Solar Modes in Seismic Spectra

Presented by Ross Caton

Background: Free Oscillations

- Alternative to travelling wave model (mathematically equivalent solution) (Stein & Wysession 2003)

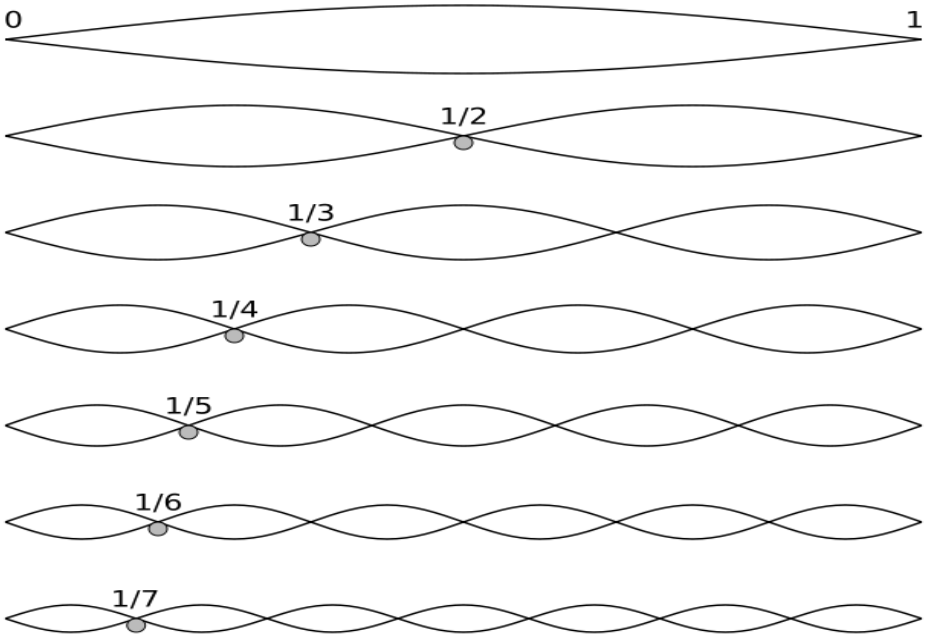


Image source (left): https://en.wikipedia.org/wiki/File:Standing_wave.gif

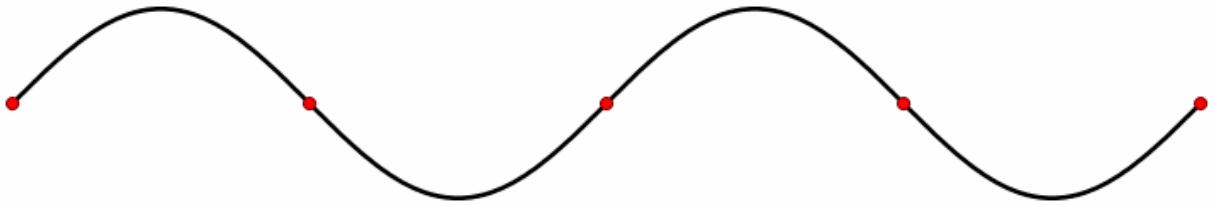


Image source (above): https://en.wikipedia.org/wiki/File:Harmonic_partials_on_strings.svg

Background: Free oscillations

- Earth's free oscillations are described by spherical harmonics
- Very ugly mathematics
- Characterized by "quantum numbers" l ,m, and n (Stein & Wysession 2003)

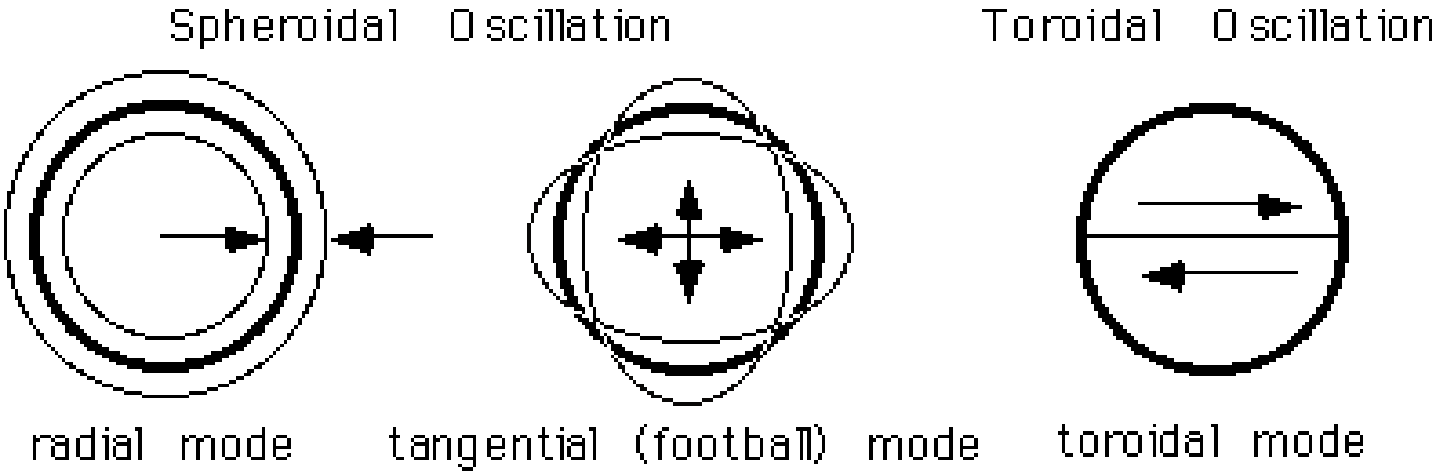
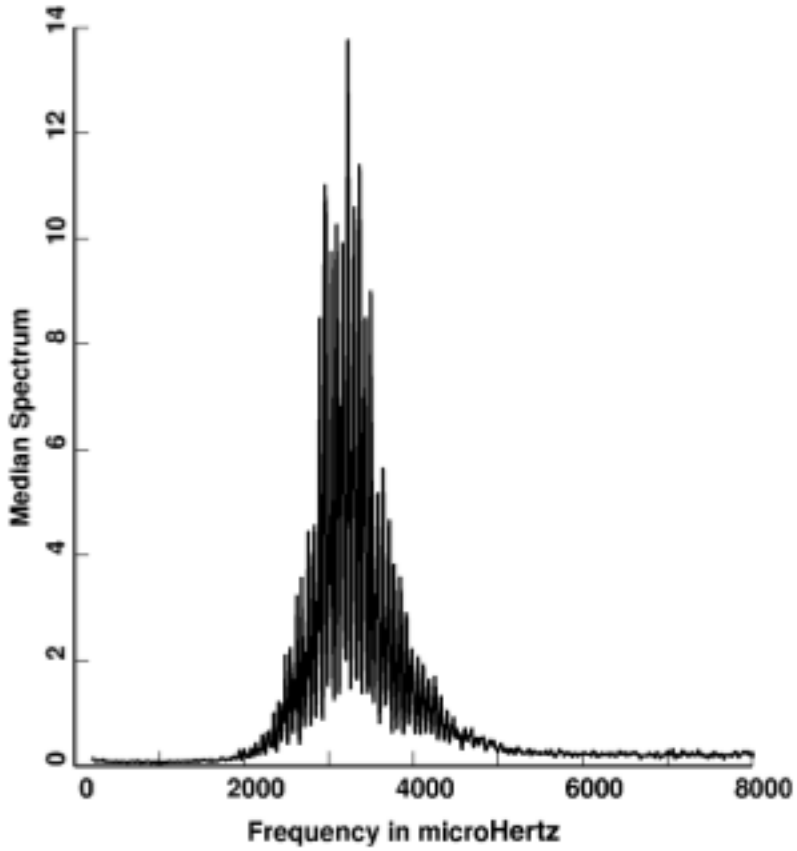


Image source: http://www.geology.illinois.edu/people/hsui/classes/geo350/lectures/earthquakes/earthquakes.html

Background: History of Study

- Earth's "hum" was first found in the 90's
- Numerous small earthquakes cannot account for this
- Some lines in the hum are not Earth modes (Suda et al. 1998)
- Suggested mechanisms
  - Atmospheric turbulence (Suda et al. 1998)
  - Ocean forcing (Watada & Masters 2001)
  - Solar forcing (Thomson et al. 2007)
- Some peaks have very high Q (~10^3) (Thomson et al. 2007)

Solar Hypothesis



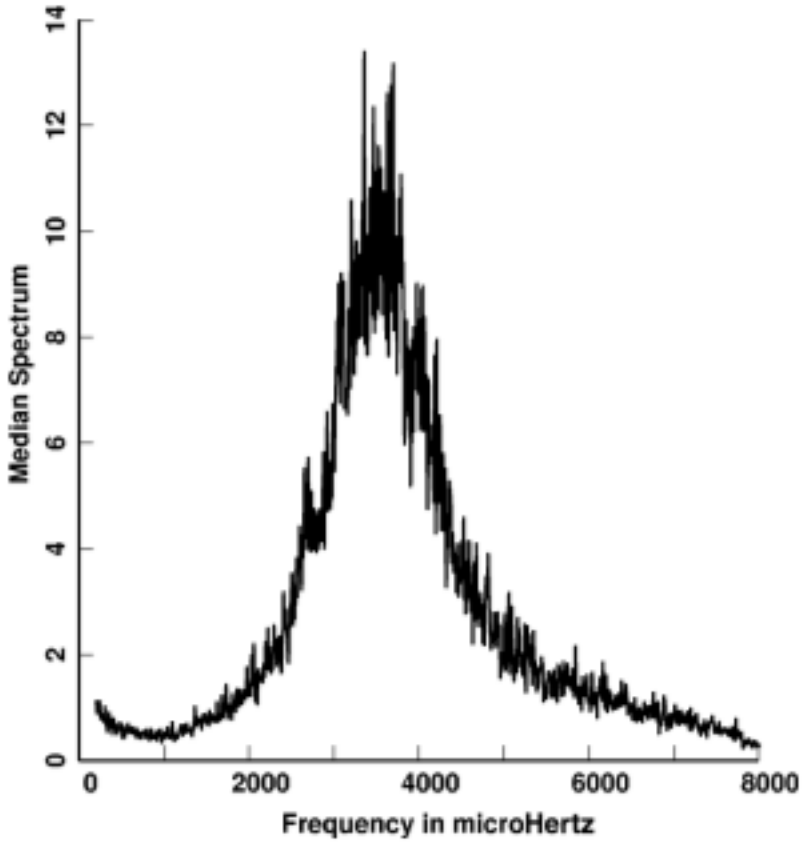
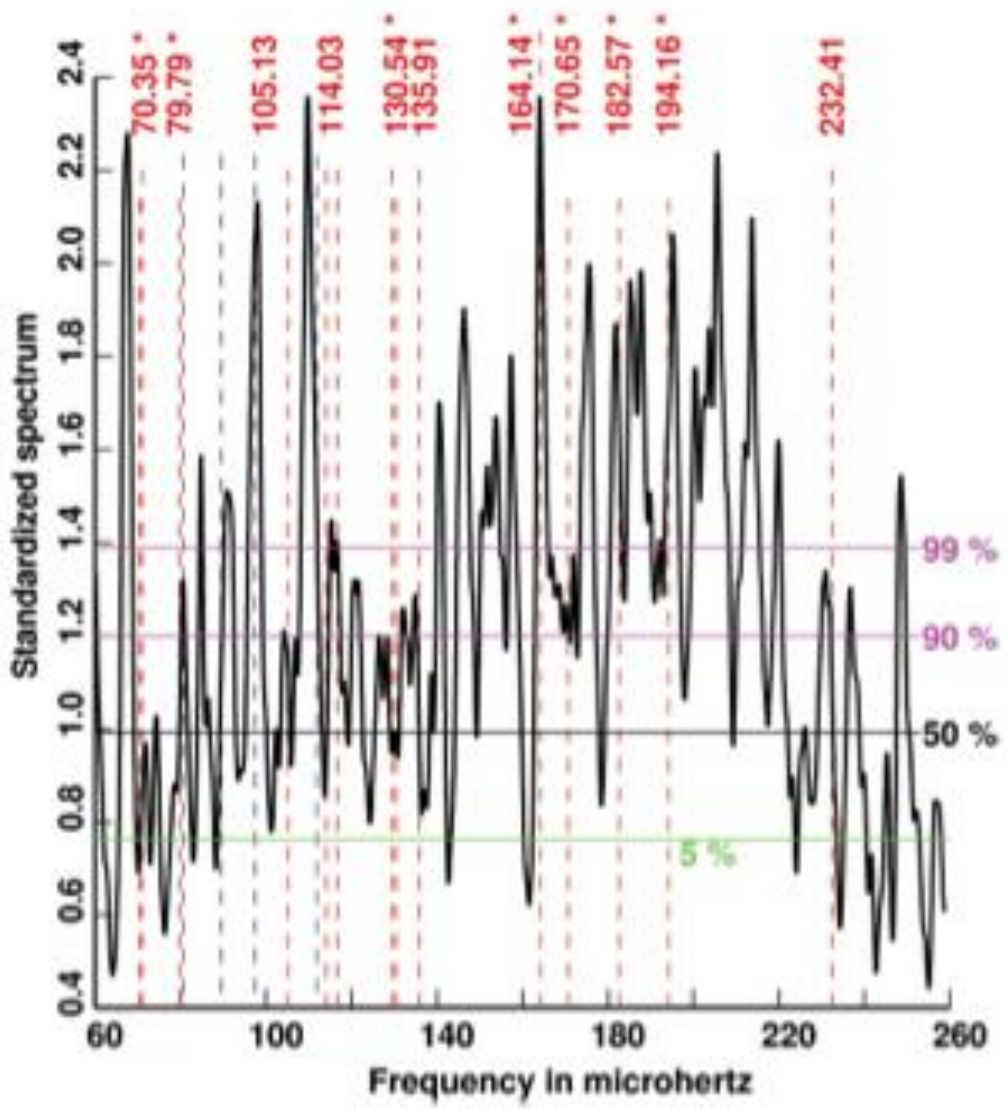


Image source: Thomson et al (2007)

Solar Hypothesis

Image source: Thomson et al. 2007



Solar Hypothesis

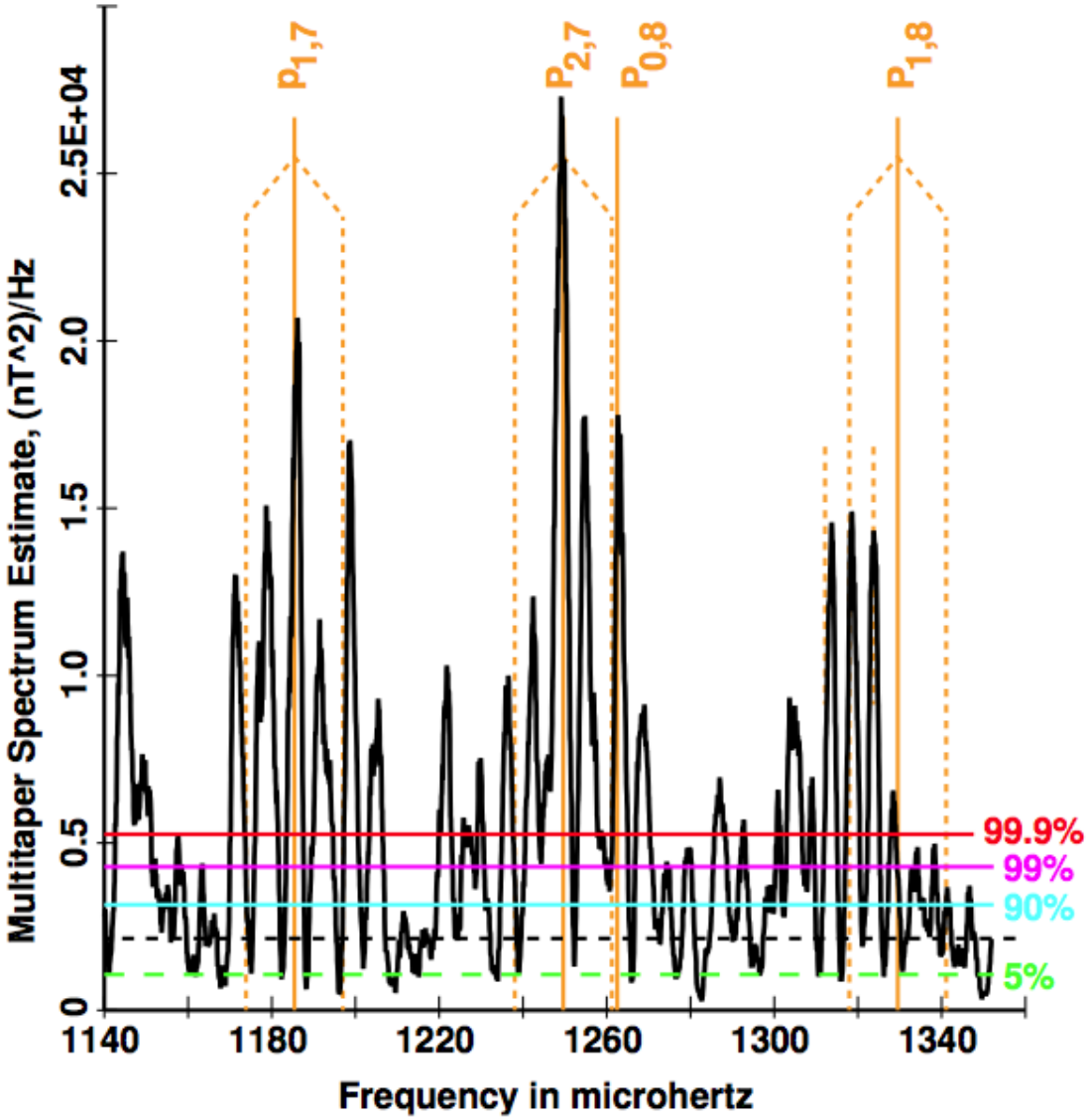
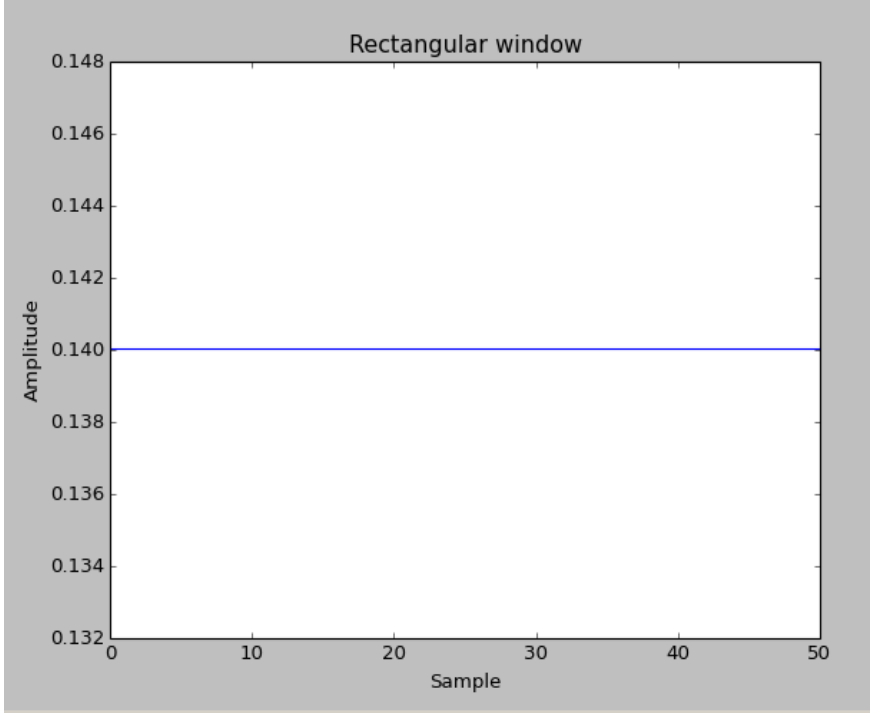


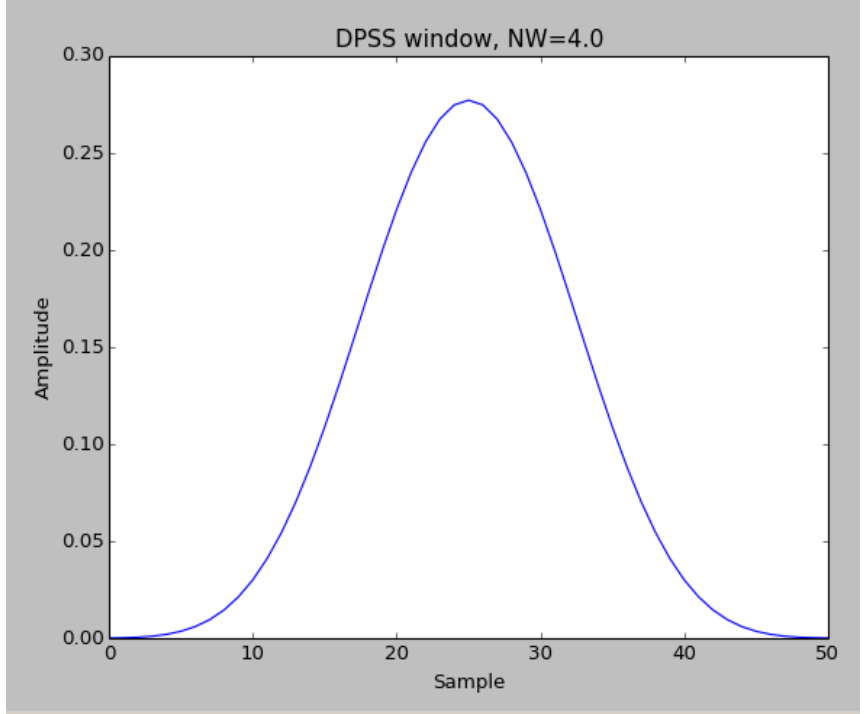
Image source: Thomson & Vernon 2015

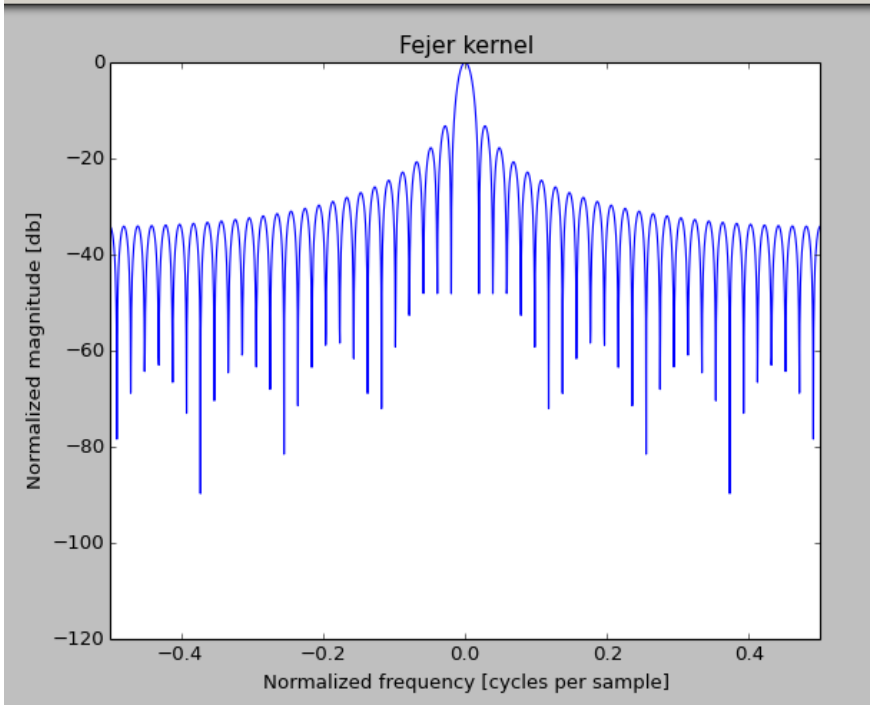
Spectrum estimation: tapering

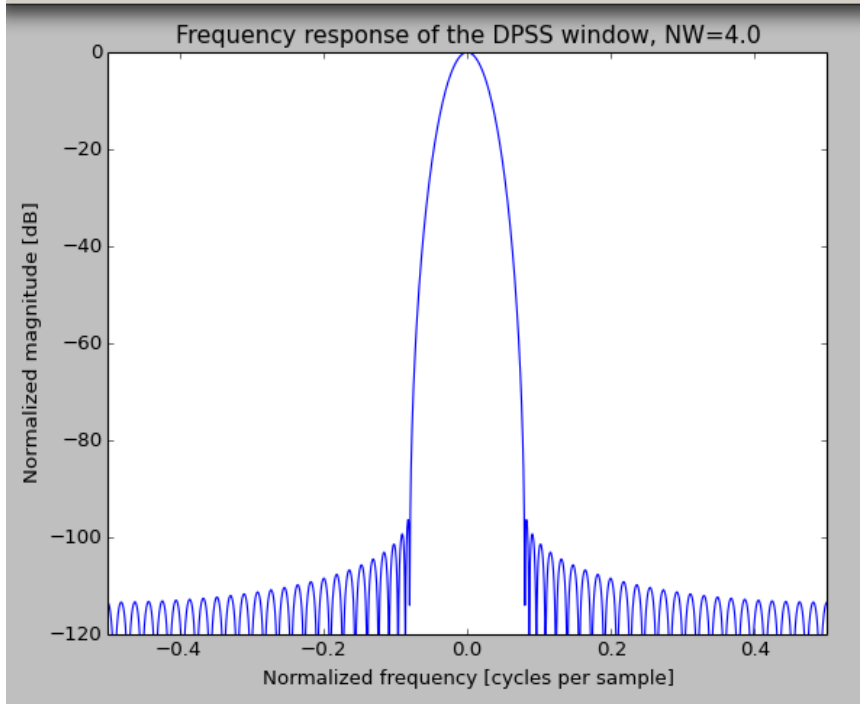
$$Y_k(f) = \sum_{n=1}^{N} w_n^k x_n e^{i 2 \pi f_n \Delta t}$$

- Y(f) = spectrum
- x = time series data
- N = number of data points
- w = taper (window) (Prieto et al. 2007)
- I'll explain the "k" indices shortly


Rectangular window
Amplitude
Sample
0.148
0.146
0.144
0.142
0.140
0.138
0.136
0.134
0.132
0
10
20
30
40
50


DPSS window, NW=4.0
Amplitude
Sample
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
50


Fejer kernel
Normalized magnitude [db]
Normalized frequency [cycles per sample]
0
−20
−40
−60
−80
−100
−120
−0.4
−0.2
0.0
0.2
0.4


Frequency response of the DPSS window, NW=4.0
Normalized magnitude [dB]
Normalized frequency [cycles per sample]
0
−20
−40
−60
−80
−100
−120
−0.4
−0.2
0.0
0.2
0.4

Tapering & leakage

- Our estimate is the convolution (moving average) of the true spectrum and the spectrum of the window (Thomson 1977)

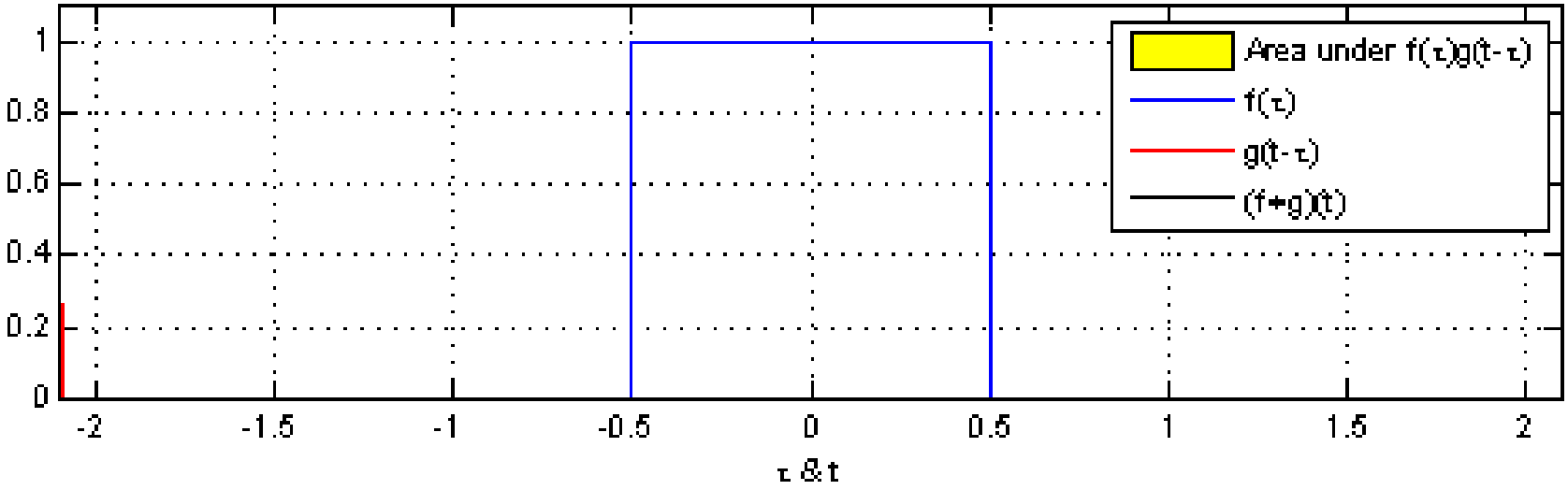


Image source: https://en.wikipedia.org/wiki/File:Convolution_of_box_signal_with_itself2.gif

Spectrum estimation: multitaper

- Several of the low-order DPSS tapers all have good leakage properties, and are orthonormal
- We average estimates based off several of these to reduce variance (Prieto et al. 2007)

Multitaper & DPSS

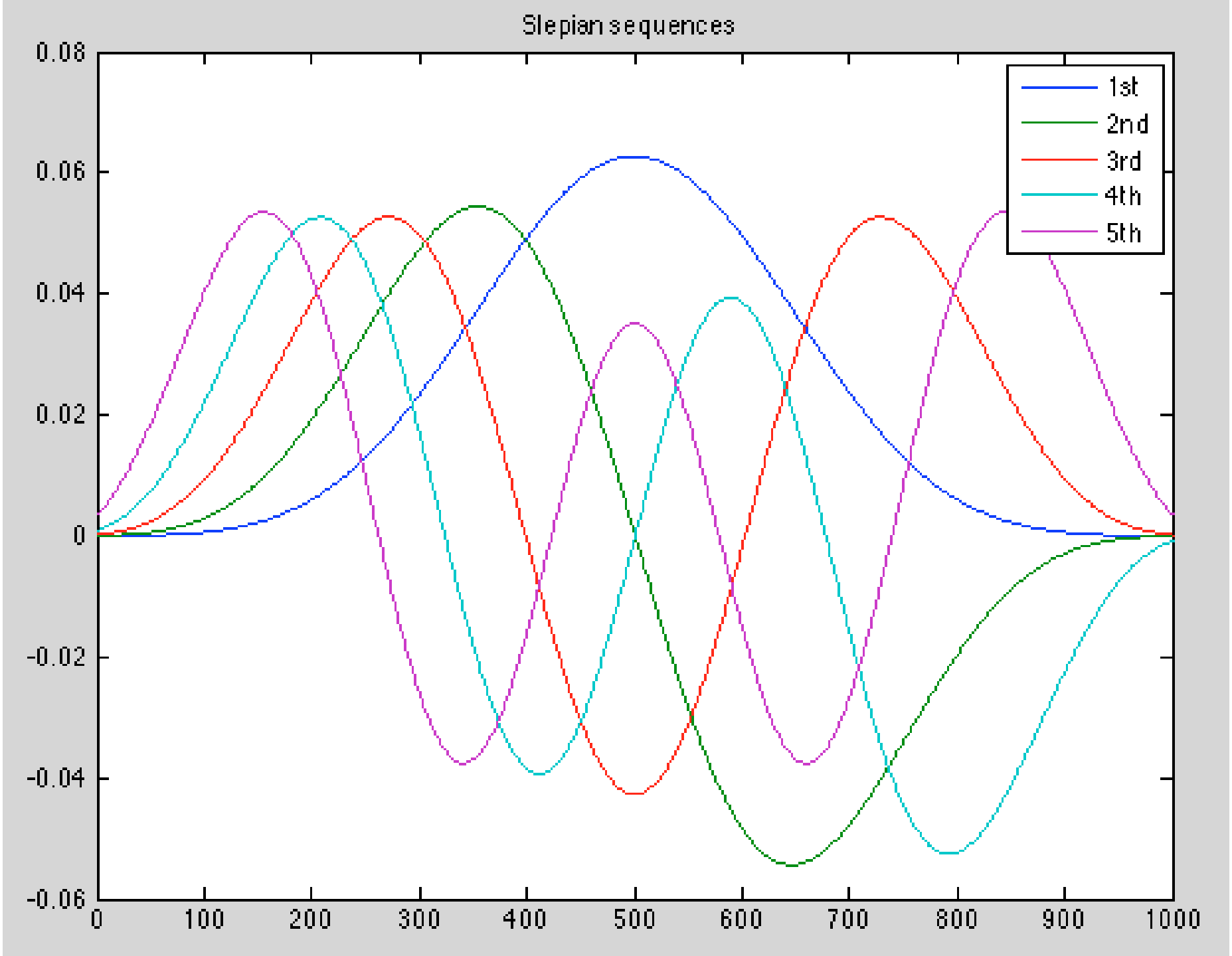
- The DPSS tapers maximize the energy in the main lobe

$$\lambda(N,W) = \frac{\int_{-W}^{W} |Y(f)|^2 df}{\int_{-\frac{1}{2}}^{\frac{1}{2}} |Y(f)|^2 \, df}$$

2007)

$$\mathbf{D} \cdot \mathbf{a} - \lambda \mathbf{a} = 0$$

$$\mathbf{D}_{t,t'} = \frac{\sin 2\pi W(t-t')}{\pi(t-t')} \quad t, t' = 0, 1, \ldots, N-1$$


Slepian sequences
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
100
200
300
400
500
600
700
800
900
1000
1st
2nd
3rd
4th
5th

Spectral estimation: multitaper

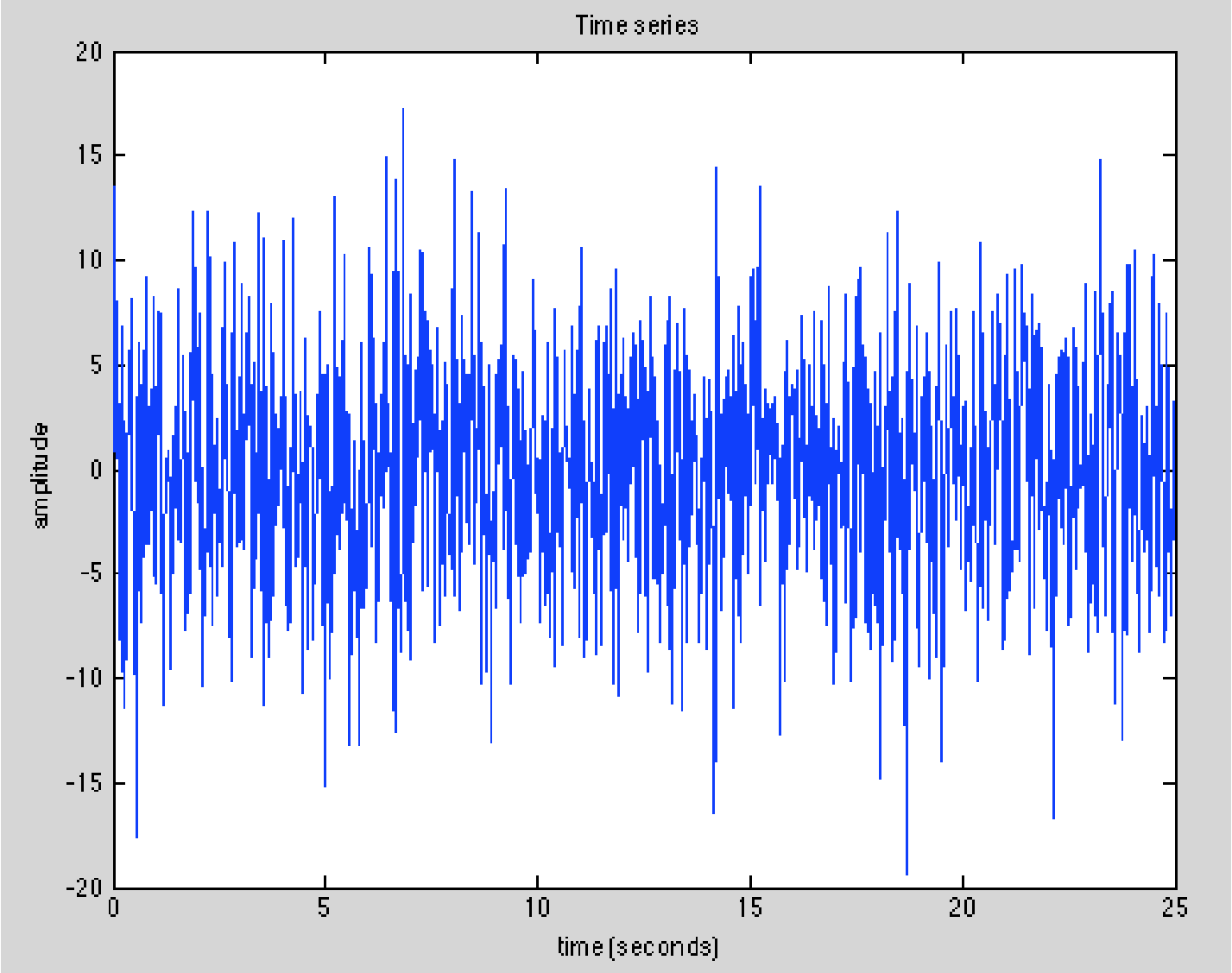
- The final multitaper estimate is given by

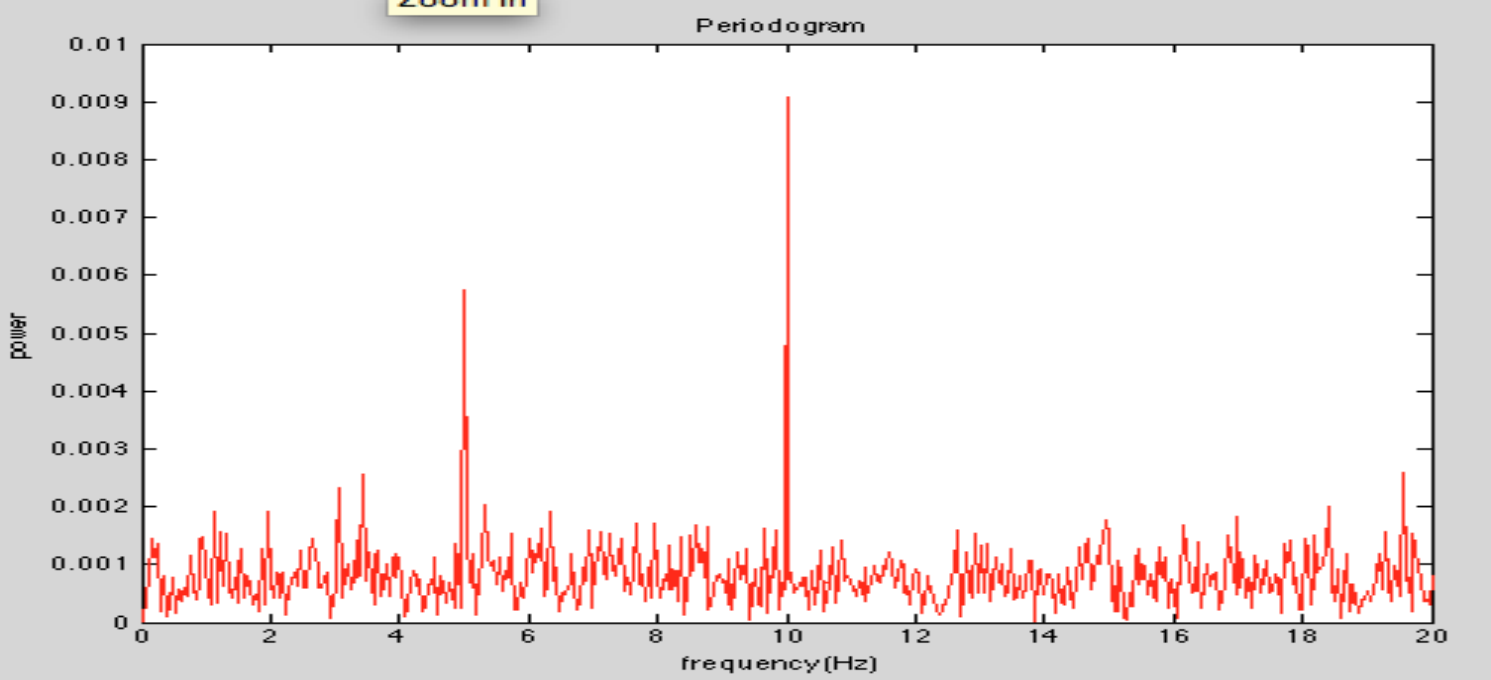
$$S_{est}(f) = \frac{\sum_{k=0}^{K-1} d_k^2 |Y_k(f)|^2}{\sum_{k=0}^{K-1} d_k^2}$$

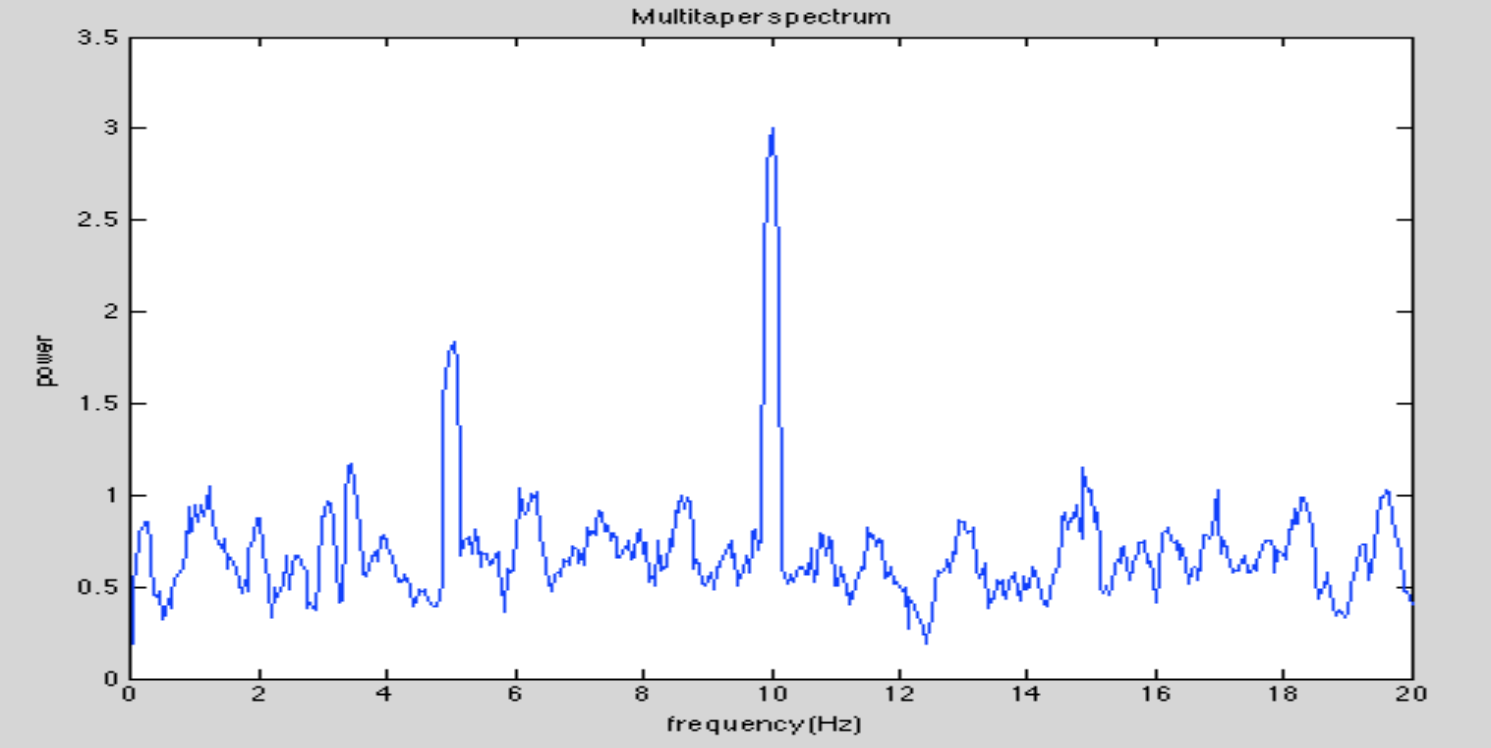
d_k are given by (Prieto et al., 2007)

$$d_k(f) = \frac{\sqrt{\lambda_k} S(f)}{\lambda_k S(f) + (1 - \lambda_k)\sigma^2}$$

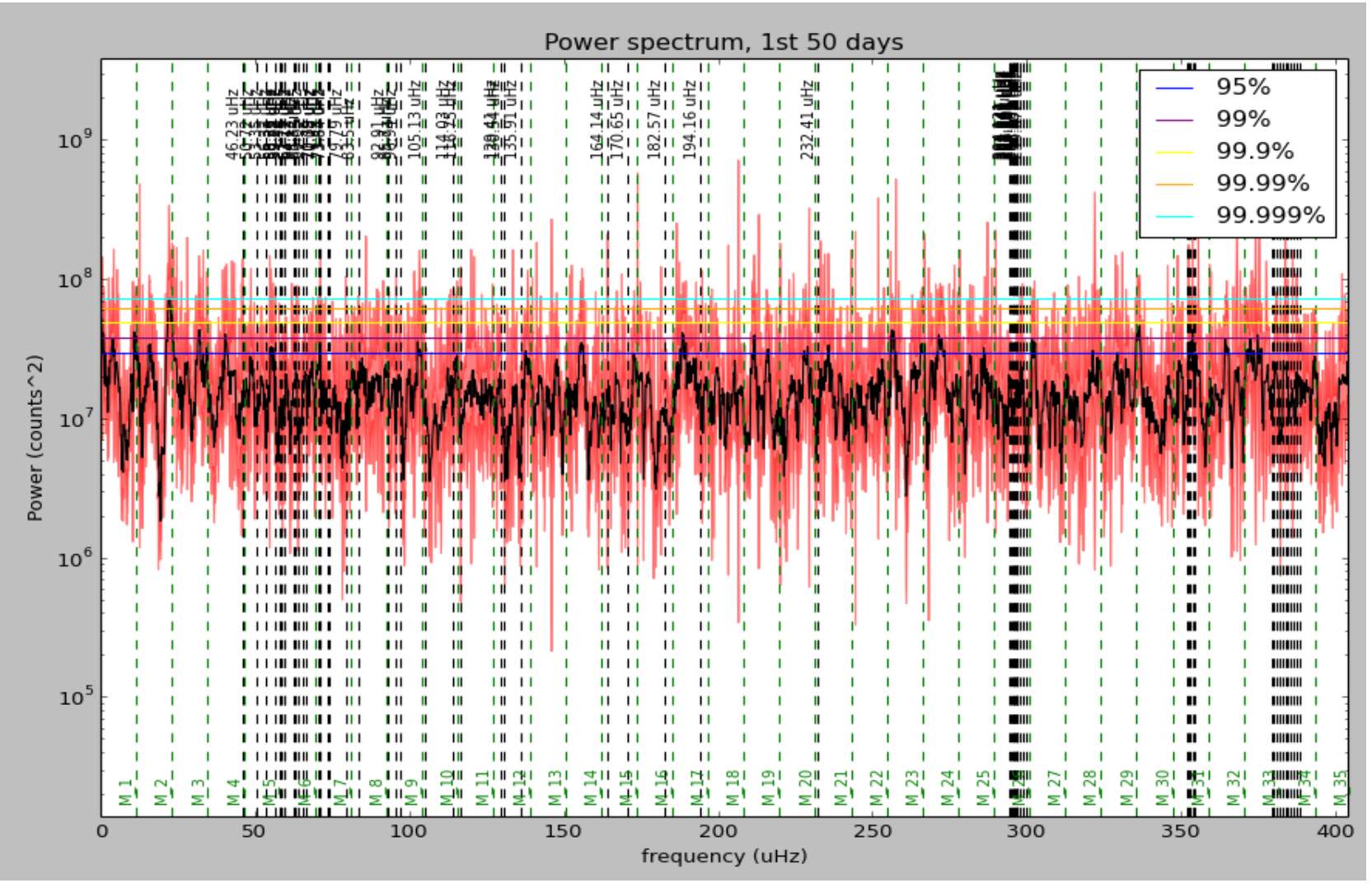
Multitaper Example




Periodogram
power
frequency(Hz)
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0
2
4
6
8
10
12
14
16
18
20

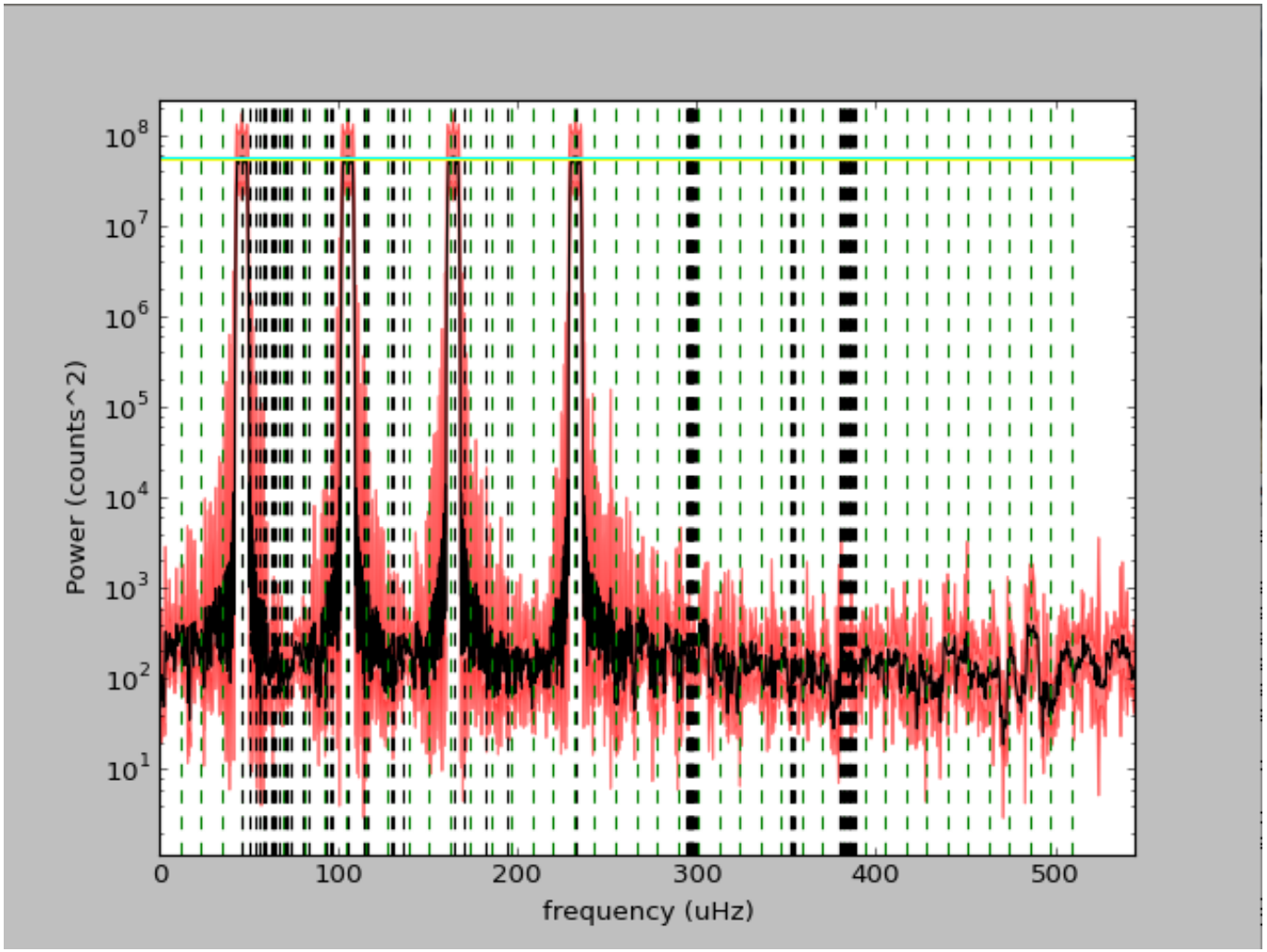

Multitaper spectrum
power
frequency(Hz)
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20

My work on this problem (or, “A cautionary tale on filters”)



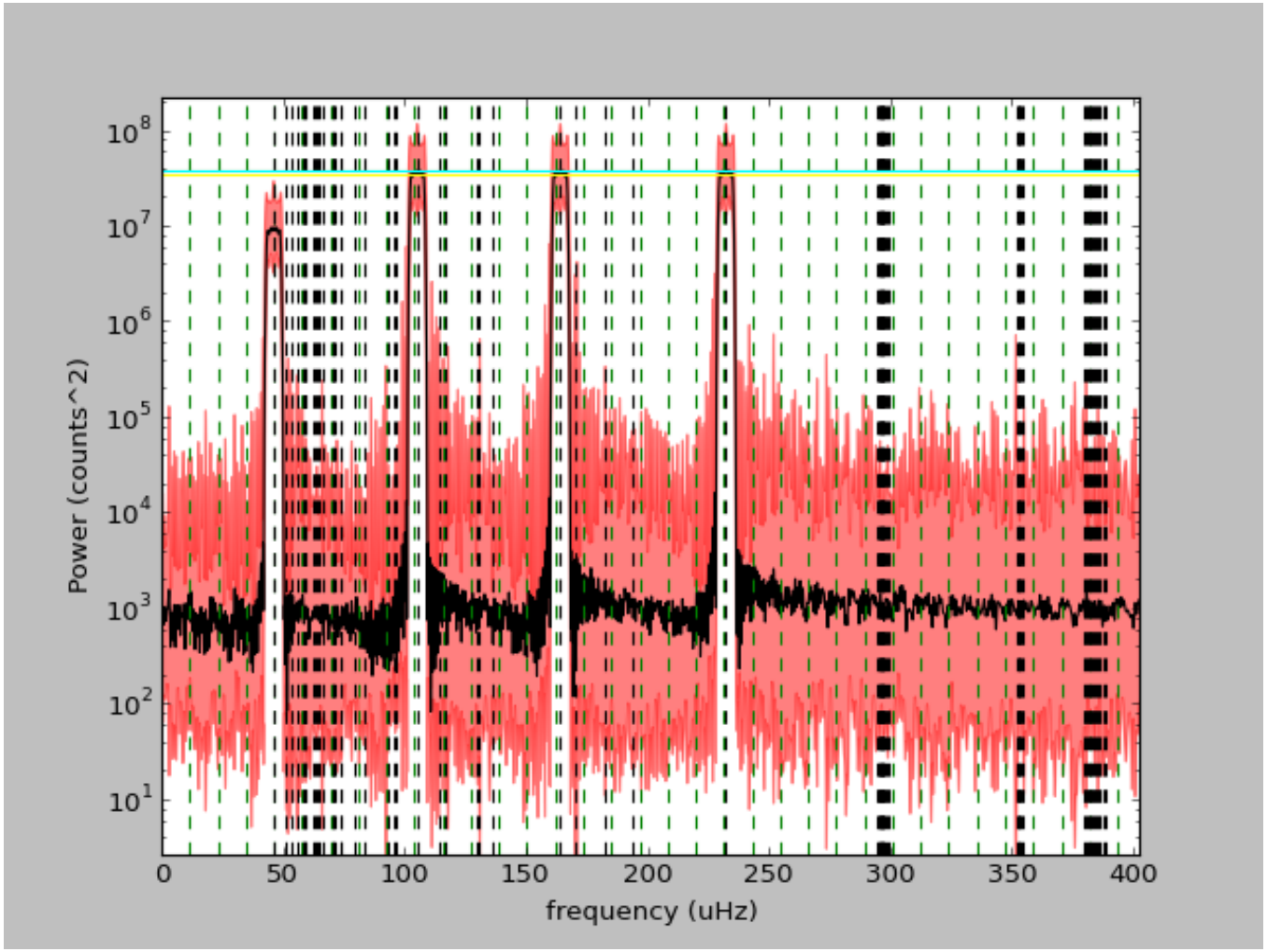
Hmm…looks like nothing…

How to destroy a periodic signal



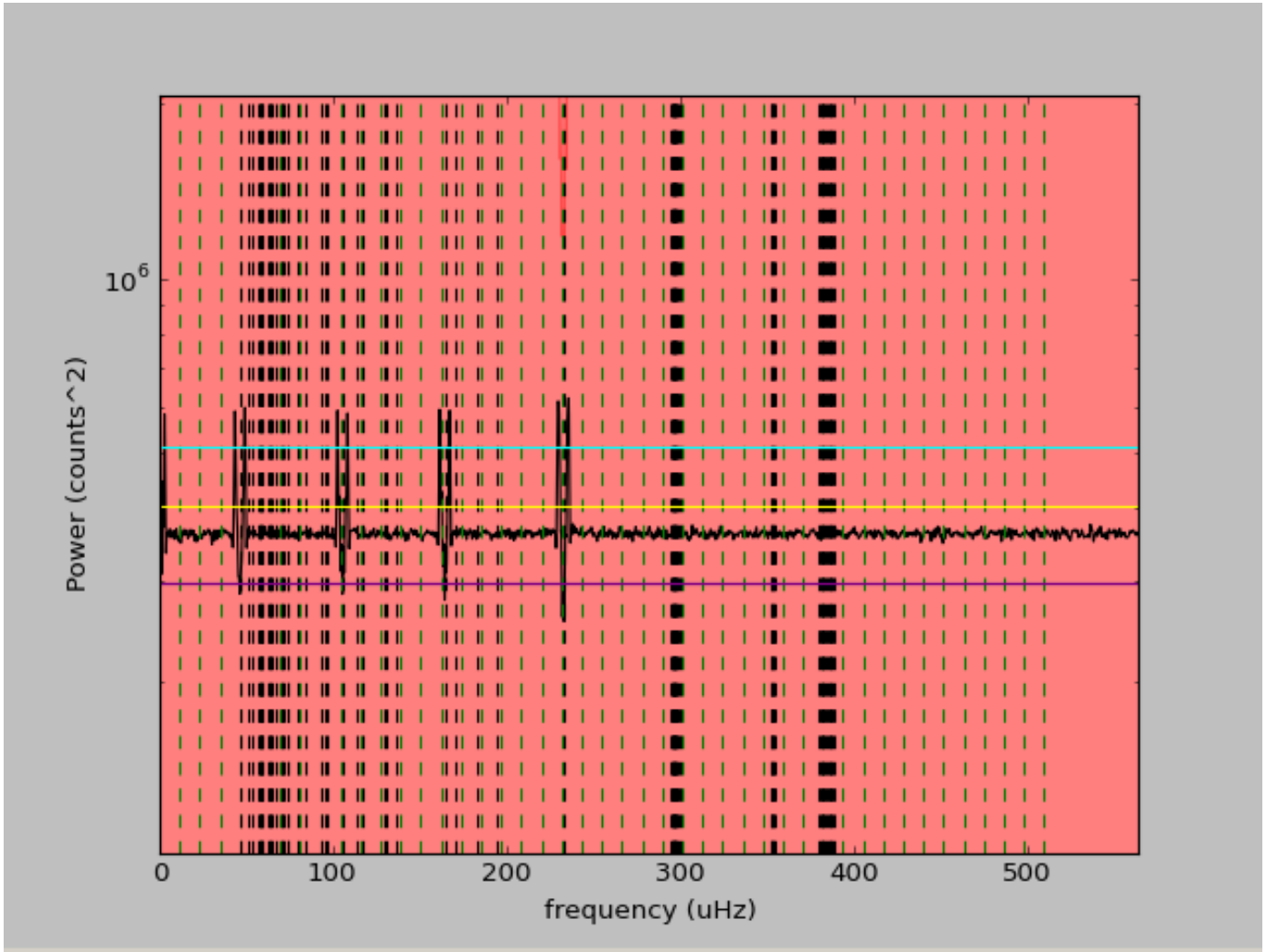
Prewhitened with AR(6)

How to destroy a periodic signal



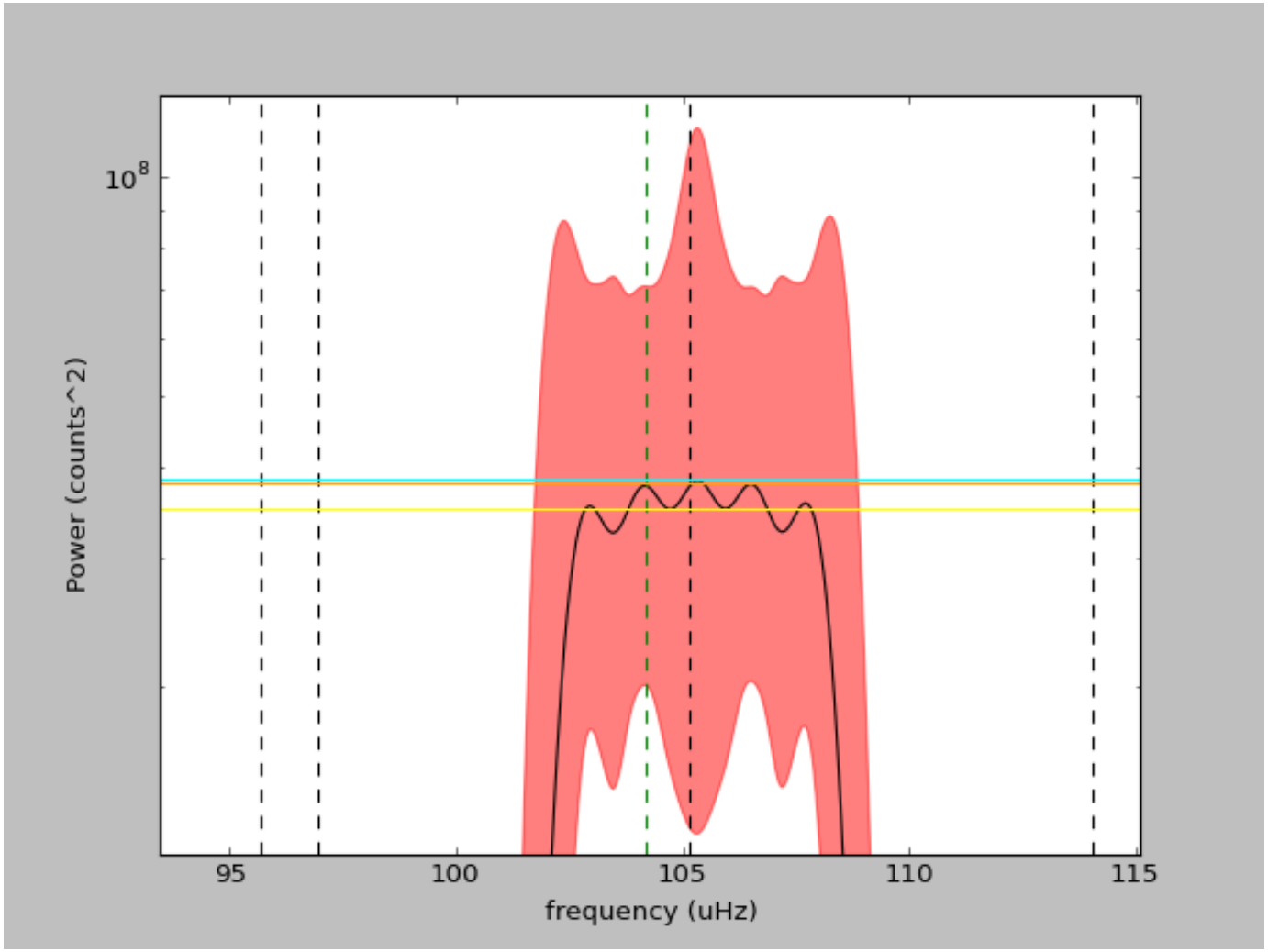
Prewhitened with AR(8)…looks funny

How to destroy a periodic signal



Prewhitened with AR(9)…oops.

How to destroy a periodic signal



The peaks were 9 samples wide, and were completely destroyed by an AR(9) filter. I will investigate a possible empirical relationship between peak width and max AR filter order.

Homestake's relationship with this work

- The solar signal is expected to be weak, thus we need relatively quiet data
- TA data may not cut it
- Homestake's broadband array has very little noise due to the absence of surface effects
- I'm still wrestling with the data, however

References

•Prieto, G. A., Parker, R. L., Thomson, D. J., Vernon, F. L., & Graham, R. L., 2007. Reducing the bias of multitaper spectrum estimates, *Geophysical Journal International*, **171**, 1269-1281.

•Stein, S., & Wysession, M., 2003. *An Introduction to Seismology, Earthquakes, and Earth Structure*, Blackwell Publishing, Malden, MA.

•Suda, N., Nawa, K., & Fukao, Y., 1998. Earth's background free oscillations, *Science*, **279**, 2089-