Discussion Problem Solution

October 17, 2013

1 Up the HIll

Our target quantity is the initial speed of our car: v_{car}. The plan is to use conservation of energy for this problem. Our initial energy is given by $E_i = \frac{1}{2}mv_{car}^2$. The final energy is given by $E_f = 0$. We are stopped! The forces acting on the car are gravity, F_g, and friction, f.

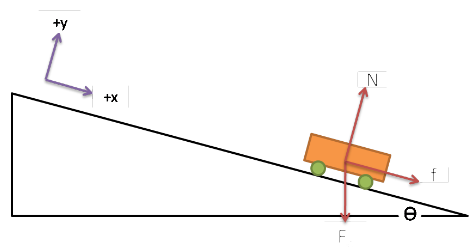

Figure 1: Free Body Diagram of the car on a hill

Looking at the free body diagram shown in Figure 1, it's clear that in the y direction we have

$$\begin{aligned} \Sigma F_y &= N - F_g \cos\theta = 0 \\ \Rightarrow N &= mg\cos\theta. \end{aligned}$$

Now, in the x direction we sum the forces as well, then make the substitution that $f = \mu_k N$:

$$\begin{aligned} \Sigma F_x = f + F_g \sin\theta &= ma_x \\ \Rightarrow mg(\mu_k \cos\theta + \sin\theta) &= ma_x. \end{aligned}$$

The important step here is that we have now identified the forces acting opposite our motion, since we identify our motion as being in the $-x$ direction. We are told that the marks on the road last for 100ft, but for now we'll call the distance over which the forces are applied d. Therefore:

$$\begin{aligned} W_{forces} &= \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} \\ &= -mg(\mu_k \cos\theta + \sin\theta) \int_{x_i}^{x_f} dx \\ &= -mg(\mu_k \cos\theta + \sin\theta)[x_f - x_i] \\ W_{forces} &= -mgd(\mu_k \cos\theta + \sin\theta). \end{aligned}$$

Note that the minus sign here tells us that the forces are in the direction opposite that of our motion. From here we use conservation of energy:

$$\begin{aligned} E_f - E_i &= W_{forces} \\ \frac{1}{2} m v_{car}^2 &= mgd(\mu_k \cos\theta + \sin\theta) \\ \Rightarrow v_{car} &= \sqrt{2gd(\mu_k \cos\theta + \sin\theta)}. \end{aligned}$$

Using that $g = 32$ ft/s^2, $d = 100$ ft, $\mu_k = 0.6$, and $\theta = 10^\circ$ we find that

$$v_{car} = 70 \frac{\text{ft}}{\text{s}}.$$

Converting to miles per hour we find that

$$v_{car} = 48 \text{ mph}.$$

I would probably choose not to fight this one.

2 Down the Hill

One could alternatively choose to do this problem as the car is going down the hill! In this case, friction acts in the opposite direction, and we have

$$v_{car} = \sqrt{gd(\mu_k \cos\theta - \sin\theta)}.$$

For this case, then we'd have that

$$v_{car} = 36.5 \frac{\text{ft}}{\text{s}} = 25\text{mph}.$$

In this case you might want to fight the ticket.