Homestake workshop
Oct 2015

$$\vec{T} = \bar{\bar{\sigma}}\,\vec{\nu}$$

traction — stress tensor — normal vector on a surface

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

displacement (2nd Newt.)

Isotropic:

$$\sigma_{ij} = \mu e_{ij} + \lambda \delta_{ij} e_{kk} \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

strain — strain

$$e_{11} + e_{22} + e_{33} = \frac{\Delta V}{V}$$ — volumetric strain

Acoustic $\mu = 0 \Rightarrow$ all there is is volumetric strain $\Rightarrow$ pressure waves

For $\sigma_{21} = \mu e_{21} \sim \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

$$\Rightarrow \frac{\partial^2 \sigma_{12}}{\partial x_j} \approx \mu \frac{\partial^2 u_{12}}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 \sigma_{12}}{\partial x^2} = \frac{\rho}{\mu} \frac{\partial^2 u}{\partial t^2}$$

$$\Rightarrow V = \beta = \frac{\mu}{\rho}$$

Shear wave speed

Anisotropic

$$\sigma_{ij} = C_{ijkl} e_{kl}$$

$$\left.\begin{array}{l} \sigma_{ij} = \sigma_{ji} \\ e_{ij} = e_{ji} \\ C_{ijkl} = C_{klij} \end{array}\right\} \Rightarrow$$ 27 independent constants in C_{ijkl}

$$\Rightarrow \frac{\partial}{\partial x_j} C_{ijkl} \frac{\partial u_i}{\partial x_l} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

?

Assuming elastic material not true near surface

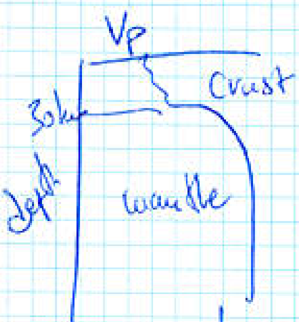
- layered anisotropic media – studied in Chris Chapman "Fundamentals of Seismic Wave Propagation"
  - case of Homestake

- anisotropic medium
  - P not purely long.
  - S not purely transverse
  - birefringent: ~~changes from place to place~~
    n different for different polarizations, so S-wave splits into 2.

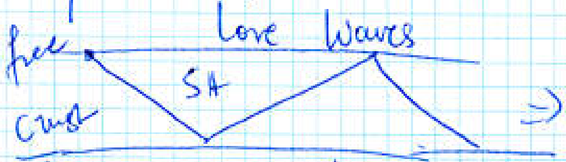
⇒ P becomes quasi-P
S ⟶ quasi S

⇒ still can speak of different wave speeds for primary and secondary, but they are not longer long and transverse ...

Rayleigh: Isotropic, elastic, half-space with free surface

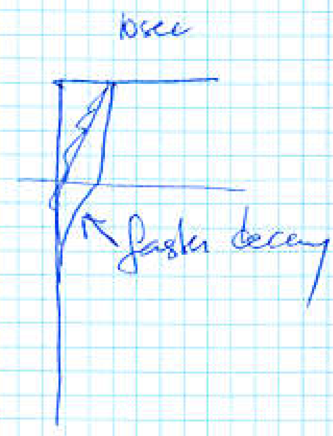


⇒ Step-like layer



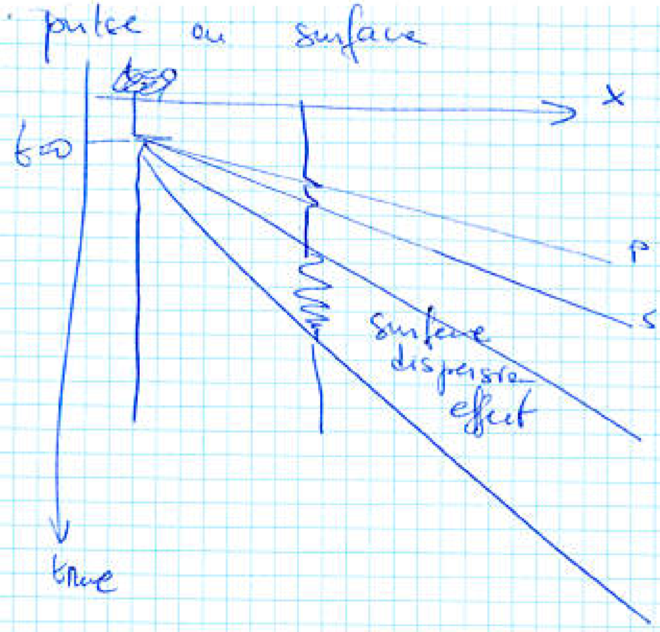
⇒ waveguide for SH

Shear wave, horizontal motion



Similar at higher frequency
- Soil: V jumps from ~300 m/s ⇒ 3 km/s
- sediment
- upper crust
- lower crust
- mantle (35 km depth)

⇒ leads to stratification and change in amplitude
- freq. dependent



decay depends on ~~...~~ wavelength and wavespeed

active exp.

S wave: 3300 - 3900 m/s

⇒ 20% variation

P-wave: 5700 m/s ~~surface~~

⇒ for sand: P-waves are ~ 1000 m/s

at 10kHz $\lambda \sim 10$ cm

⇒ cultural noise is in the surface layer of 10 m - rubble

$\frac{\sin i}{v_s} = \frac{\sin j}{v_p}$ ⇒ Different speeds for 2 different waves, but they propagate through the medium in different ways.

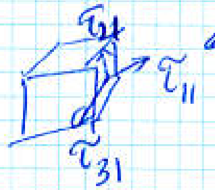
↑ frequency dependent statement

⇒ Surface effect: speed goes down ⇒ amplitude goes up.

- Love waves could be of similar amplitude as R-waves, but it is not clear. Depends of sources, fewer sources generate shear.
  → Would have another decay/speed and amplitude

Victor

$$\underbrace{\rho \ddot{u}_i}_{\text{force per unit volume}} = \underbrace{\frac{\partial \tau_{ij}}{\partial x_j}}_{\text{differential stress}} = \frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3}$$



$$\tau_{ij} = C_{ijkl} \cdot e_{kl}$$ ← strain

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Simplest case: $\tau_{ij} = \lambda(e_{11} + e_{22} + e_{33})\delta_{ij} + 2\mu\, \varepsilon_{ij}$

volumetric strain

Lamé parameters λ and μ ← position dependent

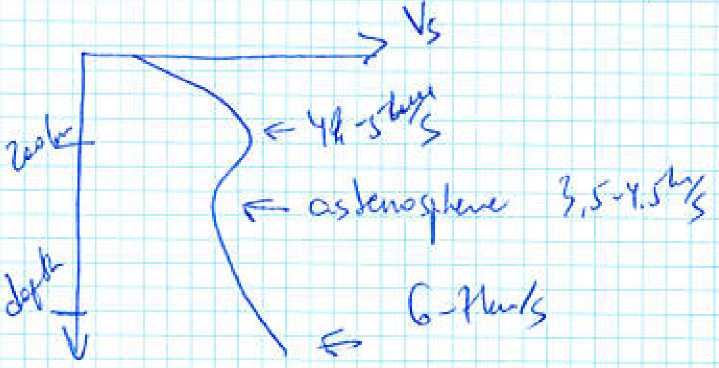
← shear modulus

Bulk modulus ~~$K = \lambda + 2\mu$~~ (not sure)

$K + \frac{4}{3}\mu = \lambda + 2\mu \Rightarrow \boxed{K = \lambda + \frac{2}{3}\mu}$?

In general, λ, μ can depend on position. But Earth can be modelled as 1D case (λ, μ depend on depth only): $\lambda(z), \mu(z)$.

$V_s = \sqrt{\mu/\rho}$, $V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$



$$\Rightarrow \rho \ddot{u}_i = \frac{\partial}{\partial x_i}\left(\lambda(e_{11} + e_{22} + e_{33})\right) + \frac{\partial}{\partial x_j}\left(2\mu e_{ij}\right)$$

Love waves: displacement in y-direction, propagate in $\hat{x}$, and depth dependent

$$u = u_1 = 0$$
$$v = u_2 = l_1(z, k, \omega) \cdot \cos(kx - \omega t)$$
$$w = u_3 = 0$$

Rayleigh: alternative assumption: displacements in x and z direction traveling a in $\hat{x}$

$$u = r_1(z, k, \omega) \cdot \sin(kx - \omega t)$$
$$v = 0$$
$$w = r_2(z, k, \omega) \cdot \cos(kx - \omega t)$$

l_1, r_1, r_2 are eigenfunctions

⇒ Love: only have diff. in $\hat{y}$, assume $\lambda(z), \mu(z), \rho(z)$

$$\rho(z) \frac{\partial^2 v}{\partial t^2} = \cancel{\frac{\partial}{\partial y}\left(\lambda(e_{11} + e_{22} + e_{33})\right)} + \frac{\partial}{\partial x}\left(\mu(z) \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu(z) \frac{\partial v}{\partial z}\right)$$

$$-\omega^2 \rho(z) l_1 \cancel{\cos} = -\mu(z) k^2 l_1 \cancel{\cos} + \frac{\partial}{\partial z}\left(\mu(z) \cdot l_1'(z)\right) \cancel{\cos}$$

⇒ ODE in l_1 for known μ, λ, ρ

If $\mu = \text{const}$, there are no solutions! So, no love-waves in half-space

→ note that this gives a relation k-ω ⇒ only some speeds work $c_L = \frac{\omega}{k}$

⇒ at 0.1Hz, $\lambda \sim$ 40km ≈ thickness of the crust, so only pressure matters and chemistry does not matter as much as at 1Hz.

⇒ depth-only dependence is fine @ 0.1Hz, but probably not at 1Hz.

Rayleigh

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right] + \cancel{\frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right]}$$

$$+ \frac{\partial}{\partial x}\left[2\mu \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$-\rho\omega^2 r_1 \cancel{\sin} = -\lambda k^2 r_1 \cancel{\sin} - \lambda k \frac{\partial r_2}{\partial z} \cancel{\sin} + \dots$$

ODE in both r_1 and r_2

But you have the second equation from

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial z}(\dots)$$

$\Rightarrow$ Combine two equations: 4^{th} order ODEs for r_1 and r_2

but again for certain k-ω dependence

$\Rightarrow$ there are plane-wave solutions, use Green's functions to connect to real effects (point forces etc)

$\rightarrow$ Low speed $\Rightarrow$ H/V large $\Rightarrow$ horizontal motion is larger

– half space: $V(f)$ but not on depth, in homogeneous case. (μ = const)

– Typically can assume $V \sim z^{-\alpha}$

$\Rightarrow$ if assuming $\mu(z) \sim z^{-\alpha}$ can solve the ODEs

$\Rightarrow$ how do we measure α?

if we know $V_S(z) \sim z^{\alpha}$ (or $\mu \sim z^{\alpha}$)

can get $l(z)$, $r_1(z)$, $r_2(z)$

single exp. — double exponentials

$\Rightarrow$ then everything else follows for radiometer.

$\lambda \sim 1m$, $v = 500 \, m/s$

$\lambda f = v \Rightarrow f \simeq \frac{v}{\lambda} \sim 500 Hz$

- Q: can you detect P + reflected P, since they are correlated?
- Deep Earthquake under Japan - no surface effects on the source, and no surface effect on receivers
  $\Rightarrow$ observe reflections, can resolve initial waveform relative to reflection

$\Rightarrow$ Cross correlation between stations A and B is equivalent to Green's function from A to B, assuming noise sources are isotropic.

$\rightarrow$ get Rayleigh wave speed as a function of frequency which gives V^{shear} vs depth.
- may also be able to measure body waves directly if we use underground stations.
- could tell presence of love waves.

$\Rightarrow$ Q: If you look at narrow-band seismic noise, can you see the retrograde elliptical motion in the data?

$\Rightarrow$ Glitch finding algorithms from LIGO might be of interest in geophys.
- should talk to Nelson, Michael etc.
- could use STAMP
- Gary started an event catalog that could serve as a starting point to identify glitches

- transient searches
  - omicron (Michael should be able to do this) < 10sec
  - Gary: saw ~1sec transients, don't know what it is
    - identify those lines
  - Michael: could use freq., duration to identify groups.

=> long time-scale not as useful
  -> could run STAMP on an example event to see how things look like.