Discussion Problem #2 Solution

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To start this problem, let's first assume that there's friction. We assume that friction opposes a velocity that would go up the ramp, and for this reason we'd have friction pointing DOWN the ramp. Assuming that the angle is with respect to the horizontal, we write down the equations from Newton's second law in the horizontal and vertical directions (note that in this case these are a better coordinate system than picking $+x$ down the ramp like one might be initially inclined to do):

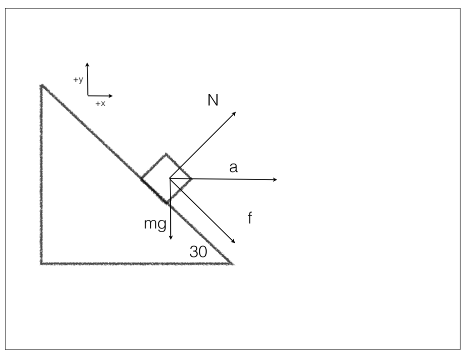

Figure 1: Free body diagram for James Bond's car

y-direction: $\Sigma F_y = N\cos\theta - f\sin\theta - mg = 0$ (no acceleration!)

x-direction: $\Sigma F_x = N\sin\theta + f\cos\theta = \dfrac{mv^2}{r}$ (centripetal acceleration).

If we assume we don't *actually* have motion in the direction anti-parallel to friction, but we are *just* on the verge of having motion, we can assume that $\mu = \mu_s$. Therefore, using that $f = \mu_s N$ we can substitute into the y direction and find

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}.$$

Substituting now into the x equation we find

$$mg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{mv^2}{r}.$$

This then gives us

$$\boxed{v = \sqrt{gr\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}.}$$

Substituting in values we find that $v = 40\text{m/s} \approx 144\text{km/hr}$.

Next, if set $\mu_s = 0$ we will have

$$\boxed{v = \sqrt{gr\tan\theta}}$$

and find that $v = 20.6\text{m/s} \approx 75\text{km/hr}$.