Analysis of Depth-dependent Behavior of Shear Waves

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Claim

With the aide of Boore and Joyner's analysis of site amplifications [3], and my analysis of the geologic composition of the Homestake Mine [4], I claim their velocity model for depth-dependent shear waves can be applied to the Homestake environment without loss of specificity.

Overview of Boore and Joyner (1997)

A power law representation for the velocity (as a function of depth) of shear waves is calculated from borehole data—we could use the data from active site and other underground experiments.

The authors use this and studies of crustal velocities to compute frequency-dependent amplifications for zero attenuation for use in simulations of strong ground motion. [1] [See Appendix]

Power Law Representation of Depth-dependent Shear Waves

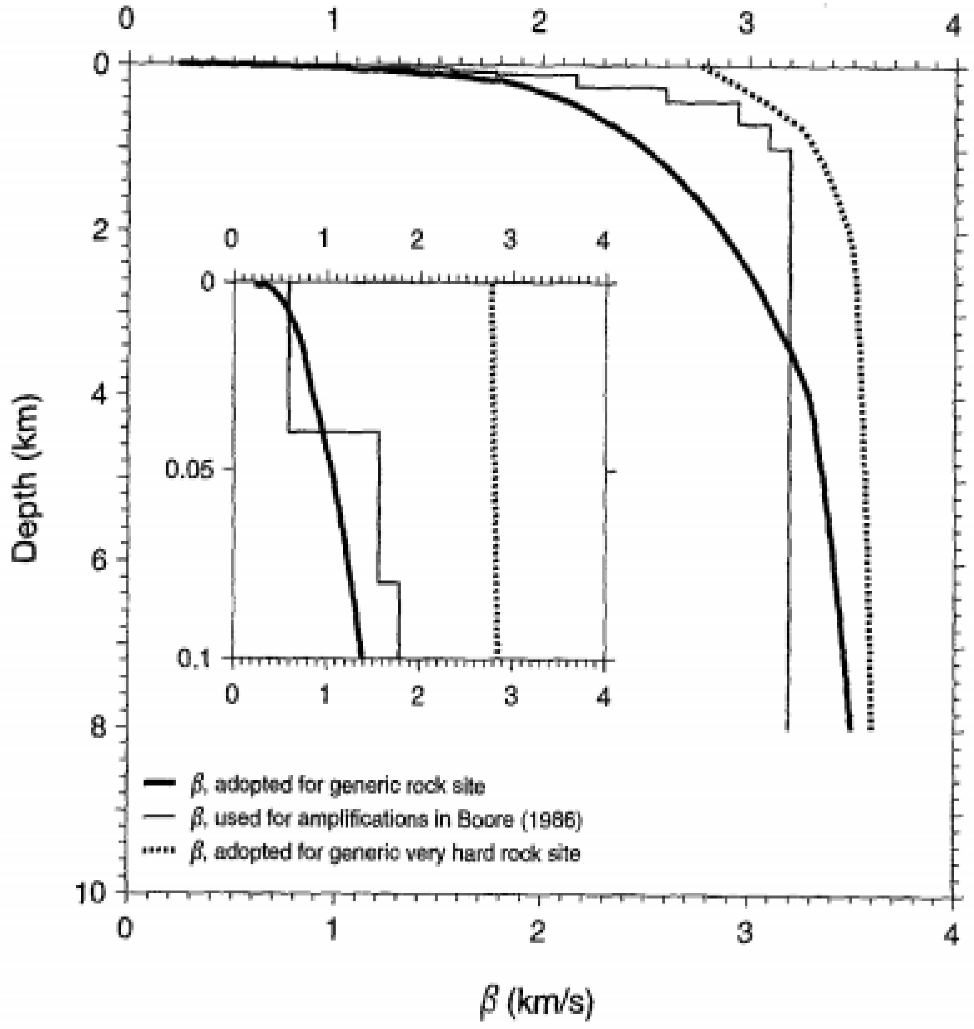


Figure 5. *S* velocity versus depth adopted in this article for generic rock sites (heavy solid line) and generic very hard rock sites (heavy broken line). For comparison, the light line is the velocity model used to obtain the amplifications published in Boore (1986).

Note(s): Found on p.7 of [1]
Compare this with Victor's slide from the October conference i.e. velocity of depth-dependent P-waves.

The difference between so-called generic rock and very hard rock sites is significant.

Functional Values for Power-Law Representation

Velocity for Generic Rock Site

Depth (km)	Shear Velocity (km/sec)*
$z \leqq 0.001$	0.245
$0.001 < z \leqq 0.03$	$2.206z^{0.272}$
$0.03 < z \leqq 0.19$	$3.542z^{0.407}$
$0.19 < z \leqq 4.00$	$2.505z^{0.199}$
$4.00 < z \leqq 8.00$	$2.927z^{0.086}$

*$\bar{V}_{30}$ = 0.618 km/sec.

Velocity for Generic Very Hard Rock Site

Depth (km)	Shear Velocity (km/sec)*
0.00	2.768
0.05	2.808
0.10	2.847
0.15	2.885
0.20	2.922
0.25	2.958
0.30	2.993
0.35	3.026
0.40	3.059
0.45	3.091
0.50	3.122
0.55	3.151
0.60	3.180
0.65	3.208
0.70	3.234
0.75	3.260
$0.75 < z \leqq 2.20$	$3.324z^{0.067}$
$2.20 < z \leqq 8.00$	$3.447z^{0.0209}$

*$\bar{V}_{30}$ = 2.88 km/sec.

Tables 1 (above) and 2 (right): Found on ppg. 6 and 8 of [1], respectively.

These values were calculated [1] with average velocities from the borehole data (<30 m.)

Rock Sites v. Very Hard Rock Sites

- Rock:

  Described by terms such as ‘granite,’ ‘diorite,’ ‘gneiss,’ ‘chert,’ ‘graywacke,’ ‘limestone,’ ‘sandstone,’ or ‘siltstone,’ etc. [2] [6]

Table 3

Rock	Hardness (on Mohs scale)
granite	5.0 - 7.0
diorite	4.8 - 6.2
gneiss	5.3 - 6.5
chert	7.0
limestone	2.0 - 5.0
sandstone	2.0 - 7.0

- Very Hard Rock

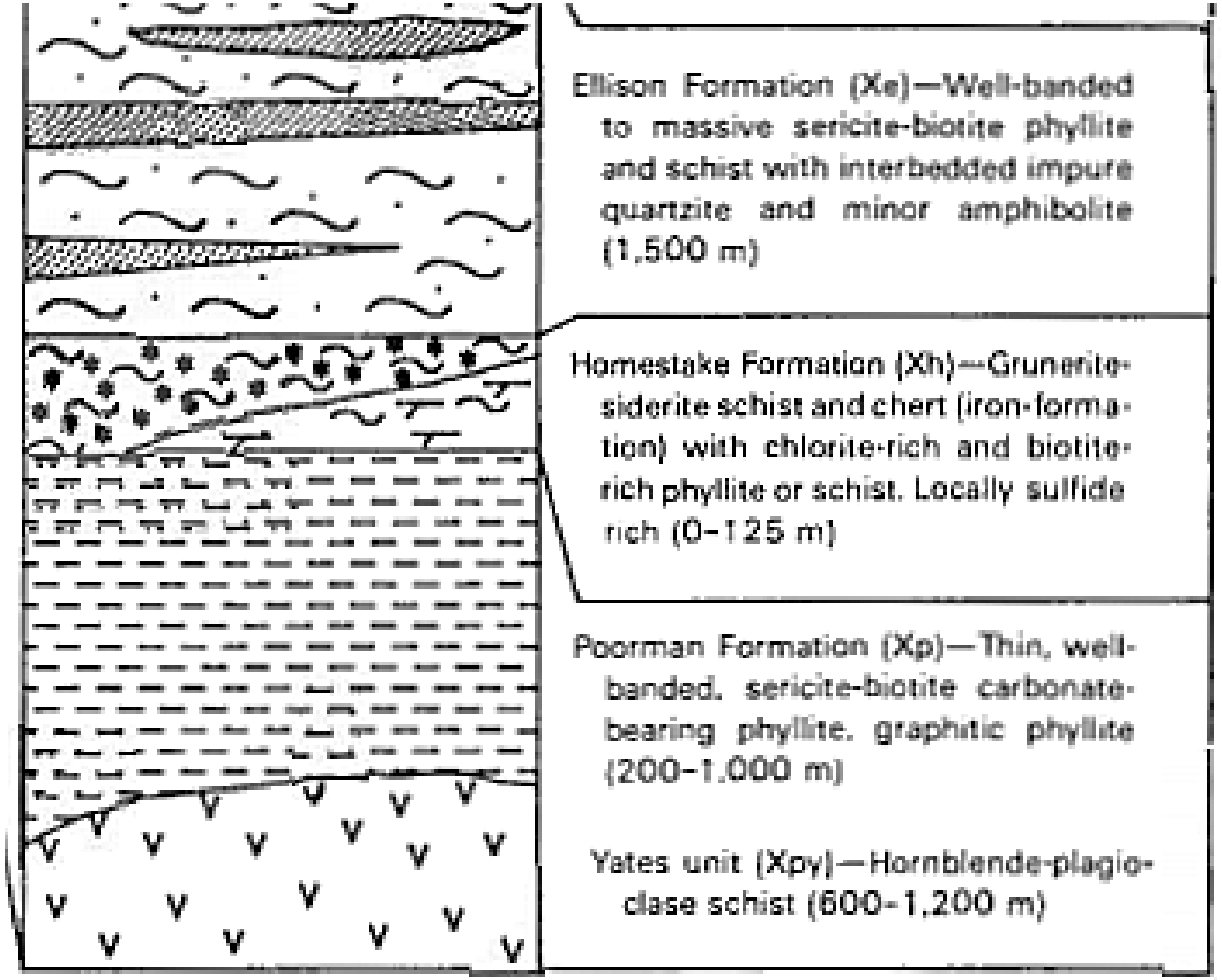
  Typical of rocks found in glaciated regions in large areas of eastern North America or in portions of western North America [1] e.g. geology of the Appalachians.

Homestake: Rock or Very Hard Rock?

Fig. 2 (right): From p. J6 of [3]

This diagram shows/lists the major mineral constituents of the major formations within the Homestake Mine. Tables containing modal percentages of representative minerals found in each formation [3] is in the Appendix.

Next: Estimate the hardness of each formation in question.



Methodology

- Main assumption: hardness of materials is an additive quantity [5]
- Using Tables J1, J3, and J5—shown in Appendix [3]:
  - Estimate hardness ($\bar{H}_M$) of each site using a normalized weighted average; i.e.

$$\bar{H}_M = \sum_{i \in S} w_i (H_M)_i \tag{1}$$

where S spans the sample space consisting of the pertinent minerals in each table, w_i is the percent mineral composition, and $(H_M)_i$ is the hardness of each constituent mineral [See Appendix]

Poorman Formation: Composition and Hardness

HPS: hornblende plagioclase schist

CS: carbonate-rich schist

HBCS: hornblende-biotite-carbonate schist

GQSP: graphitic quartz-sericite phyllite

SCQP: sericite-carbonate-quartz phyllite

BQCP: biotite-quartz-carbonate phyllite

Note(s):

- The Poorman Lower Unit is almost exclusively composed of amphibolite [3]
- The Poorman Upper Unit is dominated by calcite and ankerite containing a significant pelitic component along with minor amounts of dolomite [3]

Table 4

Rock Type	Hardness (on Mohs scale)	Location
HPS	5.2 - 6.1	3800 level, Yates Shaft area
HPS	5.2 - 6.1	4100 level, Yates Shaft area
HPS	5.3 - 6.2	4850 level, Yates Shaft area
CS	4.2 - 4.4	7700 level, No. 6 Winze
HBCS	3.8 - 4.4	4100 level, Yates Shaft area
GQSP	4.9 - 5.4	8000 level, 21 Ledge
GQSP	4.2 - 4.6	8000 level, 19 Ledge
GQSP	3.5 - 4.1	4850 level, 15 Ledge
SCQP	4.2 - 4.6	4100 level, Ross Shaft area
SCQP	3.7 - 4.1	4850 level, 4 Winze area
SCQP	3.9 - 4.3	6800 level, near Main Ledge
BQCP	4.1 - 4.4	4850 level, 15 Ledge
BQCP	3.6 - 4.0	7700 level, 6 Shaft area

Homestake Formation: Composition and Hardness

GDS: grunerite-dominant schist

SDP: siderite-dominant phyllite

CQS: chlorite-quartz schist

Notes:

- In the Homestake, in upper greenschist facies, siderite phyllite is dominant, whereas in lower amphibolite facies, grunerite is schist is dominant. [3]
- Chlorittic schist is important as a “translational” phase into the neighboring formations. [3]
- The central mine is determined solely by the presence of both iron-carbonate and iron-silicate mixtures, while the east and west mine are composed of iron-carbonates and iron-silicates, respectively [3]

Table 5

GDS	4.1 - 4.9	4550 level, Main Ledge
GDS	5.3 - 6.0	4550 level, 9 Ledge
GDS	4.8 - 5.7	6800 level, 21 Ledge
GDS	3.6 - 4.2	6800 level, 21 Ledge
GDS (ore)	5.3 - 6.2	7200 level, 9 Ledge
GDS	5.0 - 5.9	8300 level, Pierce Structure (Main Ledge)
SDP (ore)	4.0 - 4.5	800 level, 7 Ledge
SDP	4.4 - 4.8	1700 level, 7 Ledge
SDP (ore)	3.1 - 3.8	6650 level, 9 Ledge
SDP	4.1 - 4.5	5750 level, 17 Ledge
SDP	4.3 - 4.6	5900 level, 17 Ledge
SDP (ore)	4.2 - 4.6	6800 level, 21 Ledge
CQS	5.3 - 5.8	800 level, 7 Ledge
CQS	4.5 - 4.9	5600 level, 11 Ledge
CQS	4.9 - 5.4	6950 level, 21 Ledge

Ellison Formation: Composition and Hardness

QMS: quartzite-mica schist

SQP: sericite-quartz phyllite

BQP: biotite-quartz phyllite

Note(s):

- The Ellison Formation consists mainly of phyllite, quartz-mica schist (QMS), and quartzite. [3]

Table 6

Quartzite	6.3 - 6.4	4550 level, 11 Ledge
Quartzite	7.0	6500 level, Main Ledge
Quartzite	6.8	6800 level, 9 Ledge
QMS	5.4 - 5.6	5900 level, 13 Ledge
SQP	4.1 - 4.4	2600 level, east of Yates Shaft
SQP	3.9 - 4.2	6800 level, Main Ledge
SQP	3.1 - 3.5	6800 level, 13 Ledge
SQP	4.4 - 4.7	6800 level, 15 Ledge
BQP	4.5 - 4.8	2600 level, east of Yates Shaft
BQP	4.0 - 4.4	6500 level, Main Ledge
BQP	4.9 - 5.2	6800 level, 9 Ledge
Amphibolite	5.1 - 5.9	Drill hole north of Lead, S. Dak.

Potential Issues

- "Directionality of incoming seismic wave"
  - "…incidence angles of 30 ° and 45 ° were used to approximate the range of angles that would exist for events not directly under the site (the incidence angles would be smaller for input at shallower depths because of refraction)." [1]

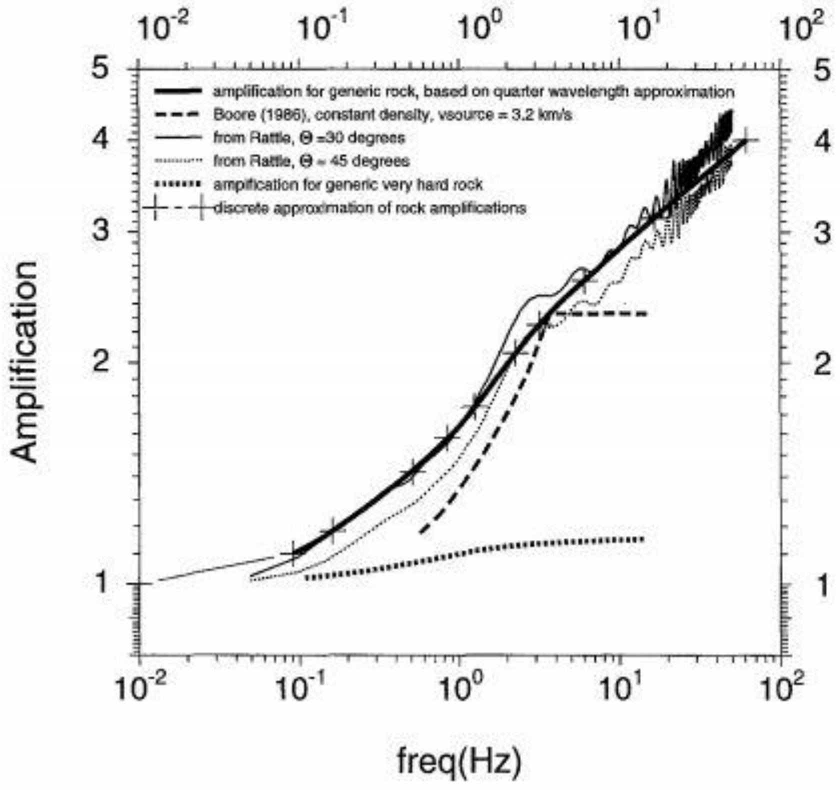


Figure 7. Amplification versus frequency. The heavy solid line is computed using the quarter-wavelength approximation and the velocity profile shown by the heavy solid line in Figure 5. The long-dashed line is from Boore (1986), also based on the quarter-wavelength approximation but using the "Boore (1986)" velocities shown in Figure 5. The results from plane *SH* waves incident at the base of a 8-km-thick stack of constant-velocity layers (with Q = 10,000) closely approximating the adopted continuous shear-wave velocity are shown by the light lines for angles of incidence of 30° and 45°; the results were computed from the Haskell matrix method, as implemented by program *Rattle* by C. Mueller. The quarter-wavelength amplification for generic very hard rock is given by the heavy short-dashed line.

It may be difficult to see in the figure on the left (found on pg. of [1]), but despite using a range of incident angles for an event, the amplification for generic rock sites are remarkably similar—especially at lower frequencies.

This hints that we may be able to conclude the shear waves are very similar in this range, but we may have to make more assumptions based on Equation 2 [See Appendix].

Conclusion

Comparison of the Homestake geology [3] and what the authors [1] deem generic rock and very hard rock: Clearly, the major formations within Homestake can be considered a “generic rock site”

Thus, the power-law representation for depth-dependent shear waves can be applied (possibly with modification) to the Homestake Mine without loss of specificity.

Appendix: Geology

Note(s):

All of these values were found in [4] unless otherwise denoted.

"[Graphite is considered to be] the only carbon phase at metamorphic conditions of middle greenschist through middle amphibole facies." [3]

*: indicates values obtained from the associated Wikipedia article.

˚: indicates subgroup in which each constituent shares properties with each other constituent

Table 7

Mineral	Hardness (on Mohs scale)
Quartz	7.0
Hornblende	5.0 - 6.0
Biotite	2.5 - 3.0
Sericite/Muscovite	2.5 - 3.0
*Mg-chlorite aka Clinochlore	2.0 - 2.5
˚Intermediate Plagioclase	6.0 - 6.5
Rutile	6.0 - 6.5
Graphite	1.0 - 2.0
Siderite	4.0 -4.5
Ankerite	3.5 - 4.0
Calcite	3.0
Pyrrhotite	3.5 - 4.5
Pyrite	6.0 - 6.5
*Grunerite	5.0 - 6.0
˚Na-amphibole	5.0 - 6.0
*Fe-chlorite aka Chamosite	2.0 -2.5
*Garnet	6.5 - 7.5
Albite	6.0 - 6.5
Arsenopyrite	5.5 - 6.0
Epidote/Clinozoisite	6.0 - 6.5
Magnetite	5.5 - 6.5

Poorman Formation Compostion

Fig. 1 (right): The modal mineral percentages of representative Poorman Formation as found on p. J11 of [3].

Note: Not all of these values are normalized.

Table J1. Modal mineral percentages in thin sections of representative Poorman Formation, Homestake mine

[Data from unpublished Homestake reports. Chemical data on table J2 are for different samples than shown here. Trace amounts of unusual minerals are not shown. HPS, hornblende-plagioclase schist (Yates unit); HBCS, hornblende-biotite-carbonate schist; CS, carbonate-rich schist; GQSP, graphitic quartz-sericite phyllite; SCQP, sericite-carbonate-quartz phyllite; BQCP, biotite-quartz-carbonate phyllite; X, <1 percent]

Rock type	Matrix quartz	Grunerite	Hornblende	Na-amphibole	Biotite	Sericite (muscovite)	Fe-chlorite	Mg-chlorite	Clinochlore	Garnet	Albite	Intermediate plagioclase	Tourmaline	Titanite or "leucoxene"	Epidote or clinozoisite	Zircon	Ilmenite or rutile	Magnetite	"Graphite"	Siderite	Ankerite	Calcite	Pyrrhotite	Arsenopyrite	Pyrite	Location
HPS	2		84									12					1	1			X					3800 level, Yates Shaft area
HPS	4		77									18		X	X		X	X			1					4100 level, Yates Shaft area
HPS	5		75									20					X				X					4850 level, Yates Shaft area
HBCS			40		40												5					15				7700 level, No. 6 Winze
CS	33				20	3													3		9	30	2			4100 level, Yates Shaft area
GQSP	38				4	18													11		12		13		9	8000 level, 21 Ledge
GQSP	38				9	28													9		15		1			8000 level, 19 Ledge
GQSP	28				10	30													20		10		12			4850 level, 15 Ledge
SCQP	30					27													2		40		X			4100 level, Ross Shaft area
SCQP	18					45													1		35		X		1	4850 level, 4 Winze area
SCQP	22				5	30										X			3		27		2		1	6800 level, near Main Ledge
BQCP	35				56														3		6					4850 level, 15 Ledge
BQCP	25				55			15													X		5			7700 level, 6 Shaft area

Homestake Formation Compostion

Fig. 2 (right): The modal mineral percentages of representative Homestake Formation as found on p. J16 of [3].

Note: Not all of these values are normalized.

Table J3. Modal mineral percentages in thin sections of representative Homestake Formation, Homestake mine

[Data from unpublished Homestake reports. Chemical data on table J4 are for different samples than shown here. Trace amounts of unusual minerals are not shown. GDS, grunerite-dominant schist; SDP, siderite-dominant phyllite; CQS, chlorite-quartz schist; X, <1 percent; *, equivalent mine level encountered in drill core. No visible gold present]

Rock type	Matrix quartz	Grunerite	Hornblende	Na-amphibole	Biotite	Sericite (muscovite)	Fe-chlorite	Mg-chlorite	Clinochlore	Garnet	Albite	Intermediate plagioclase	Tourmaline	Titanite or "leucoxene"	Epidote or clinozoisite	Zircon	Ilmenite or rutile	Magnetite	"Graphite"	Siderite	Ankerite	Calcite	Pyrrhotite	Arsenopyrite	Pyrite	Location
GDS	4	51			34					5									4				2			4550 level, Main Ledge
GDS	30	56			5						2								3	3			1			4550 level, 9 Ledge
GDS	2	78			6		3			3	6								2							6800 level, 21 Ledge
GDS	4	38			13		36												1	8						6800 level, 21 Ledge
GDS (ore)		40			5				5	40													5	5		7200 level, 9 Ledge*
GDS	8	75		8	8					1											X		X			8300 level, Pierce Structure (Main Ledge)*
SDP (ore)	6				4		4				6								4	72			4			800 level, 7 Ledge
SDP	24				10		4				X								2	60						1700 level, 7 Ledge
SDP (ore)							24			2									13	50			11			6650 level, 9 Ledge
SDP	18						18												4	60			X			5750 level, 17 Ledge
SDP	28				2		24												2	41			3			5900 level, 17 Ledge
SDP (ore)	26						20												6	42			6			6800 level, 21 Ledge
CQS	38				15		36			16	1								2	11			X			800 level, 7 Ledge
CQS	14						49													12			6			5600 level, 11 Ledge
CQS	26				4		37													33						6950 level, 21 Ledge

Ellison Formation Compostion

Fig. 3 (right): The modal mineral percentages of representative Ellison Formation as found on p. J19 of [3].

Note: Not all of these values are normalized.

Table J5. Modal mineral percentages in thin sections of representative Ellison Formation, Homestake mine

[Data from unpublished Homestake reports. Chemical data on table J6 are for samples different from those shown here. Trace amounts of unusual minerals are not shown. SQP, sericite-quartz phyllite; BQP, biotite-quartz phyllite; QMS, quartz-mica schist; X, <1 percent]

Rock type	Matrix quartz	Grunerite	Hornblende	Na-amphibole	Biotite	Sericite (muscovite)	Fe-chlorite	Mg-chlorite	Clinochlore	Garnet	Albite	Intermediate plagioclase	Tourmaline	Titanite or "leucoxene"	Epidote or clinozoisite	Zircon	Ilmenite or rutile	Magnetite	"Graphite"	Siderite	Ankerite	Calcite	Pyrrhotite	Arsenopyrite	Pyrite	Location
Quartzite	83				3	5										X			3			3				4550 level, 11 Ledge
Quartzite	95					X						X				X			X		X	X			X	6500 level, Main Ledge
Quartzite	91				X	5				X							X		X				X			6800 level, 9 Ledge
QMS	60				20	15				4		1	X						X		X		X			5900 level, 13 Ledge
SQP	30				X	65				5			X	X									X			2600 level, east of Yates Shaft
SQP	30				20	50																				6800 level, Main Ledge
SQP	12				15	70																	3			6800 level, 13 Ledge
SQP	35					35															30		X			8000 level, 15 Ledge
BQP	45				45	10		X		X										X	X		X			2600 level, east of Yates Shaft
BQP	23				30														3		35		10			6500 level, Main Ledge
BQP	40				45					15											X					6800 level, 9 Ledge
Amphibolite			72		4							20		X	3		X									Drill hole north of Lead, S. Dak.

Notes on Amplification

From Boore and Joyner (1997):

- [T]he S travel time S_{tt}(z) from the surface to depth z either is taken from downhole surveys or is computed using shear velocity as a function of depth; the average velocity to depth z, $\overline{\beta(z)}$, is z/S_{tt}(z) and the frequency corresponding to the depth, $f(z)$, is 1/[4 × S_{tt}(z)]; a travel-time-weighted average is taken of the density, $\overline{\rho(z)}$; and the amplification is given by:

$$A[f(z)] = \sqrt{\frac{\rho_s \beta_s}{\overline{\rho(z)}\,\overline{\beta(z)}}} \qquad (2)$$

Resources

[1] Boore and Joyner. *Site Amplifications for Generic Rock Sites*, BSSA, Vol. 87, No.2, pp. 327 – 341, April 1997.

[2] Caddey, Bachman, and Otto. (1990). *15 Ledge Ore Discovery, Homestake Mine, Lead, South Dakota*, Retrieved from: http://homestake.sdsmt.edu/Protected/Lead1990meeting/15%20Ledge%20discovery.pdf

[3] Caddey, S., & Geological Survey. (1992). *The Homestake Gold Mine : An Early Proterozoic Iron-formation-hosted Gold Deposit, Lawrence County, South Dakota*. Print.

[4] Company, Chemical Rubber. "CRC Handbook of Chemistry and Physics." *CRC Handbook of Chemistry and Physics*. (1975). Print.

[5] Szymański, Andrzej, and Janusz Mikołaj Szymański. *Hardness Estimation of Minerals, Rocks and Ceramic Materials*. Amsterdam: Elsevier, 1989. Print.

[6] Winkler, Erhard M. *Stone--properties, Durability in Man's Environment*. New York: New York : Springer-Verlag. (1975). Print.