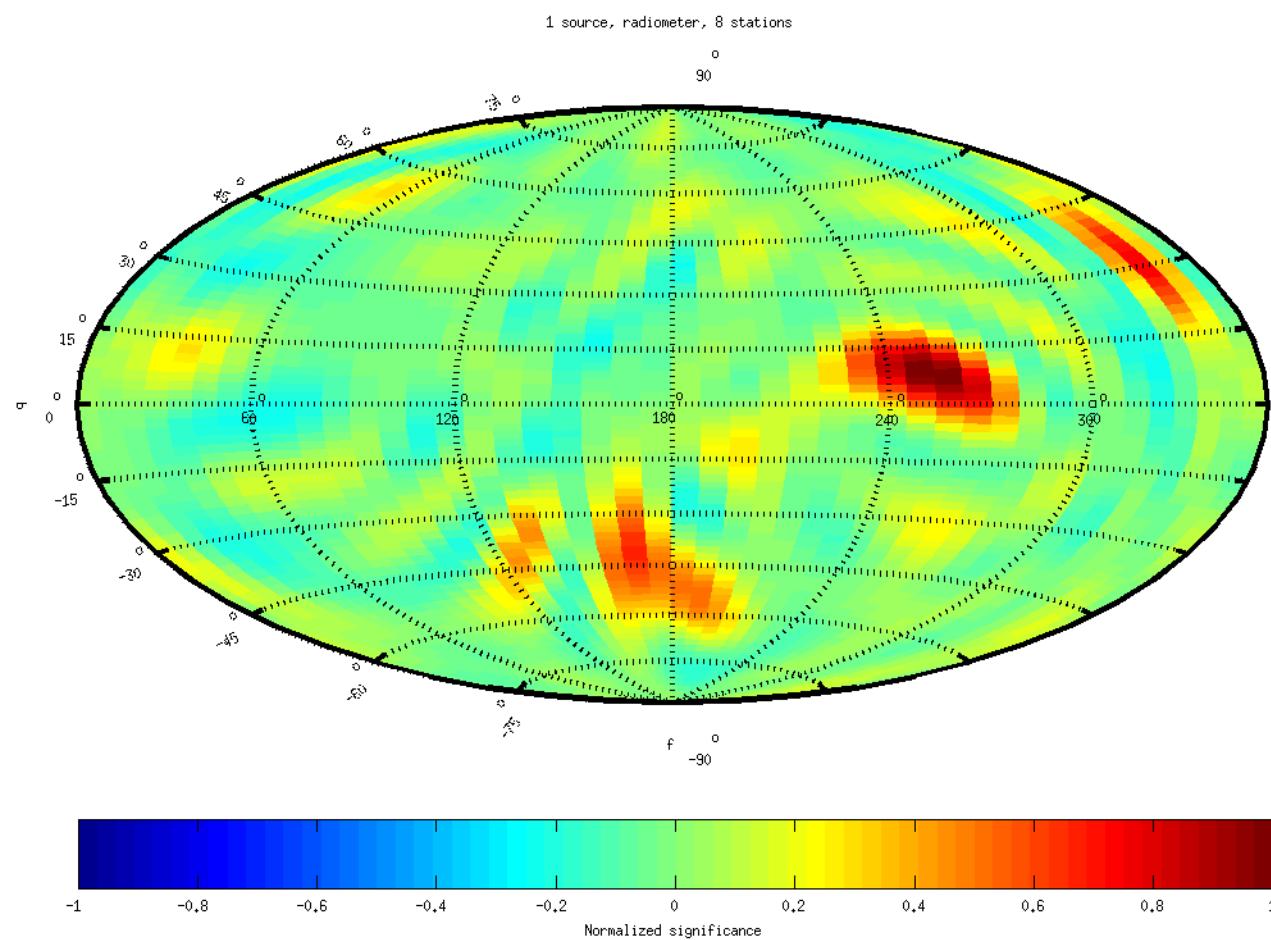
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 104.2868$ nstest39



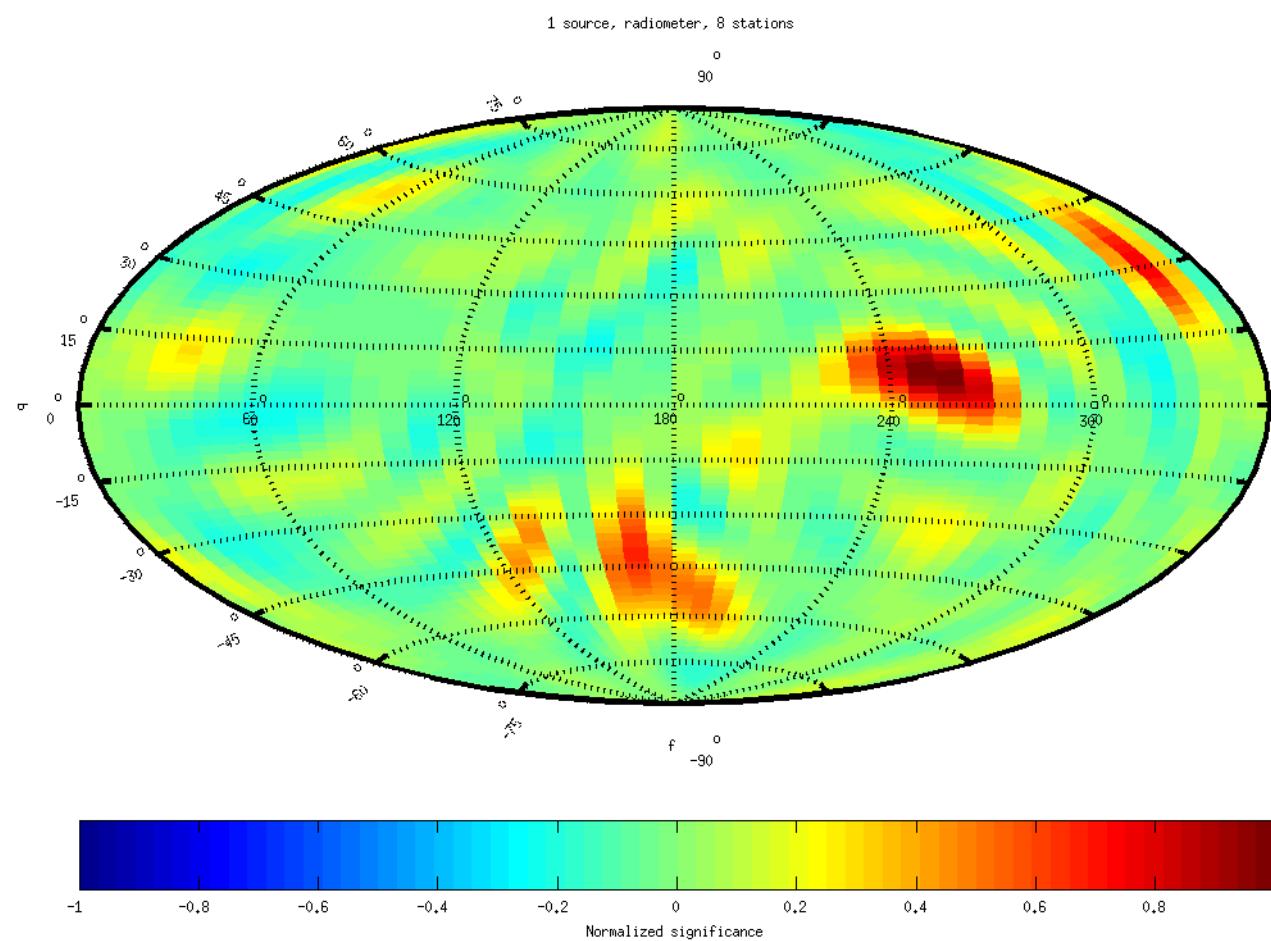
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 66.0591$ nstest40



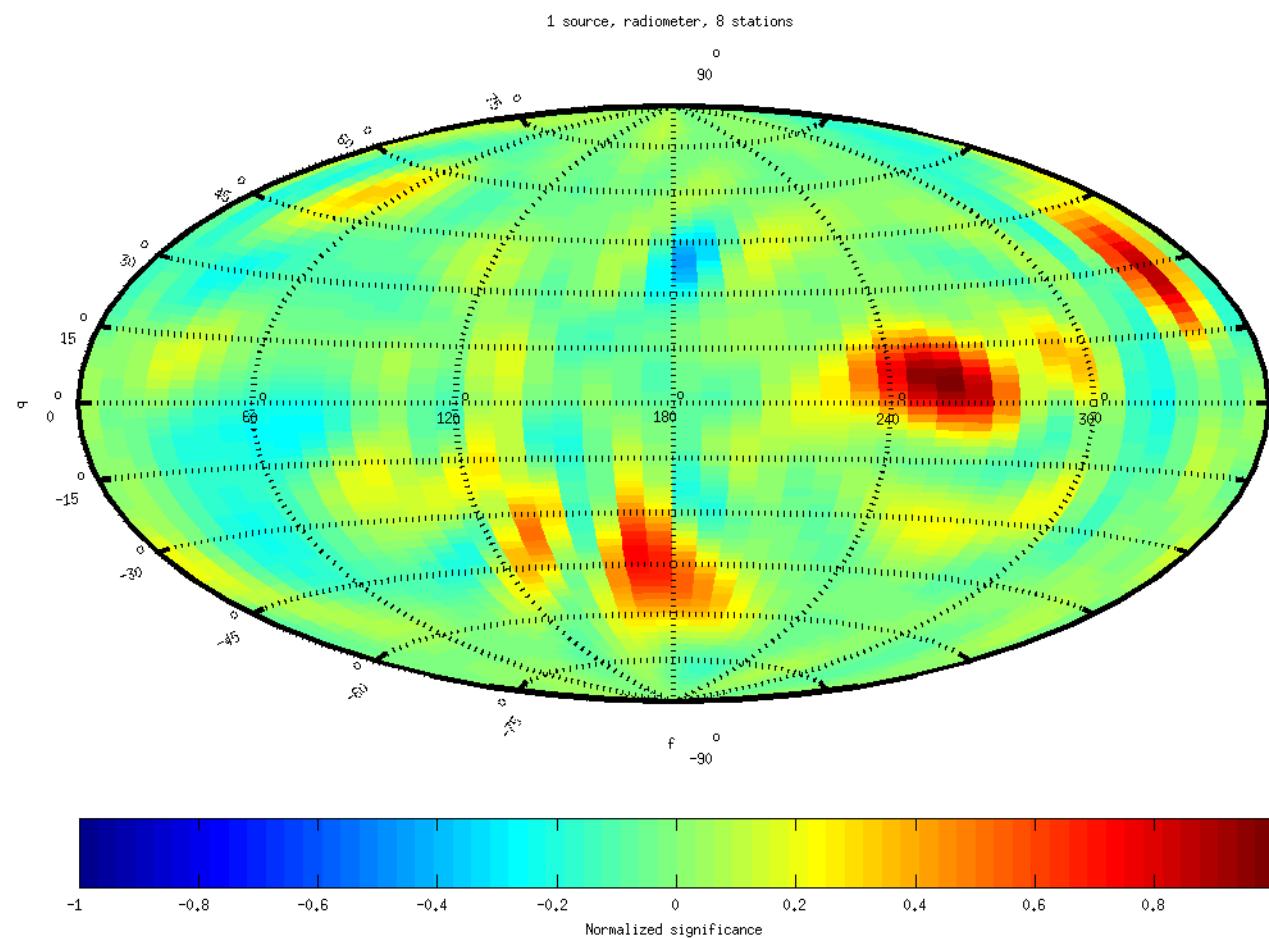
$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 31.6273$ nstest41



$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 65.2960$ nstest41



$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 97.7315$ nstest42



Number of Detectors/Polarization Testing

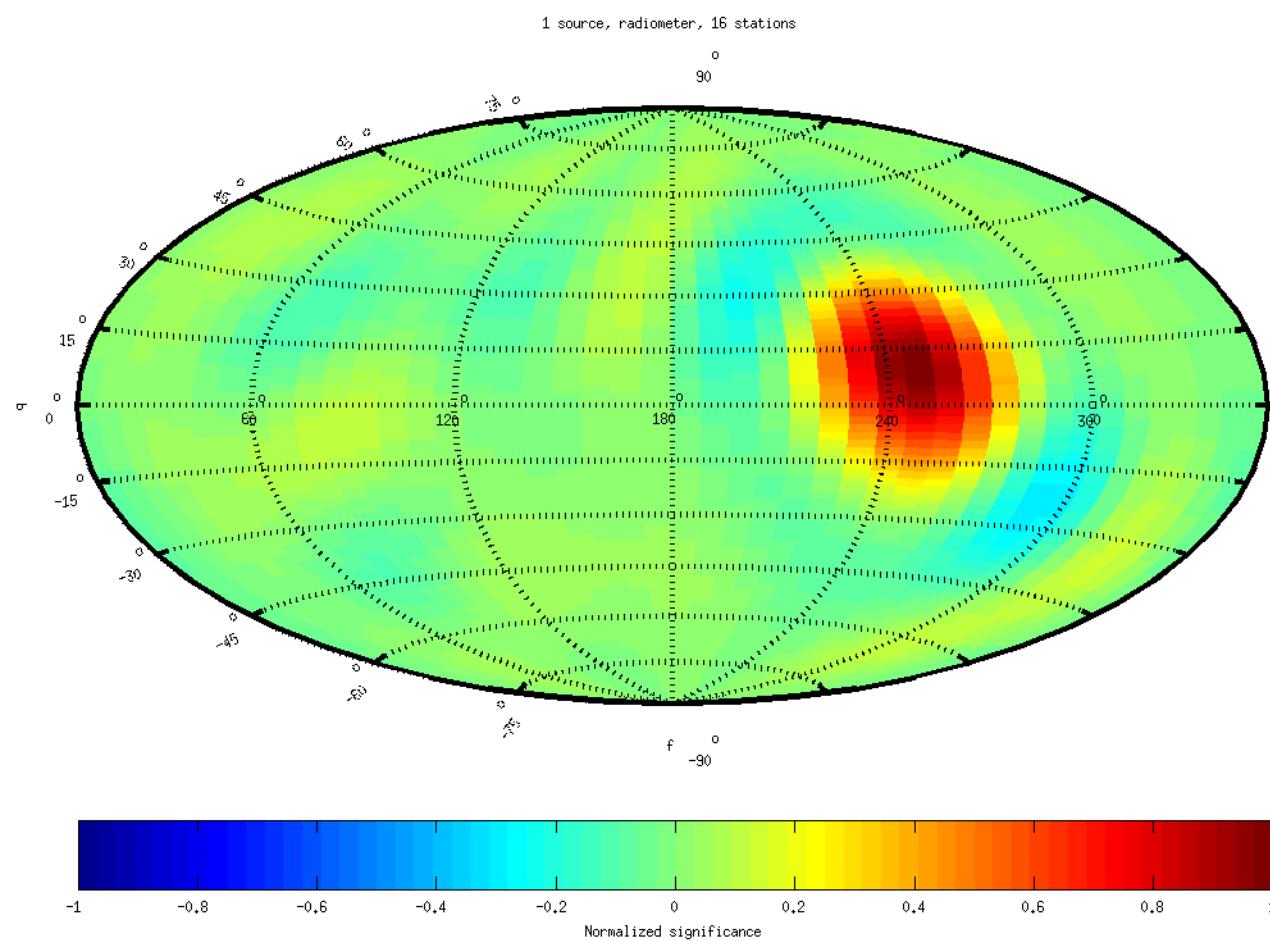
- Returning to a monochromatic source of frequency 1 or 3, for a certain number of detectors, the injection signal is equipped with random polarizations. All other parameters are the same as before.

16 Detectors

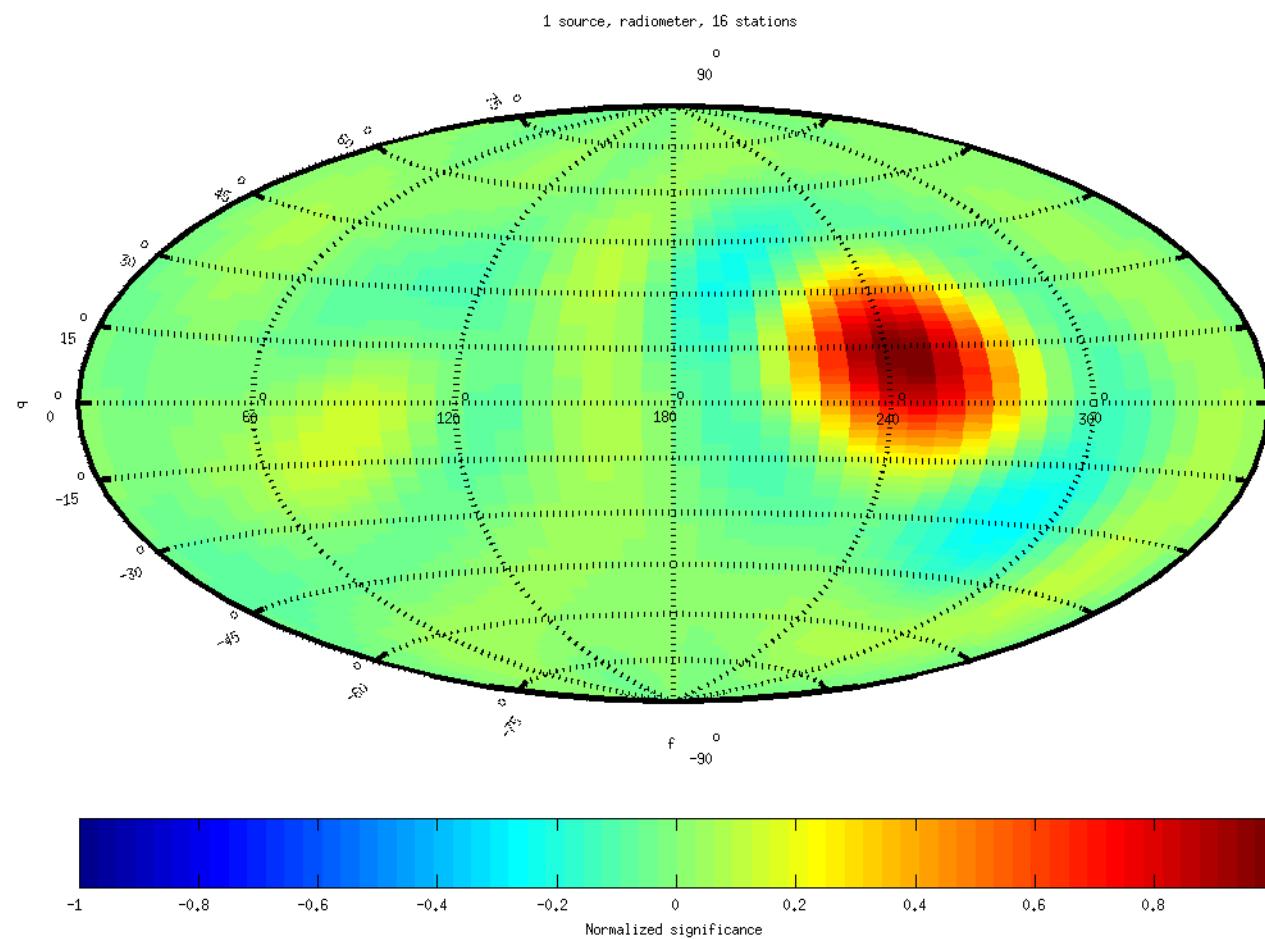
Multiplying this array by
1000 m gives the detector
locations

1.0049	0.8696	1.5024
0.9540	1.4846	0.5536
1.7282	1.7792	1.8847
0.8537	0.4295	1.0085
0.7172	0.3935	1.1054
1.4601	1.3522	1.1130
0.8472	0.0663	1.3949
0.8863	0.5722	0.4442
0.0771	0.9183	0.2785
0.7568	0.8304	0.0098
0.6074	1.1076	1.0030
0.0887	0.9303	0.1124
1.9672	0.7449	1.2506
1.3730	1.7486	1.5732
0.5502	1.8365	1.0232
1.1078	1.7061	0.5809

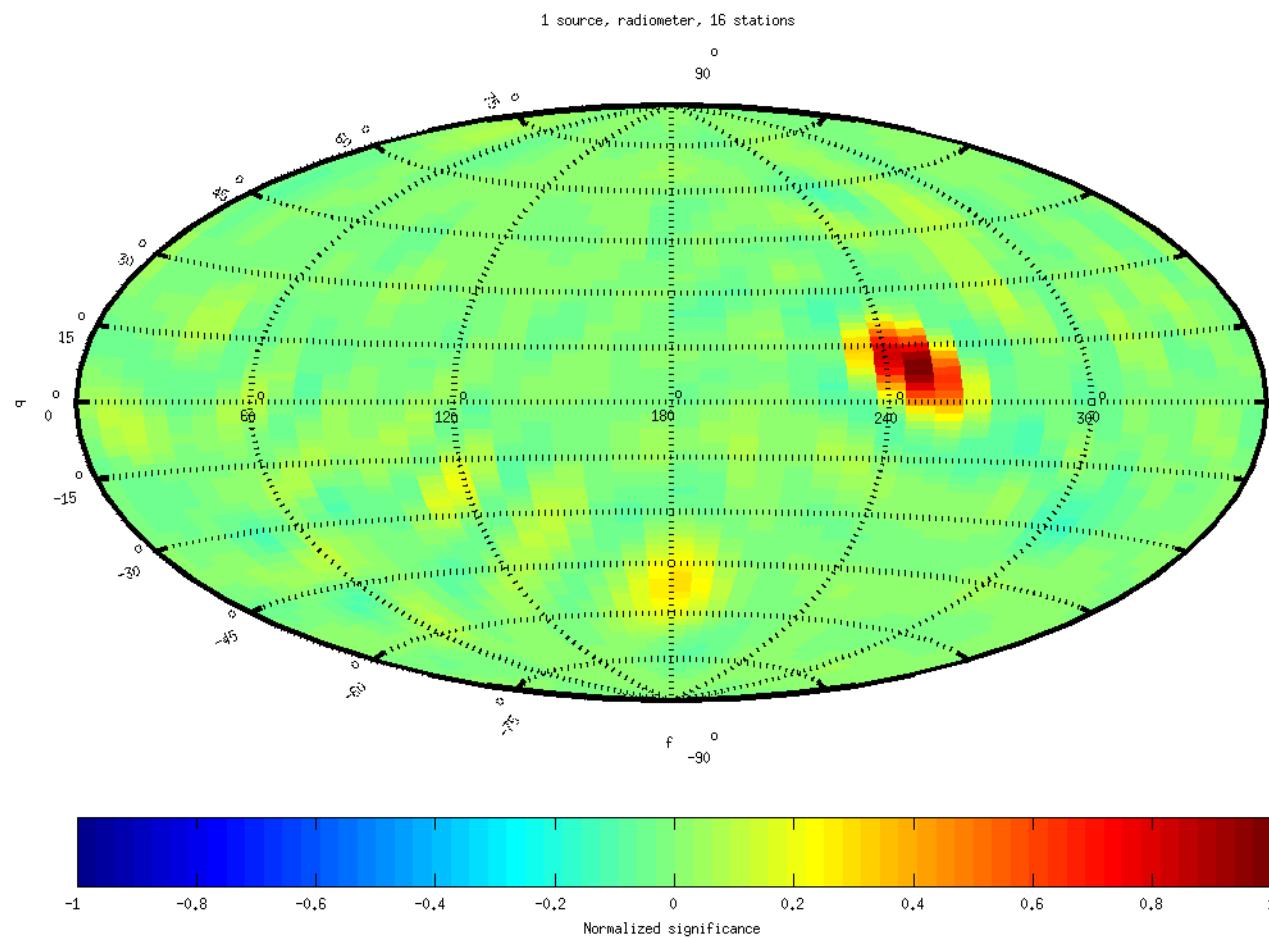
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 321.7864$ nstest44



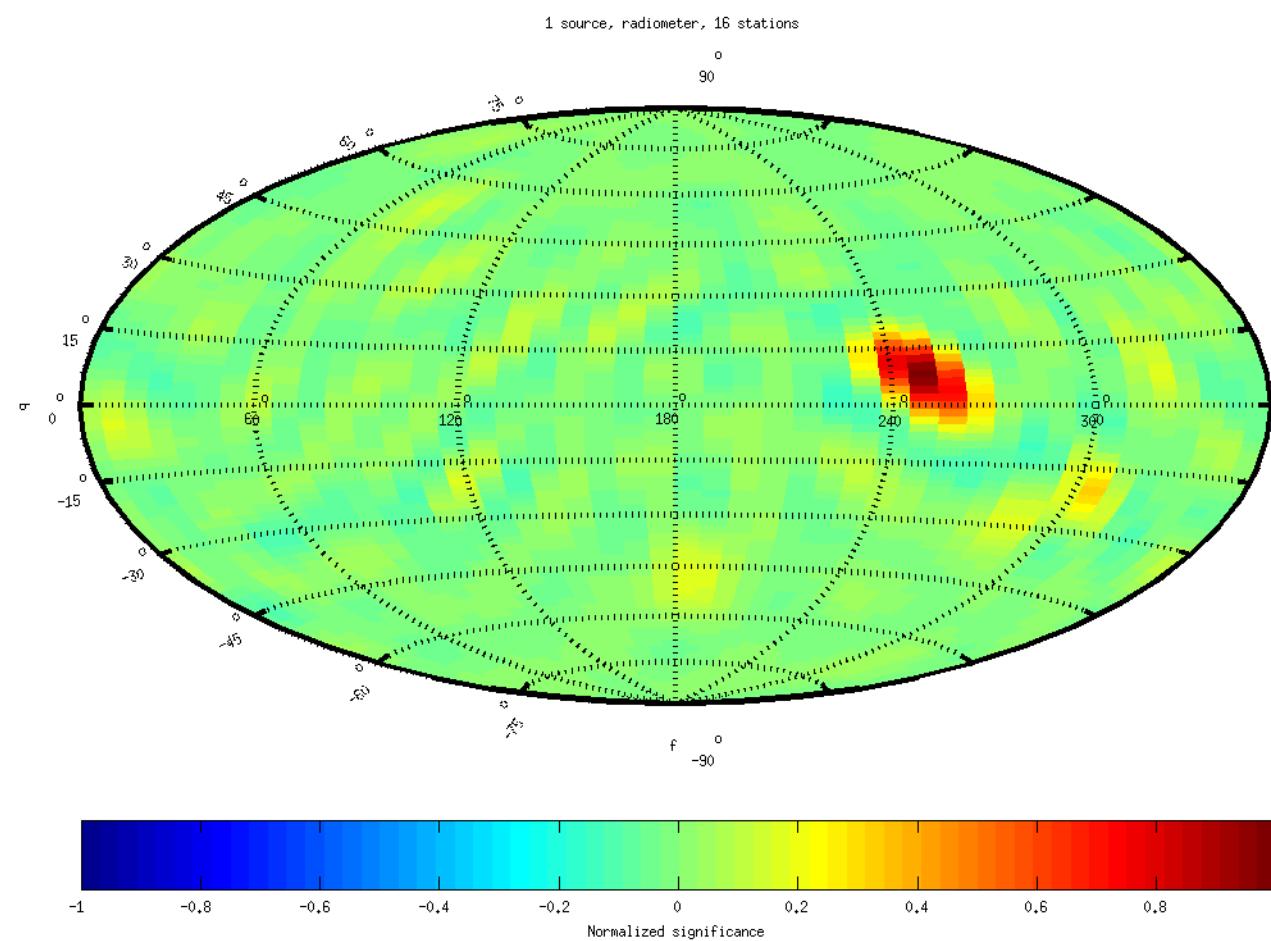
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 281.0784$ nstest45



$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 295.4467$ nstest46



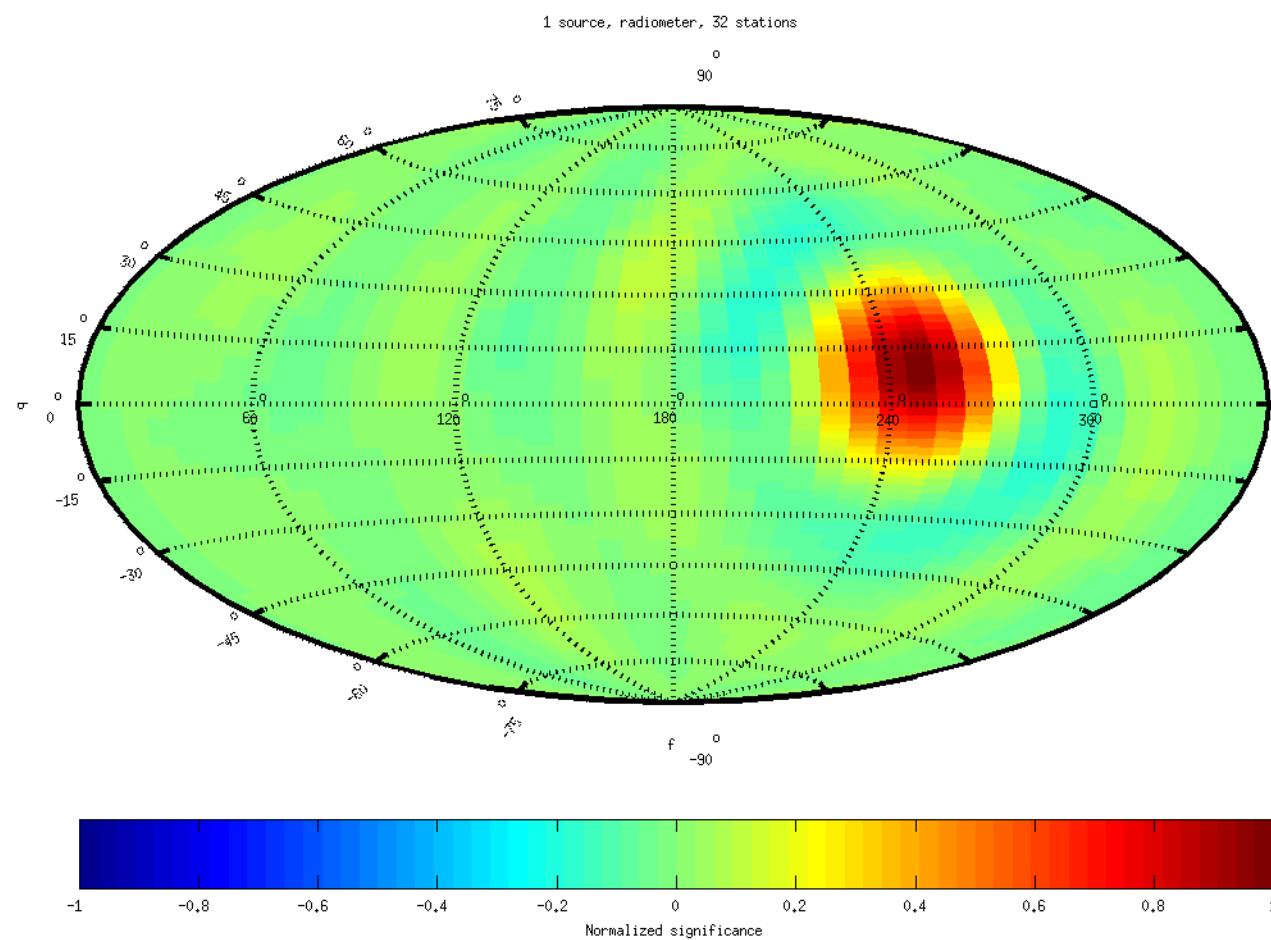
$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 90.0454$ nstest47



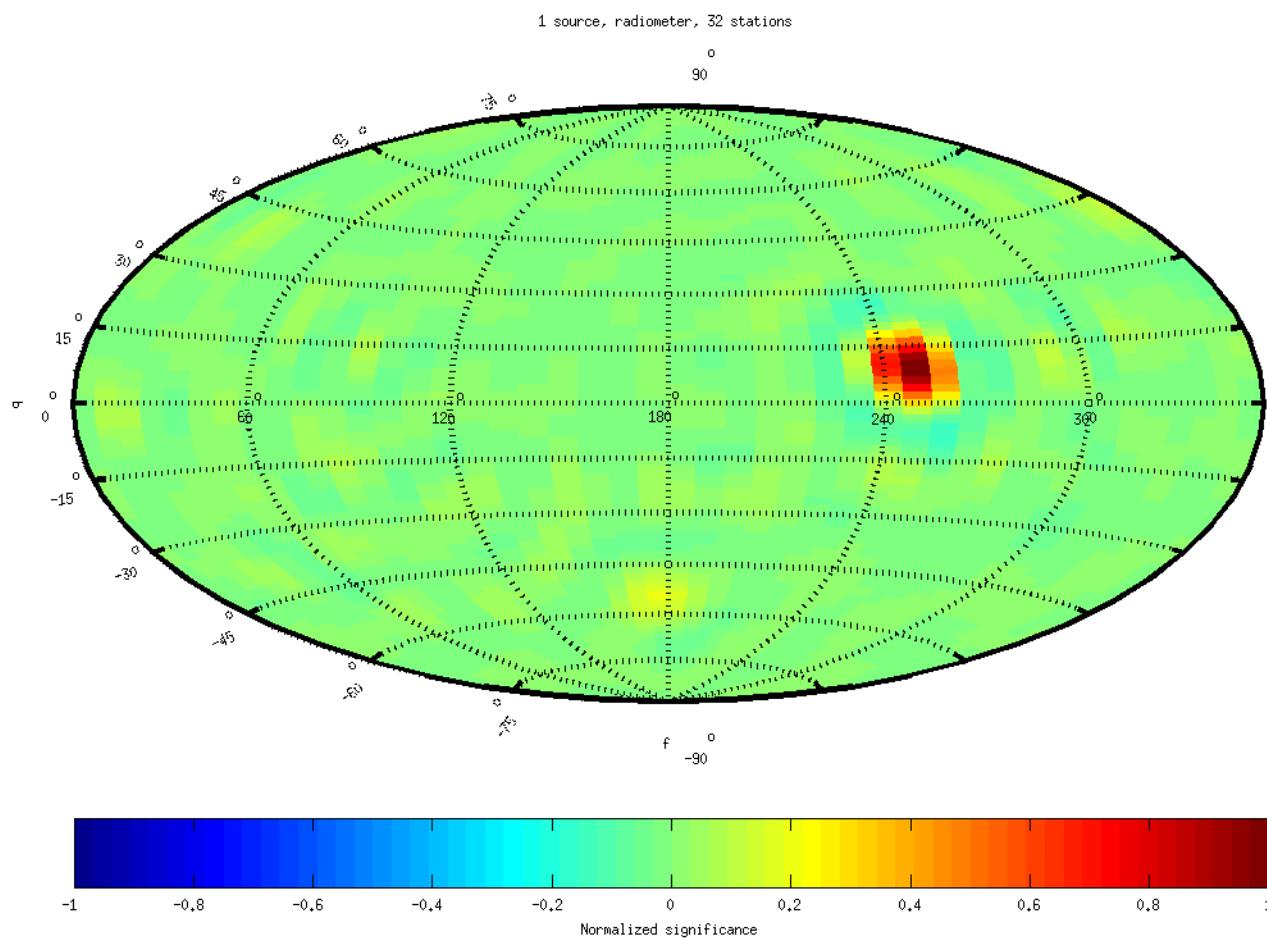
32 Detectors; Multiplying this array by 1000 m gives the detector locations

1.0049	0.8696	1.5024	0.9884	1.3599	0.5643
0.9540	1.4846	0.5536	0.8514	1.4977	1.3285
1.7282	1.7792	1.8847	1.0969	1.6837	0.9787
0.8537	0.4295	1.0085	1.7185	0.4925	1.6350
0.7172	0.3935	1.1054	1.9911	1.9055	1.7076
1.4601	1.3522	1.1130	0.9881	1.3445	1.0704
0.8472	0.0663	1.3949	0.1055	1.9273	1.7278
0.8863	0.5722	0.4442	1.8164	0.8485	1.8223
0.0771	0.9183	0.2785	1.1277	1.3868	1.5947
0.7568	0.8304	0.0098	1.0463	1.9557	1.9045
0.6074	1.1076	1.0030	0.3735	1.4004	1.9817
0.0887	0.9303	0.1124			
1.9672	0.7449	1.2506			
1.3730	1.7486	1.5732			
0.5502	1.8365	1.0232			
1.1078	1.7061	0.5809			
0.0094	0.9413	1.6537			
1.9372	1.9708	0.2618			
0.6468	0.7042	1.9955			
1.7509	1.6374	1.1217			
0.0904	0.2564	0.0551			

$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 321.7864$ nstest50



$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 295.4467$ nstest51



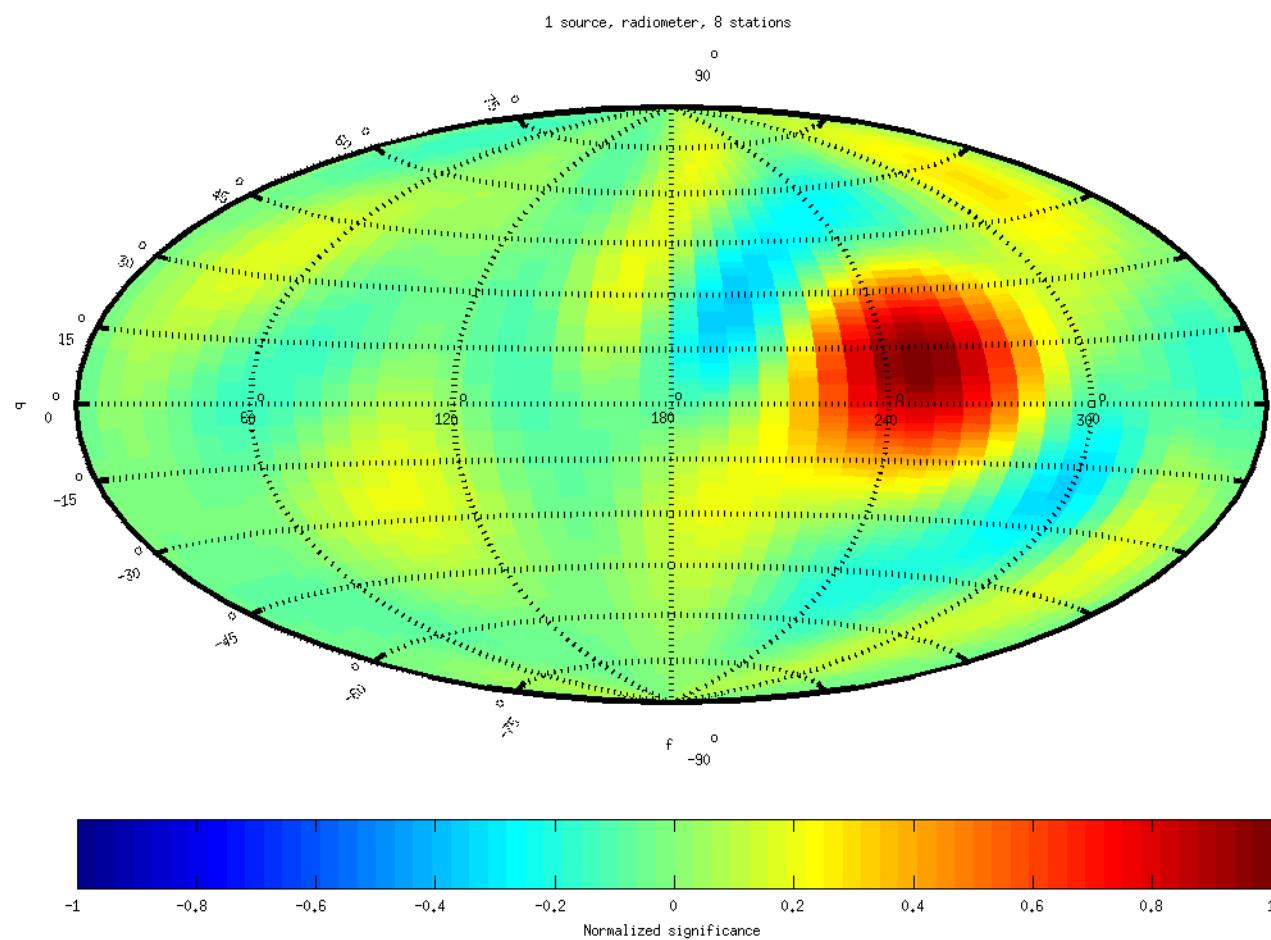
Broadband Number of Detectors/Polarization Testing

- This next series of tests are the same as before except that the injection signal is now broadband instead of monochromatic. The detector arrays are also the same.

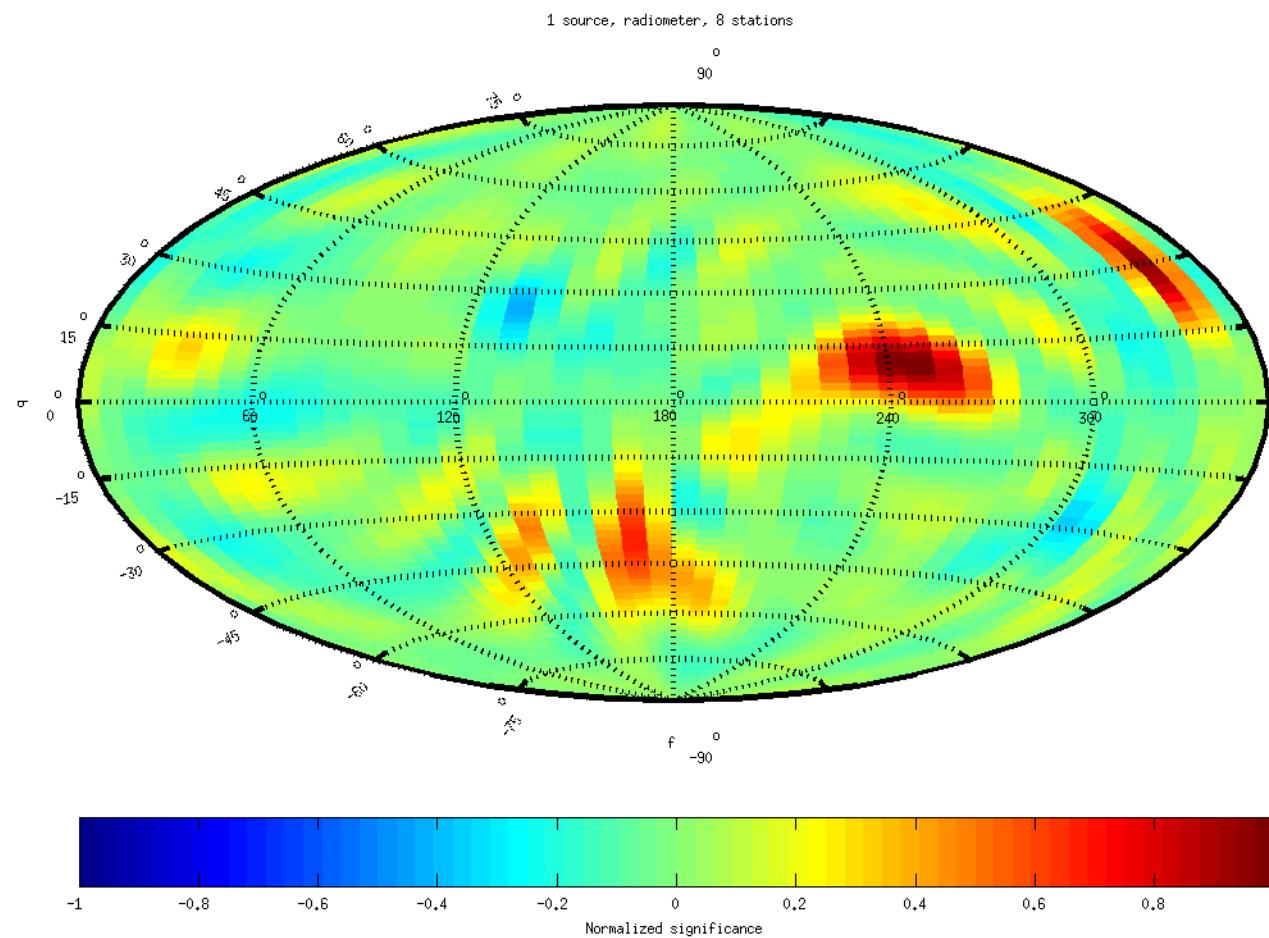
8 Detectors

- See previous 8 detector array

$f_{\text{analyse}} = 1 \text{ Hz}$ $\psi = 321.7864$ nstest52



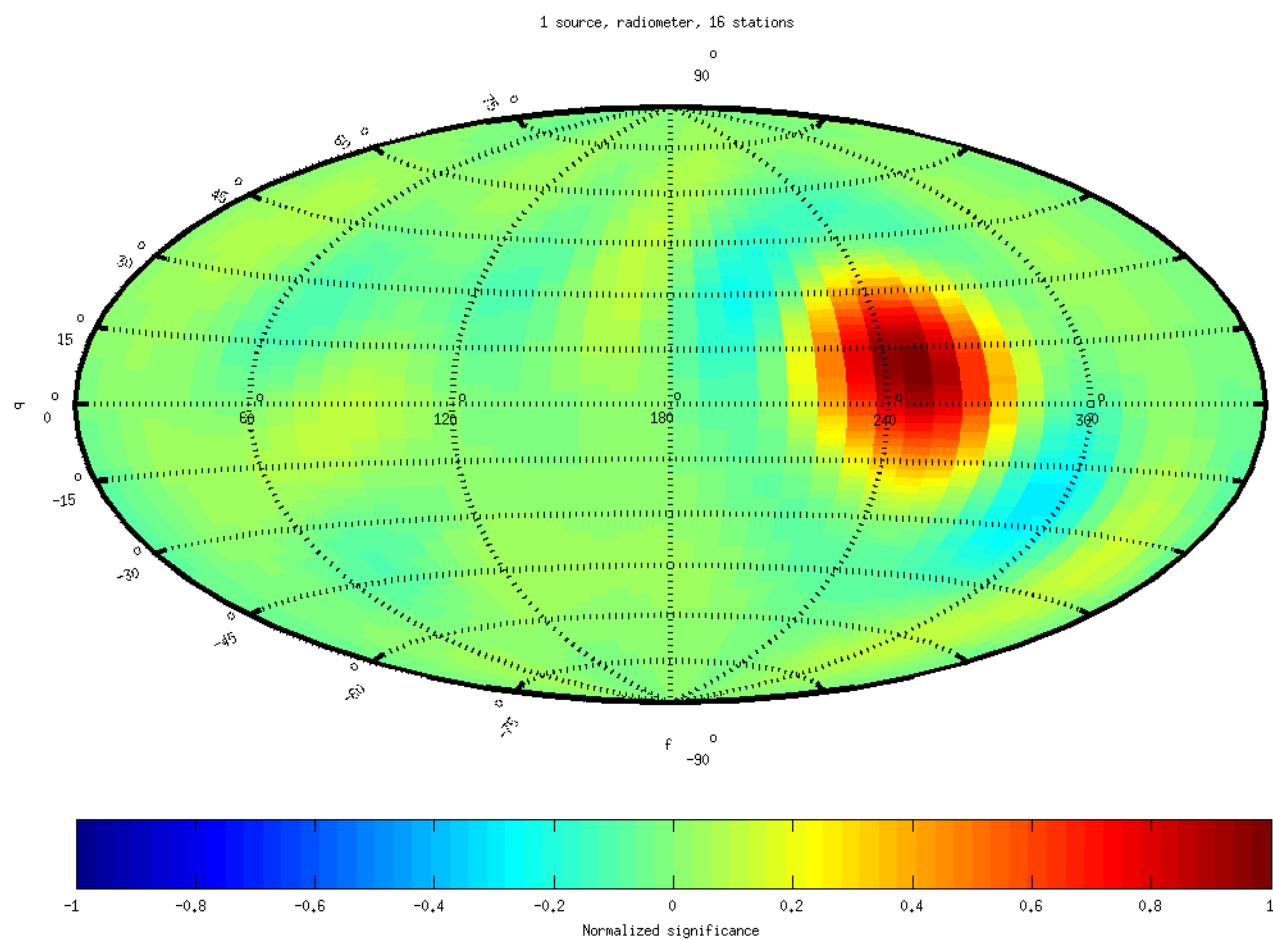
$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 295.4467$ nstest53



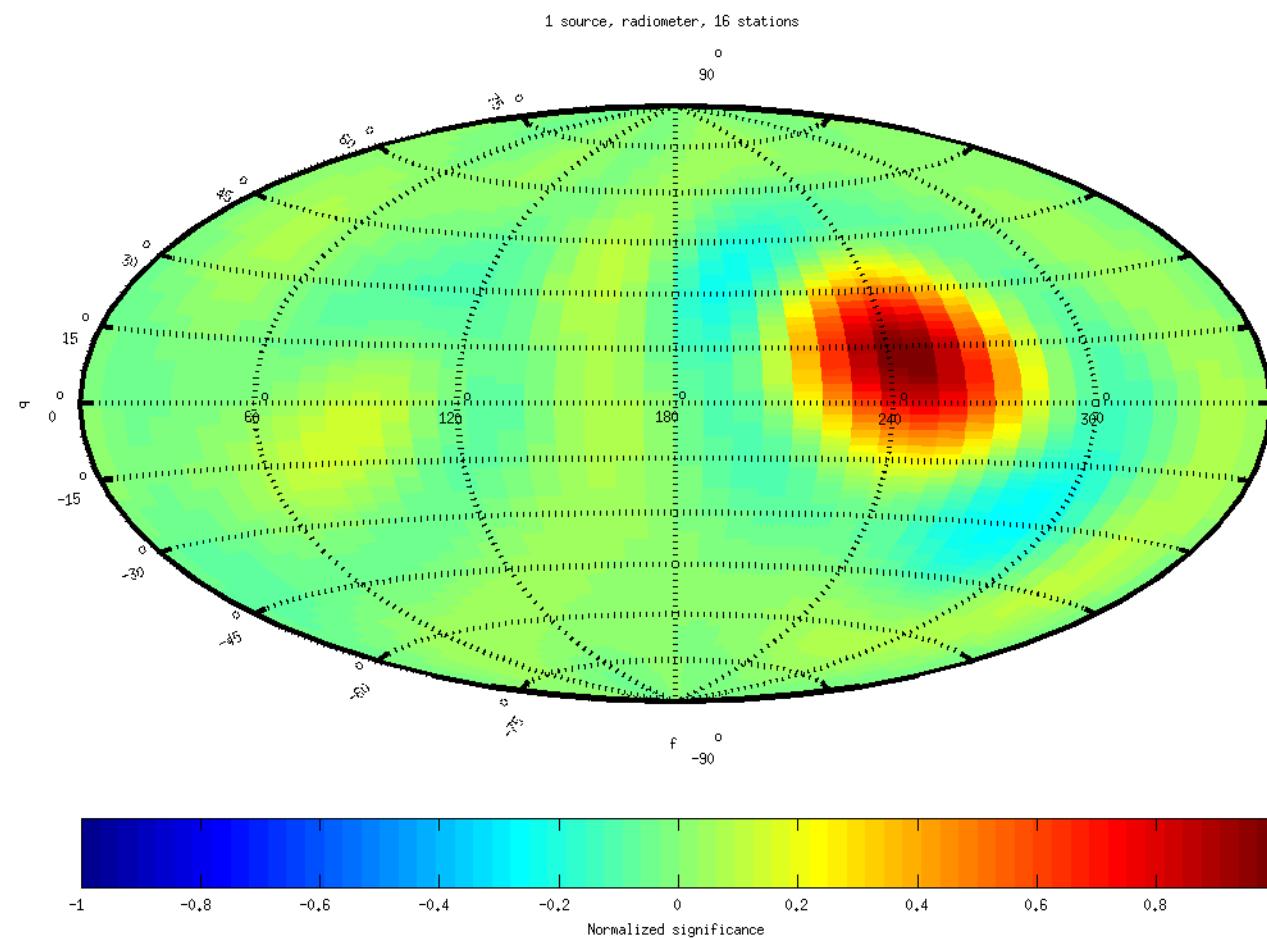
16 Detectors

- See previous 16 detector array

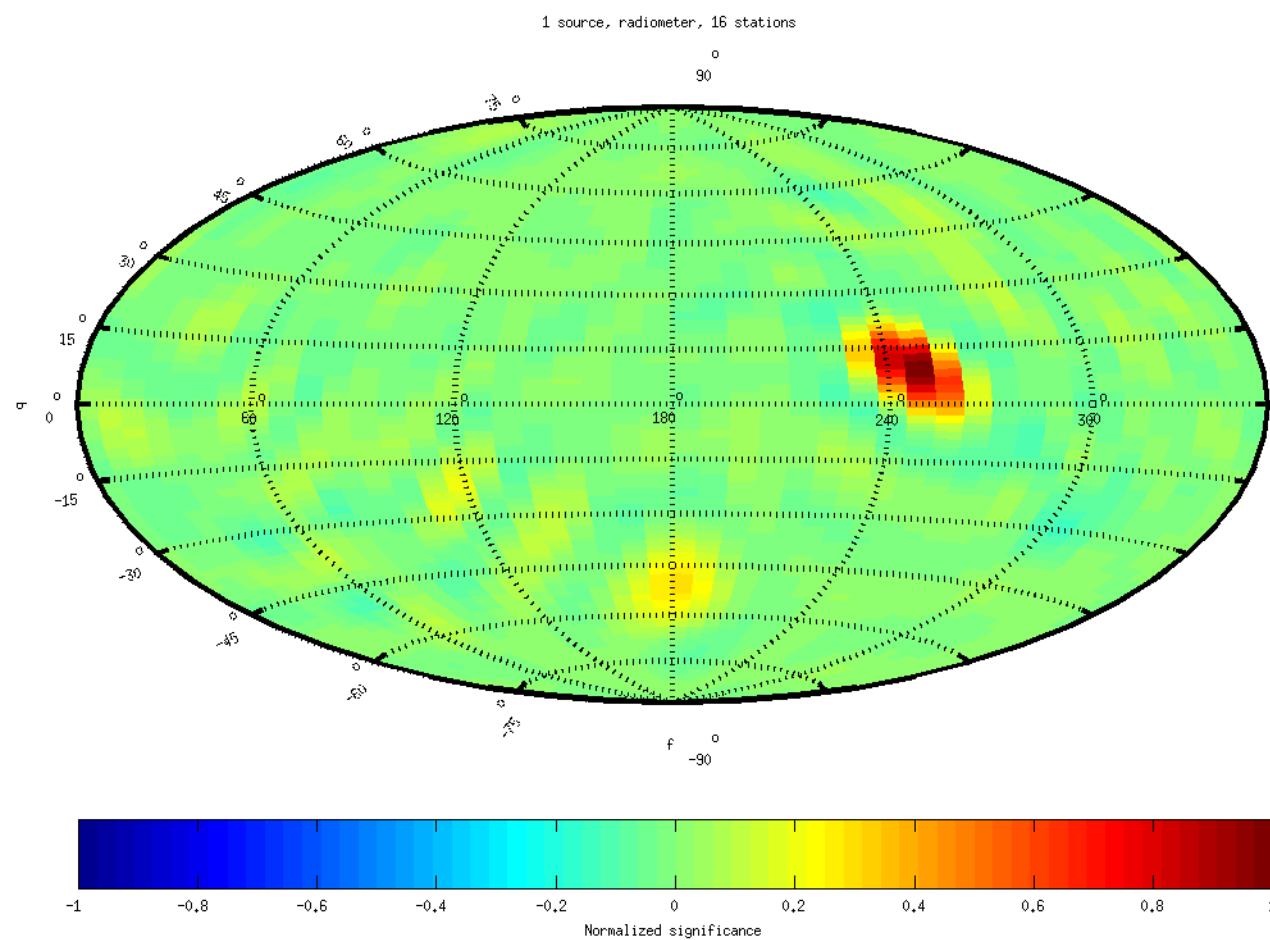
f_analyse = 1 Hz, psi = 321.7864 nstest54



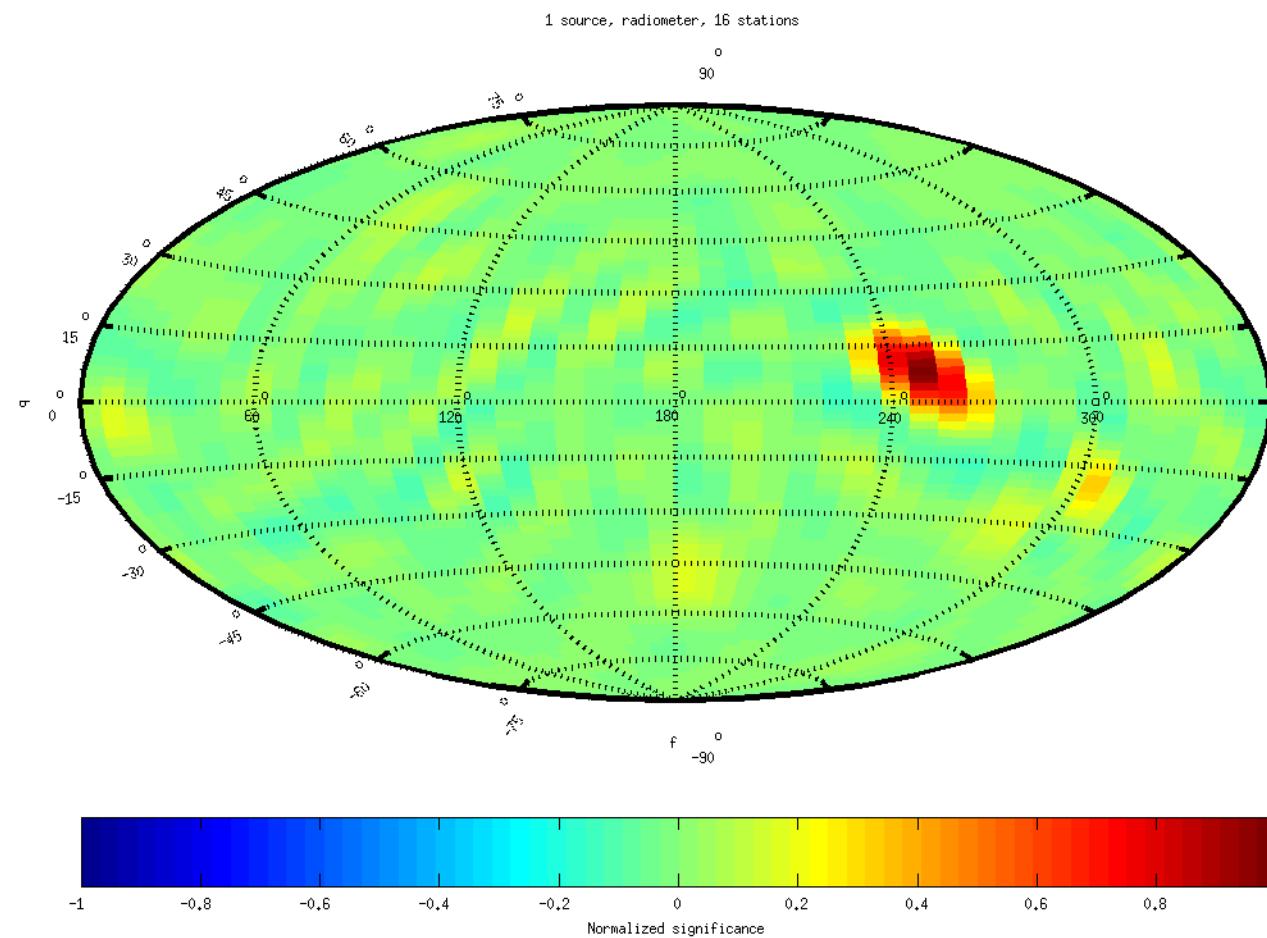
$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 281.0784$ nstest55



$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 295.4467$ nstest56



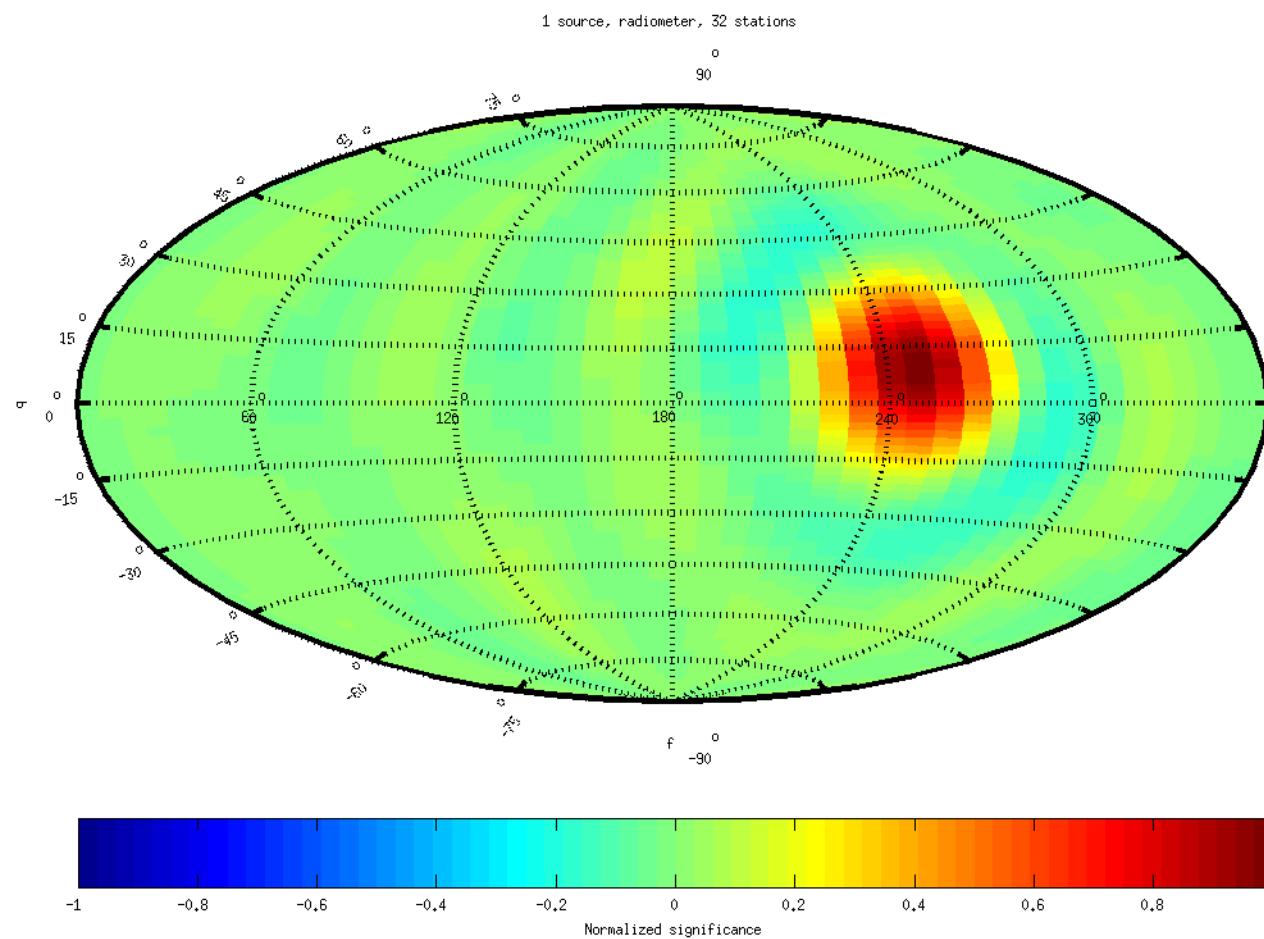
$f_{\text{analyse}} = 3 \text{ Hz}$, $\psi = 90.0454$
nstest57



32 Detectors

- See previous 32 detector array

$f_{\text{analyse}} = 1 \text{ Hz}$, $\psi = 321.7864$ nstest58



f_analyse = 3 Hz, psi = 295.4467 nstest59

