

# WELCOME TO THE PHYSICS LABORATORY

Physics is our human attempt to explain the workings of the world. The success of that attempt is evident in the technology of our society. You have already developed your own physical theories to understand the world around you. Some of these ideas are consistent with accepted theories of physics while others are not. This laboratory manual is designed, in part, to help you recognize where your ideas agree with those accepted by physics and where they do not. It is also designed to help you become a better physics problem solver.

You are presented with contemporary physical theories in lecture and in your textbook. In the laboratory you can apply the theories to real-world problems by comparing your application of those theories with reality. You will clarify your ideas by: answering questions and solving problems *before* you come to the lab room, performing experiments and having discussions with classmates *in the lab room*, and occasionally by writing lab reports *after you leave*. Each laboratory has a set of problems that ask you to make decisions about the real world. As you work through the problems in this laboratory manual, remember: **the goal is not to make lots of measurements**. The goal is for you to examine your ideas about the real world.

The three components of the course - lecture, discussion section, and laboratory section - serve different purposes. The laboratory is where physics ideas, often expressed in mathematics, meet the real world. Because different lab sections meet on different days of the week, you may deal with concepts in the lab before meeting them in lecture. In that case, the lab will serve as an introduction to the lecture. In other cases the lecture will be a good introduction to the lab.

**The amount you learn in lab will depend on the time you spend in preparation before coming to lab.**

Before coming to lab each week you must read the appropriate sections of your text, read the assigned problems to develop a fairly clear idea of what will be happening, and complete the prediction and warm-up questions for the assigned problems.

Often, your lab group will be asked to present its predictions and data to other groups so that everyone can participate in understanding how specific measurements illustrate general concepts of physics. You should always be prepared to explain your ideas or actions to others in the class. To show your instructor that you have made the appropriate connections between your measurements and the basic physical concepts, you will be asked to write a laboratory report. Guidelines for preparing lab reports can be found in the lab manual appendices and in this introduction. An example of a good lab report is also in the appendix. Please do not hesitate to discuss any difficulties with your fellow students or the lab instructor.

Relax. Explore. Make mistakes. Ask lots of questions, and have fun.

## WHAT TO DO TO BE SUCCESSFUL IN THIS LAB:



Safety comes first in any laboratory. If in doubt about any procedure, or if it seems unsafe to you, STOP. Ask your lab instructor for help. **YOU MUST READ ALL WARNINGS FOUND IN THE LAB MATERIALS.**

### A. What to bring to each laboratory session:

1. Bring an 8" by 10" graph-ruled lab journal, to all lab sessions. Your journal is your "extended memory" and should contain everything you do in the lab and all of your thoughts as you are going along. Your lab journal is a legal document; you should **never** tear pages from it. Your lab journal **must** be bound (as *University of Minnesota 2077-S*) and must **not** allow pages to be easily removed (as spiral bound notebooks).
2. Bring a "scientific" calculator.
3. Bring this lab manual.

### B. Prepare for each laboratory session:

Each laboratory consists of a series of related problems that can be solved using the same basic concepts and principles. Sometimes all lab groups will work on the same problem, other times groups will work on different problems and share results.

1. Before beginning a new lab, carefully read the Introduction, Equipment, Warm Up and Prediction sections. Read sections of the text specified.
2. Each lab contains several different experimental problems. Before you come to a lab, complete the assigned *Prediction* and *Warm-up*. The Warm-up helps you build a prediction for the given problem, so it is usually helpful to complete the Warm-up before making the prediction. **These individual predictions will be collected 1-2 days before the lab and checked (graded) by your lab instructor. They will be returned to you immediately at the beginning of each lab session.**

This preparation is crucial if you are going to get anything out of your laboratory work. There are at least two other reasons for preparing:

- a) There is nothing duller or more exasperating than plugging mindlessly into a procedure you do not understand.
- b) The laboratory work is a **group** activity where every individual contributes to the thinking process and activities of the group. Other members of your group will be unhappy if they must consistently carry the burden of someone who isn't doing his/her share.

### C. Laboratory Reports

At the end of every lab (about once every two weeks) you will be assigned to write up one of the experimental problems. Your report must present a clear and accurate account of what you and your group members did, the results you obtained, and what the results mean. A report must not be copied or fabricated. (That would be scientific fraud.) Copied or fabricated lab reports will be treated in the same manner as cheating on a test, and will result in **a failing grade for the course and possible expulsion from the University**. Your lab report should describe your predictions, your experiences, your observations, your measurements, and your conclusions. A description of the lab report format is discussed at the end of this introduction. **Each lab report is due within two days of the end of that lab.**

### D. Attendance

Attendance is required at all labs **without exception**. If something disastrous keeps you from your scheduled lab, contact your lab instructor **immediately**. The instructor will arrange for you to attend another lab section that same week. **There are no make-up labs in this course.**

### E. Grades

Satisfactory completion of the lab is required as part of your course grade. **Those not completing all lab assignments by the end of the quarter at a 60% level or better will receive a quarter grade of F for the entire course.** The laboratory grade makes up 15% of your final course grade. Once again, we emphasize that each lab report is due within two days of the end of that lab.

There are two parts of your grade for each laboratory: (a) your laboratory journal, and (b) your formal problem report. Your laboratory journal will be graded by the lab instructor during the laboratory sessions. Your problem report will be graded and returned to you in your next lab session.

If you have made a good-faith attempt but your lab report is unacceptable, your instructor may allow you to rewrite parts or all of the report. A rewrite must be handed in again within two days of the return of the report to you by the instructor.

**F. The laboratory class forms a local scientific community. There are certain basic rules for conducting business in this laboratory.**

1. *In all discussions and group work, full respect for all people is required. All disagreements about work must stand or fall on reasoned arguments about physics principles, the data, or acceptable procedures, never on the basis of power, loudness, or intimidation.*
2. *It is OK to make a reasoned mistake. It is in fact, one of the most efficient ways to learn. This is an academic laboratory in which to learn things, to test your ideas and predictions by collecting data, and to determine which conclusions from the data are acceptable and reasonable to other people and which are not.*

What do we mean by a "reasoned mistake"? We mean that after careful consideration and after a substantial amount of thinking has gone into your ideas you simply give your best prediction or explanation as you see it. Of course, there is always the possibility that your idea does not accord with the accepted ideas. Then someone says, "No, that's not the way I see it and here's why." Eventually persuasive evidence will be offered for one viewpoint or the other.

"Speaking out" your explanations, in writing or vocally, is one of the best ways to learn.

3. *It is perfectly okay to share information and ideas with colleagues. Many kinds of help are okay. Since members of this class have highly diverse backgrounds, you are encouraged to help each other and learn from each other.*

**However, it is never okay to copy the work of others.**

Helping others is encouraged because it is one of the best ways for you to learn, but copying is inappropriate and unacceptable. Write out your own calculations and answer questions in your own words. It is okay to make a reasoned mistake; it is wrong to copy.

No credit will be given for copied work. It is also subject to University rules about plagiarism and cheating, and may result in dismissal from the course and the University. See the University course catalog for further information.

4. *Hundreds of other students use this laboratory each week. Another class probably follows directly after you are done. Respect for the environment and the equipment in the lab is an important part of making this experience a pleasant one.*

The lab tables and floors should be clean of any paper or garbage. Clean up your area before you leave the lab. Return equipment to the lab instructor or leave it neatly at your station, as appropriate.

If any lab equipment is missing or broken, submit a problem report by sending an email to **labhelp@physics.umn.edu**. Be sure to include a complete description of the problem. You can also file a report containing comments about this lab manual (for example, when you discover errors or inconsistencies in statements).

In summary, the key to making any community work is **RESPECT**.

*Respect* yourself and your ideas by behaving in a professional manner at all times.

*Respect* your colleagues (fellow students) and their ideas.

*Respect* your lab instructor and his/her effort to provide you with an environment in which you can learn.

*Respect* the laboratory equipment so that others coming after you in the laboratory will have an appropriate environment in which to learn.

# 1101 LAB 1: DESCRIPTION OF MOTION IN ONE DIMENSION

In this laboratory you will measure and analyze one-dimensional motion; that is, motion along a straight line. With digital videos, you will measure the positions of moving objects at regular time intervals. You will investigate relationships among quantities useful for describing the motion of objects. Determining these kinematic quantities (position, time, velocity, and acceleration) under different conditions allows you to improve your intuition about their quantitative relationships. In particular, you should identify which relationships are only valid in some situations and which apply to all situations.

There are many possibilities for one-dimensional motion of an object. It might move at a constant speed, speed up, slow down, or exhibit some combination of these. When making measurements, you must quickly understand your data to decide if the results make sense. If they don't make sense to you, then you have not set up the situation properly to explore the physics you desire, you are making measurements incorrectly, or your ideas about the behavior of objects in the physical world are incorrect. In any of the above cases, it is a waste of time to continue making measurements. You must stop, determine what is wrong and fix it.

If your ideas are wrong, this is your chance to correct them by discussing the inconsistencies with your partners, rereading your text, or talking with your instructor. Remember, one of the reasons for doing physics in a laboratory setting is to help you confront and overcome your incorrect ideas about physics, measurements, calculations, and technical communications. Pinpointing and working on your own difficulties will help you in other parts of this physics course, and perhaps in other courses. Because people are faster at recognizing patterns in pictures than in numbers, the computer will graph your data **as you go along**.

## OBJECTIVES:

After you successfully complete this laboratory, you should be able to:

- Describe completely the motion of any object moving in one dimension using position, time, velocity, and acceleration.
- Distinguish between average quantities and instantaneous quantities for the motion of an object.
- Write the mathematical relationships among position, time, velocity, average velocity, acceleration, and average acceleration for different situations.
- Graphically analyze the motion of an object.
- Begin using technical communication skills such as keeping a laboratory journal and writing a laboratory report.

# 1101 LAB 1: DESCRIPTION OF MOTION IN ONE DIMENSION

## PREPARATION:

Read the following: Knight, Jones & Field: Chapter 2.

The **Video and Analysis** appendix. You will be using the cameras and software throughout the semester, so please take the time now to become familiar using them.

The appendices **Significant Figures**, **Review of Graphs** and **Accuracy, Precision and Uncertainty** to help you take data effectively.

Before coming to the lab you should be able to:

- Define and recognize the differences among these concepts:
  - Position, displacement, and distance.
  - Instantaneous velocity and average velocity.
  - Instantaneous acceleration and average acceleration.
- Find the slope and intercept of a straight-line graph.
- Determine the slope of a curve at any point on that curve.
- Use the definitions of  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  for a right triangle.

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# 1101 Lab 1 Problem 1: INTRODUCTION TO MEASUREMENT AND UNCERTAINTY

Welcome to 1101 Physics Laboratory! This lab exercise is meant to introduce you to measurement procedures, uncertainties in measurement, and the computer software that you will be using throughout the course. It will be worth your time to read through this entire lab and the next one as there are many helpful tips and references that you may want to use in later labs.

## PART I—Measurement and Uncertainty

You perform quality control for BuggyMagic, the leading manufacturer of toy constant velocity buggies. Recently the company started manufacturing buggies at a second factory. Coincidentally, the customer service department started fielding complaints at about the same time the new factory came on line. The customer service department couldn't elaborate many specifics, only that many customers felt the buggies offered from the second factory were "inconsistent" and had "less magic" than the original buggies. The customer service manager at your company has decided that you need to check out the buggies ASAP.

You fly to the new factory with all sorts of sophisticated measurement and analysis equipment and a brand new buggy from the original factory, but sadly, when you arrive, all your luggage is lost by the airline. All you have is a smartphone (stopwatch) and a small flag affixed to a wooden dowel that is handed to you as you get off the airplane by a local welcoming committee. You'll need to do your best assessing the buggies with what you have.

### EQUIPMENT

You have a stopwatch, wood dowel and a toy buggy.



### WARM UP

*These are your first lab "Warm-ups", to be done before the lab meets, written in your lab notebook, and turned into your TA as specified by the course syllabus. You may want to refer back to the appendices during the lab.*

- 1) Read the appendix **Significant Figures**. Do the exercises at the end and write the results in your lab notebook under a section called "Warm-ups".
- 2) Read the appendix **Accuracy, Precision and Uncertainty** and write the answers to the exercises in your lab notebook.
- 3) Read the appendix **Review of Graphs**.
- 4) Work out a plan as specified in the Prediction.

You should also start reading the appendix **Video and Analysis**. You will be using the software and equipment described at the very end of this lab more extensively later on.

### PREDICTION

With the limited tools on hand, what sorts of measurements can you make? Devise a plan to measure the dimensions and velocity of the buggies that you can compare to verify the 'sameness' of the buggies and later translate to metric units.

### EXPLORATION

*Each lab will have an "Exploration" section before the "Measurement" section. This is where you can run informal trials to develop your procedure and see how the equipment responds to the activity. The data from these exploratory trials do not need to be included in your final data set.*

Can you determine the length of the dowel without using a meterstick or ruler? If you had to say, is it really a guess? Can you determine if something is .5 dowel, or .75 dowel? How accurate can you be with it? If you measure the buggy with the dowel, what qualities of your dowel and buggy can help you estimate the uncertainty in your measurement?

How accurate is your stopwatch? If you measure the time it takes for the buggy to travel 1 dowel length, how accurate can you be?

### MEASUREMENT

#### 1) Length

Use the dowel to measure the dimensions of the buggy. How long is it? How tall is it? Have each person in the group measure the buggy using the dowel in successive turn, then do it again. This should give 6-8 measurements per group per dimension. Individually record measurements and then combine them after everyone is done. Record your procedure and associated measurements in your lab notebook.

What is your estimated uncertainty in your measurements?

Using the instructions in the appendix, calculate the mean and average deviation of the combined data set for the length and height of the buggy. Compare your estimated uncertainty to your average deviation. Do they agree within significant figures?

**Note on Assumptions:** *When physicists are trying to solve a problem, they often make assumptions about the situation. Depending on how accurate the results need to be (i.e. how small the uncertainty), making estimates saves a lot of time if it turns out to be 'good enough' for the task. You will see phrases such as 'friction is negligible', 'ignoring air resistance', or 'assuming that earth is a sphere' in your textbook or in class. The assumptions made must always be stated since it gives the audience important information about the precision of the results.*

#### 2) Time

Have each member of your group use a stopwatch simultaneously to measure the time it takes

the buggy to travel 1 dowels length. Do the times agree? Did you start the buggy, then set it down? Or, did you start the buggy already sitting on the table? Does it matter?

Record your procedure and associated measurements in your lab notebook. Calculate the mean and average deviation of the times to travel one dowel length.

**Note on rejecting data:** *One must be very careful about rejecting data. In general, you should keep all of your data even if it does not seem to match with what you are expecting. For this class, the only reason you might 'throw away' data is if you can say EXACTLY what was wrong with it. For example, if you just did a run with the cart and someone forgot to say "Go!" at the right time, then you know that time measurement is wrong. You may not, however, ignore the data points that just seem too big or too small. Hopefully you see by now that ALL MEASUREMENTS HAVE UNCERTAINTY. This is nothing to apologize for as it is expected for any measurement.*

### 3) Constant Velocity

Now, have each member measure and record the time it takes the buggy to travel two, then three, and finally four dowel lengths.

Did the measurements become more or less consistent as each person did more trials? Did you "formalize" the procedure after the first couple trials (e.g. agree upon the start procedure, decide what viewing angle to measure from)? Could you make the average deviation smaller with this equipment or are you close to the limit of the accuracy that can be expected? You should redo any trials as needed.

Take at least 4 time measurements **for each** of your 4 distances. You will want to format the data in a 4x4 table. Find the average time and the average deviation of times for each distance.

Which point on the buggy are you using for your measurement? This kind question might seem trivial, but it is an example of the amount to detail you should be recording in your notebook.

ANALYSIS
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Use an entire page of your lab notebook to make a graph with time along the vertical axis and (the more accurate) distance in dowels along the horizontal axis. (This does not make your graph look like those in the **Review of Graphs** appendix; usually we put time along the horizontal axis.) Plot your average time for each distance with the 'error bars' on the graph. The error bars are the range of the average deviation of the measurement.

**Example:** If your time is  $3.40 \pm .4$  seconds, then you should put a dot at 3.4, a vertical line through the dot that extends from 3.0 to 3.8, and 'cross' the line at the top and bottom.

Now draw your best fit line through the four data points, as directed in the **Review of Graphs** appendix. You are now able to find the average velocity from the best fit line.

To get the uncertainty of the measured velocity, make the steepest straight line that fits inside



the error bars. The slope of this line corresponds to the lowest velocity (remember we are graphing time vs. distance). Now draw a line that has the least possible slope that fits inside the error bars. This corresponds to the greatest possible velocity.

Use these values to quote your average velocity in dowels/s plus or minus the uncertainty.

You could graph the same information except with time on the horizontal axis and distance on the vertical axis. If your distance measurements are accurate but your time measurements are not, the “error bars” will lie in the horizontal direction. This is OK! If your time measurements were accurate but your distance measurements were not, then the error bars *would* lie in the vertical direction.

<b>CONCLUSION</b>
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Which of the three measurements (length, time, or velocity) gives the most uncertainty of measurement? Would you consider this uncertainty significant, moderate or insignificant? Why?

If you compare your data with a different lab group’s buggy data, would you consider the buggies to be manufactured to be the same? Why, or why not?

Using a meterstick, can you convert your earlier measurements? How accurately can you measure the dowel? Does additional error get added (propagated) into your earlier measurements if you convert the units? If so, is it substantial?

Give a value in m/s with uncertainty for the velocity of the buggy.

## PART II—THE COMPUTERS AND VIDEO CAMERAS

Back in your office at BuggyMagic headquarters, you realize that you probably could have used your smartphone and taken a few pictures and a simple video of the buggies motion and achieved similar results. You decide to compare the analysis of the buggies motion using a video to your earlier hand measurements. You would like to see if video is an effective analysis technique, one that might be used as an automated quality control to ensure that you always produce quality buggies.

### EQUIPMENT

You have a video camera and MotionLab analysis software, a wood dowel of fixed length and a toy buggy.

### IN CLASS - WARM UP

The goal of this Warm Up exercise is to gain familiarity with the video cameras and explore the uncertainty of their measurement, which could possibly show up as distortion in the image. The primary way to accidentally introduce distortion into a measurement is through perspective. If you are interested in a measurement three feet away from the camera, and you calibrate it using an object ten feet away from the camera, your results will be different than expected by an unknown factor.

Consider the relative size of the objects in the photo. If your brain didn't tell you otherwise, you would either assume that the buildings in downtown were several inches tall or that the pop can was several hundred feet tall. This illustrates the need to calibrate (or scale) your camera with items that are the same distance from the camera as the motion of the object being recorded.



Similarly, if you are interested in the motion of a cart, it is important that it moves roughly the same distance in front of the camera the whole time. In this exercise, you will explore the visible effects of perspective on meter sticks and then practice calibration.

### IN CLASS - PREDICTION

How will the results of analyzing the motion of the buggy using video compare to your earlier measurements?

### EXPLORATION

Position a meter stick in front of the video camera. Experiment with holding it in different orientations, at different heights relative to the camera, and at different distances. In what position would it best function as a smaller or larger "meter stick" for your monitor? How much distortion is visible in that position? Is the camera focused? Try focusing the lens by turning the housing around the lens.

Place the meter stick and a toy car on the table. Align them so that the minimal amount of distortion is visible.

You ALWAYS need to have a calibration object in your video at roughly the same distance from the camera as the plane of motion. Any object that has a known length will work for this. When you analyze your video, you need to select the ends of this known object using your mouse and state its length. This tells the software how big everything in the plane of motion is.

Make sure everyone in your group gets the chance to operate the camera or the computer.

### MEASUREMENT

Practice taking videos of the toy car moving across the table. When you are satisfied with your video, save it in the Lab Data folder on the desktop, use a unique name you will remember. Open MotionLab to analyze your movie.

Each member of your group is expected to analyze the same video, so pay attention as you go and work as a team! You'll need to compare results, so **each group member should try and start data collection on the same video frame**. When you are calibrating, setting the origin and taking data, try to be as consistent as possible.

Although the directions to analyze a video are given in the instructions box in the upper left corner within MotionLab, the following is a short summary that will be useful to do the exploration for this and any other lab video.

1. Once MotionLab is started you will be prompted to open a movie file.
2. With the video loaded, a calibration screen automatically opens. **Advance the video with the "Fwd >" button in the Video Controls to the frame where the first data point will be taken. This step is very important because it sets up the origin of your time axis ( $t=0$ ).**

3. To tell the analysis program the real size of the video images, select the calibration object in the plane of motion that you can measure. Drag the red cursor, located in the center of the video display, to one end of the calibration object. Make sure to use the same part of the cursor for each point selected, either the central circle or the tip of one of the cross-hairs will work the same if used consistently. Click the "Accept >" button when the red cursor is in place. Move the red cursor to the other end and select "Accept >". Enter the length of the object in the "Length" box and specify the "Units" then select "Accept >". You do not need to rotate the reference frame for this lab. Select the "Quit Calibration" button to complete the calibration sequence.
4. Enter your prediction equations for how you expect the position to behave. This is the same procedure that you used for the PracticeFit exercise, but now you will enter your prediction based on the data you took by hand earlier. For the x-position graph, use the function that matches the kinematic equation relating position, velocity and time (\*Remember! z is time!). Fill in the function with your previous measurement values. Make sure the units all agree! Once your x-position prediction is ready, select "Accept >" and repeat the procedure for the y-position. (Do you expect the cart to move in the y-direction?)
5. Once you have made predictions for the x- and y-position, a data acquisition screen will automatically open. Select a specific point on the cart. Drag the red cursor over this point and click the "Add Point" button and you will see the data on the appropriate graph on your computer screen. The video will automatically advance one frame. Again, drag the red cursor over the same point selected on the object and accept the data point. Experiment with advancing the video several frames and taking a data point. Should that change your results? Decide how many data points are necessary for reliable results.
6. Once you have added enough points, click the "Quit Data Acq" button and fit your data. Sometimes you will not see your data because the scale of the graph is not in the right place. If you click the buttons in the center of the screen called "Autorange x", "Autorange y", etc. the graph will automatically scale to the data points. This may not include the prediction equation in the newly scaled window. You may need to further re-scale the axes by highlighting the highest or lowest value on the graph and typing in values to expand the ranges. Decide which equation and constants are the best approximations for your data and accept your "x-fit" and "y-fit".
7. The program will ask you to enter your prediction for velocity in the x- and y-directions. Choose the function that matches the kinematic equation relating velocity and time. Fill in your prediction values (NOT the best fit value from the position graph). Accept your  $v_x$  and  $v_y$  predictions, and you will see the data on the last two graphs.
8. Fit your data for these velocities in the same way that you did for position. Accept your fit and click the "Print Results" button to view a PDF document of your graphs that can be e-mailed to you and your group members. You must save the file on the computer in order to send it.

*Remember, each member of your lab group is required to analyze the same video! If you make a mistake in this first try, don't worry! Make sure you have an idea about how to correct it for next time!*

### ANALYSIS

When you have finished making a fit equation for each graph, rewrite the equations in a table but now matching the *dummy letters* with the appropriate *kinematic quantities*. If you have constant values, assign them the correct units.

How did each of your lab groups fit equations for the position and velocity of the buggy compare? Who had the most consistent velocity data?

Compare the average velocity of the buggy from your earlier hand measurements and the one found with the computer analysis. Do the measurements fall within the expected uncertainty? Find the mean and average deviation of the velocities determined from each members turn analyzing the video. How do the uncertainties between methods compare?

Can you see the effects of the camera distortion in your data? Which data points have the lowest uncertainty associated with them? What other measurement uncertainty is introduced by using the computer analysis software?

Why do you have fewer data points for the velocity vs. time graph than the position vs. time graph?

### CONCLUSION

Compare the buggy's velocity measured with video analysis to the measurement using a stopwatch. Do your graphs match what you expected for constant velocity motion?

Do measurements near the edges of the video give the same velocity as that as found in the center of the image within the uncertainties of your measurement? Does this affect what will you do for future measurements?

Why is there one less data point in a *velocity vs. time* graph than in the corresponding *position vs. time* graph?

## 1101 Lab 1 Problem 2: MOTION DOWN AN INCLINE

*This lab problem is formatted in the same way as the remainder of the manual. Since this physics laboratory design may be new to you, it contains the **Motion Down an Incline** problem and also additional explanations of the various parts of the lab problem. The explanation of each part is preceded by the double vertical lines seen to the left.*

These laboratory instructions may be unlike any you have seen before. You will *not* find any worksheets or step-by-step instructions. Instead, each laboratory consists of a set of problems that you solve before coming to the laboratory by making an organized set of decisions (a problem solving strategy) based on your initial knowledge. The **prediction and warm-up** are designed to help you examine your thoughts about physics. These labs are your opportunity to compare your ideas about what "should" happen with what really happens. The labs will have little value in helping you learn physics unless you take time to predict what will happen before you do something.

While in the laboratory, try to answer all the questions in this lab manual. In particular, answering each of the **exploration** questions can save you time and frustration later by helping you understand the behavior and limitations of your equipment before you make measurements. Make sure to complete the laboratory problem, including all **analysis** and **conclusions**, before moving on to the next one.

The first paragraphs of each lab problem describe a real-world situation. Before coming to lab, you will solve a physics problem to predict something about that situation. The measurements and analysis you perform in lab will allow you to test your prediction against the behavior of the real world.

You have a summer job working with a team investigating accidents for the state safety board. To decide on the cause of a particular accident, your team needs to determine the acceleration of a car rolling down a hill without any brakes. Everyone agrees that the car's velocity increases as it rolls down the hill. Your team's supervisor believes that the car's acceleration also increases as it rolls down the hill. Do you agree? To resolve the issue, you decide to measure the acceleration of a cart moving down an inclined track in the laboratory.

Read: Knight, Jones & Field Chapter 2, Sections 2.4 to 2.6.

### EQUIPMENT

This section contains a brief description of the apparatus you can use to test your prediction. Working through the exploration section will familiarize you with the details. **If any lab equipment is missing or broken, submit a problem report by sending an email to [labhelp@physics.umn.edu](mailto:labhelp@physics.umn.edu).** Be sure to include a complete description of the problem and the **room #**. You can also file a report containing comments about this lab manual (for example, when you discover errors or inconsistencies in statements). If you are unable to, please ask your TA to submit a problem report.

You have a stopwatch, meterstick, endstop, wood block, video camera and a computer with video analysis software. You will also have a cart to roll down an inclined track.



Read the **Video and Analysis** appendix. You will be using this software throughout the semester, so please take the time now to become familiar using them.

Read the appendices **Significant Figures**, **Review of Graphs** and **Accuracy, Precision and Uncertainty** to help you take data effectively.

### WARM UP

The Warm-up section is intended to help you solve the problem stated in the opening paragraphs. The statements may help you make the prediction, help you plan how to analyze data, or help you think through the consequences of a prediction that is an educated guess. **Warm-up questions should be answered, written in your lab journal, and also turned into your TA at least 24-hours before you come to lab. Follow the class procedure for how and when you should turn in Warm-up questions.** In this case, the Warm-up helps you plan what data to take and how to analyze it.

When you are predicting behavior that you are unsure of, it is often helpful to think of the ‘extreme’ cases. What would happen if the slope were very steep? What would happen if the mass were very large or small? What would happen if the force were eight times larger? This strategy helps you use your physical intuition to predict something OR to check if your answer makes sense.

1. Sketch *instantaneous acceleration vs. time graphs* for a cart moving (1) with a constant acceleration, (2) with increasing acceleration, and (3) with decreasing acceleration. For easy comparison, draw these graphs next to each other. Write down the equation that best represents each of these graphs. (\*Note: the textbook will only address constant acceleration. You can use the kinematic relationships for constant acceleration and make an educated guess about either increasing or decreasing acceleration.) If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Which graph do you think best represents a cart rolling down an incline?
2. Write down a relationship between the acceleration and the velocity of the cart. Use this to sketch a rough graph of *instantaneous velocity vs. time* for each of the three accelerations you drew in question one. Write down an equation that best represents each of these graphs\*. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Which graph do you think best represents the velocity of a cart rolling down an incline?
3. Write down a relationship between the velocity and the position of the cart. Use this to construct *position vs. time graphs* from the instantaneous velocity graphs for the 3 situations

above. For these graphs, only write down the equation for the constant acceleration case. Can you determine the constants from the graph?

PREDICTION
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Everyone has "personal theories" about the way the world works. One purpose of this lab is to help you clarify your conceptions of the physical world by testing the predictions of your personal theory against what really happens. For this reason, you will always predict what will happen *before* collecting and analyzing the data. The "Prediction" section merely helps you identify the goal of the lab problem.

Spend the first few minutes at the beginning of the lab session comparing your prediction with those of your partners. Discuss the reasons for differences in opinion. *It is not necessary that your predictions are correct, but it is absolutely crucial that you understand the basis of your prediction.*

Sketch the *instantaneous acceleration vs. time graph* for a cart released from rest near the top of an inclined track. Do you think the cart's instantaneous acceleration **increases, decreases, or stays the same** (is constant) as it moves down the track? Explain your reasoning.

Sometimes, your prediction is an "educated guess" based on your knowledge of the physical world. In these problems, the exact calculation is too complicated and is beyond this course. However, for every problem it's possible to come up with a qualitative prediction by making some plausible simplifications. For other problems, you will be asked to use your knowledge of the concepts and principles of physics to calculate a mathematical relationship between quantities in the experimental problem.

EXPLORATION
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*This section is extremely important*—many instructions will not make sense, or you may be led astray, if you fail to carefully explore your experimental plan.

In this section you practice with the apparatus and carefully observe the behavior of your physical system before you make precise measurements. You will also explore the range over which your apparatus is reliable. Remember to always treat the apparatus with **care and respect**. Students in the next lab sections will use the equipment after you are finished with it. If you are unsure about how equipment works, ask your lab instructor.

Most equipment has a range over which its operation is simple and straightforward. This is called its range of reliability. Outside that range, complicated corrections are needed. Be sure your planned measurements fall within the range of reliability. You can quickly determine the range of reliability by making **qualitative** observations at the extremes of your measurement plan. Record these observations in your lab journal. If the apparatus does not function properly for the ranges you plan to measure, you should modify your plan to avoid the frustration of useless measurements.

At the end of the exploration you should have a plan for doing the measurements that you need. **Record your measurement plan in your journal.**



**This exploration section is much longer than most. You will record and analyze digital videos several times during the semester.**

You will use a wood block and an aluminum track to create an incline. What is the best way to change the angle of the incline in a reproducible way? How are you going to measure this angle with respect to the table? *Hint: Think about trigonometry!*

Start with a small angle and with the cart at rest near the top of the track. Observe the cart as it moves down the inclined track. Try a range of angles. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!** If the angle is too large, you may not get enough video frames, and thus enough position and time measurements to measure the acceleration accurately. If the angle is too small, the acceleration may be too small to measure accurately with the precision of your measuring instruments. Select the best angle for this measurement.

When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

Where is the best place to release the cart so it does not damage the equipment but has enough of its motion captured on video? Take a few practice videos, make sure you have captured the motion you want.

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking. Write down your measurement plan.

<b>MEASUREMENT</b>
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Now that you have predicted the result of your measurement and have explored how your apparatus behaves, you are ready to make careful measurements. To avoid wasting time and effort, make the minimal measurements necessary to convince yourself and others that you have solved the laboratory problem.

Use a meter stick and a stopwatch to determine the average acceleration of the cart. Under what condition will this average acceleration be equal to the instantaneous acceleration of the cart?

Make a video of the cart moving down the inclined track. *Don't forget to measure and record the angle of the track (with estimated uncertainty). You may use it for later labs.*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case? Try the measurement with and without a rotated coordinate system.

All the MotionLab prediction equations need to be based on theoretical values. You always need to give the exact reason for the prediction equation. When you analyze your data, you will compare the fit equations to the predicted values, then say whether or not the experimental data falls within the uncertainty of the predicted data.

Do not use the fit data from the position graphs to “predict” the velocity graph.

Why is it important to click on the same point on the car’s image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and total time to determine the maximum and minimum value for each axis before taking data.

*Make sure everyone in your group gets the chance to operate the camera and the computer.*

*Note: Be sure to record your measurements with the appropriate number of significant figures and with your estimated uncertainty. Otherwise, the data are nearly meaningless. If necessary, refer to the appendix.*

### ANALYSIS

Data alone is of very limited use. Most interesting quantities are those *derived* from the data, not direct measurements themselves. Your predictions may be *qualitatively* (behaviorally) correct but *quantitatively* (numerically) very wrong. To see this you must process your data.

Always complete your data processing (analysis) before you take your next set of data. If something is going wrong, you shouldn't waste time taking a lot of useless data. After analyzing the first collection of data, you may need to modify your measurement plan and re-perform the measurements. If you do, be sure to **record how you changed your plan in your journal**.

In MotionLab, choose a fit function to represent the *position vs. time graphs* in the x and y directions. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Choose a fit function to represent the *velocity vs. time graphs* in the x and y directions. How can you calculate the values of the constants of this function from the function representing the position vs. time graph? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent?

Why do you have fewer data points for the velocity vs. time graphs compared to the position vs. time graphs? Use the data tables generated by the computer to explain how the computer generates the graphs.

Look at your graphs in MotionLab and rewrite all of the fit equations in a table, but now matching the *dummy letters* with the appropriate kinematic quantities. If you have constant values, assign them the correct units and explain their meaning.

From the velocity vs. time graphs determine if the acceleration is constant, increasing, or decreasing as the cart goes down the ramp. Use the fit equation representing the velocity vs. time graph to calculate the acceleration of the cart as a function of time. Make a graph of the acceleration vs. time. Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

Calculate the average acceleration of the cart from your stopwatch and meter stick measurements. Compare the accelerations for the cart you found with your video analysis to your acceleration measurement using a stopwatch.

<b>CONCLUSION</b>
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After you have analyzed your data, you are ready to answer the experimental problem. State your result in the most general terms supported by your analysis. **This should all be recorded in your journal in one place before moving on to the next problem assigned by your lab instructor. Make sure you compare your result to your prediction.**

How does a cart accelerate as it moves down an inclined track? In what direction is the acceleration? State your result in the most general terms supported by your analysis. Did your measurements agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Was your team supervisor right about how a cart accelerates down a hill? If yes, state your result in the most general terms supported by your analysis. If no, describe how you would convince your supervisor.

Address the following questions. In MotionLab, how do you think the computer generates data for a velocity graph? How is this related to the effect of measurement uncertainty on velocity (compared to position) graphs? Why is there one less data point in a velocity vs. time graph than in the corresponding position vs. time graph?

Looking at these graphs, will reasonable uncertainty affect your ability to test the supervisor's statement?

## 1101 Lab 1 Problem 3: LABORATORY EXTENSION- MOTION DOWN AN INCLINE WITH AN INITIAL VELOCITY

*This exercise should be completed with your group members after you have finished **Motion Down an Incline**. You do not need to take any new data, but you will need to refer to the data that you collected from the previous lab activity.*

You have a summer job with a company designing a new bobsled for the U.S. team to use in the next Winter Olympics. You know that the success of the team depends crucially on the initial push of the team members – how fast they can push the bobsled before they jump into the sled. You need to know in more detail how that initial velocity affects the motion of the bobsled. The acceleration of the bobsled will affect the design so you need to know whether the initial velocity increases the acceleration of the bobsled or if it just increases the overall velocity of the bobsled. To solve this problem, you decide to model the situation using a cart moving down an inclined track.

Often, physicists will use well-understood situations to provide reasonable evidence for an extension of a system. In fact, most experiments in science are designed as an extension of the most current understanding of how the universe works.

Instead of repeating the experiment to answer this question, you will work with your group to use the previous problem to predict the initial velocity's effect on acceleration.

<b>EXTENSION QUESTIONS</b>
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One helpful place to start is with limiting cases. For this problem, we will think about what is expected when the initial velocity is either very small or very large.

1. If you were to reanalyze the video from Problem #2, but you started taking data points 2 frames into the cart's motion, what would you expect the *instantaneous velocity vs. time graph* to look like? What is the relationship between velocity and time that would fit the function? Write this in your journal.
2. Now imagine that the same track were 50 meters long and you had a video of the entire motion. If you started analyzing the data after the cart had gone 45 meters down the track, what would you expect the *instantaneous velocity vs. time graph* to look like? What is the relationship that would fit this function? What is different about the small initial velocity graph and the large initial velocity graph? What is similar about the two?
3. From your two limiting cases, do you have a sufficient info to make a prediction? As a check, think about one more case to confirm that it reasonably fits your answer. If you were to analyze the video that you made in **Motion Down an Incline**, but you advanced the video to the last frames of the motion and then took data, would you expect a constant acceleration or something else?
4. Is this sufficient for predicting the behavior of the cart under different circumstances? Do you need to run the experiment in order to be sure of your prediction? Why or why not?
5. What is the benefit of having an initial velocity in the bobsled race? Compare the two *instantaneous position vs. time graphs* that you made in questions 1 and 2 and explain why this is important in the race.

## MOTION DOWN AN INCLINE WITH AN INITIAL VELOCITY

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<b>PREDICTION</b>
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Make a rough sketch of the *acceleration vs. time graph* for a cart released from rest on an inclined track. On the same graph, sketch how you think the *acceleration vs. time graph* will look when the cart is given an initial velocity down the track.

Do you think the cart launched down the inclined track will have a **larger acceleration**, **smaller acceleration**, or **the same acceleration** as the cart released from rest? Explain your reasoning.

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# 1101 Lab 1 Problem 4: MASS AND THE ACCELERATION OF A FALLING BALL

The local fire station in California has enlisted your help in studying the dropping of balls of chemicals from helicopters to extinguish forest fires. The amount of chemicals in one of these balls is varied depending on the size of the fire. As a first step to your study, you assume the helicopters are stationary, hovering over a fire. You are to determine if balls of the same size with different amounts of chemicals will fall differently.

Read: Knight, Jones & Field Chapter 2 Section 2.7

## EQUIPMENT

You have a collection of balls each with approximately the same diameter. You also have a stopwatch, meterstick, camera and a computer with video analysis software.



## WARM UP

1. Sketch a graph of *instantaneous acceleration vs. time* for a falling ball. Next to this graph sketch a graph of *instantaneous acceleration vs. time* for a heavier falling ball that has the same size and shape. Explain your reasoning for each graph. Write down an equation for each graph. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
2. Use your acceleration vs. time graphs to sketch *instantaneous velocity vs. time graphs* for a light and heavy ball using the same scale for the time axes. Write down an equation for each graph. If there are constants in your equations, what kinematic quantities do they represent? How would you determine these constants from your graph? Can any of the constants be determined from the equations representing the acceleration vs. time graphs?
3. Use your velocity vs. time graphs to sketch *instantaneous position vs. time graphs* for each case using the same scale for the time axes. Write down an equation for each graph. If there are constants in your equations, what kinematic quantities do they represent? How would you determine these constants from your graph? Can any of these constants be determined from the equations representing the acceleration vs. time or velocity vs. time graphs?
4. How could you determine the acceleration of a falling ball from video data (graphs and equations for position and velocity)? Write down an outline for how to do this, based on your experiences in earlier problems.
5. Do you expect that a heavier ball will have a higher, lower, or equal acceleration as a lighter ball of the same size? Is the relationship linear, or curved? Use this to predict a graph of *acceleration vs. mass* for falling balls.

### PREDICTION

Sketch how you expect the *acceleration vs. mass* graph to look for balls dropped from rest with the same size and shape, but having different masses.

Do you think the free-fall acceleration **increases**, **decreases**, or **stays the same** as the mass of the object increases? Explain your reasoning. (Remember that the shape of the ball does not change.)

### EXPLORATION

Review your lab journal from earlier problems. Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice dropping one of the balls until you can get its motion to fill the least distorted part of the screen. Determine how much time it takes for the ball to fall and estimate the number of video points you will get in that time. Are there enough points to make the measurement? Adjust the camera position and screen size to give you enough data points. You should be able to reproduce the conditions described in the Predictions.

Although the ball might be the most obvious choice to use to calibrate the video, the image quality due to its motion might make this difficult. Instead, you might hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory, for calibration purposes. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object close to the camera and then further away. What do you notice about the size of the reference object in the video image? Determine the best place to put the reference object for calibration.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see a blurred image due to the camera's setup and design. If you cannot further decrease the exposure time of each frame, devise a plan to measure the position of the same part of the "blur" in each video frame. You should be able to adjust the camera settings to create non-blurred images of objects in motion.

Write down your measurement plan.

### MEASUREMENT

Measure the mass of the ball and make a video of its fall according to the plan you devised in the exploration section. Make sure you can see the ball clearly in the video.

Acquire the position of the ball in enough frames to accomplish your analysis. Set the scale for the axes of your graph so that you can see the data points as you take them. Use your

measurements of total distance the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

*Complete your data analysis as you go along* (before making the next video), so you can determine how many different videos you need to make and what the object's mass should be for each video. Don't waste time collecting data you don't need. Repeat this procedure for balls with different masses. Collect enough data to convince yourself and others of your conclusion.

<b>ANALYSIS</b>
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Using MotionLab, determine the fit functions that best represent the position vs. time graphs in the x and y directions. How can you estimate the values of the constants of each function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the velocity vs. time graphs in the x and y directions. Compare these functions with the position vs. time functions. Determine the acceleration of the ball for different masses. Is the average acceleration different for the beginning of the video (when the object is moving slowly) and the end of the video (when the object is moving fast)?

Determine the average acceleration of the object in free fall for each value of its mass and use this to make a graph of the acceleration vs. mass. Is the average acceleration of the ball equal to its instantaneous acceleration in this case? Do you have enough data to convince others of your conclusions about your predictions? If the accelerations turn out to be dependent on mass, what might be the reason for the difference?

<b>CONCLUSION</b>
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How does the acceleration of a freely falling object depend on its mass? Did the data from the video images support your predicted relationship between acceleration and mass? (Make sure you carefully review *the appendix A Review of Graphs* to determine if your data really supports this relationship.) If your data did not support your prediction, were your predictions wrong or were your results unreliable? Explain your reasoning.

How does the acceleration you found compare to the gravitational acceleration? Can you explain any differences? What are the limitations on the accuracy of your measurements and analysis?



## 1101 Lab 1 Problem 5: LABORATORY EXTENSION - ACCELERATION OF A BALL WITH AN INITIAL VELOCITY

You have designed an apparatus to measure air quality in your city. To quickly force air through the apparatus, you will launch it straight downward from the top of a tall building. A very large acceleration may destroy sensitive components in the device; the launch system's design ensures that the apparatus is protected during its launch. You wonder what the acceleration of the apparatus will be once it exits the launcher. Does the object's acceleration after it has left the launcher depend on its velocity when it leaves the launcher? What effect does the initial velocity have on the apparatus? You decide to model the situation by throwing balls straight down.

*If you have done the previous Laboratory Extension, Motion Down an Incline with an Initial Velocity, you can review the answers for questions 1-4 below. Questions 5-7 address the effect of initial velocity on constant acceleration motion.*

Often, physicists will use well-understood situations to provide reasonable evidence for an extension of the system. In fact, most experiments in science are designed as an extension of the most current understanding of how the universe works.

Instead of repeating the experiment to answer this question, you will work with your group to use the previous problem to predict the initial velocity's effect on acceleration.

### EXTENSION QUESTIONS

One helpful place to start is with limiting cases. For this problem, we will think about what is expected when the initial velocity is either very small or very large.

Acceleration with an initial velocity:

1. If you were to re-analyze the video from the earlier **Mass and the Acceleration of a Falling Ball**, but you started taking data points 2 frames into the ball's motion, what would you expect the *instantaneous velocity vs. time graph* to look like? What is the relationship between velocity and time that would fit the function? Write this in your journal.
2. Now imagine that you dropped the ball from 50 meters and you had a video of the entire motion. If you started analyzing the data after the ball had fallen 45 meters, what would you expect the *instantaneous velocity vs. time graph* to look like? What is the relationship that would fit this function? What is different about the small initial velocity graph and the large initial velocity graph? What is similar about the two?
3. From your two limiting cases, do you have a sufficient answer to the prediction? As a check, think about one more case to confirm that it reasonably fits your answer. If you were to analyze the video that you made in **Mass and the Acceleration of a Falling Ball**, but you advanced the video to the last frames of the motion and then took data, would you expect a constant acceleration or something else?
4. Is this sufficient for predicting the behavior of the ball under different circumstances? Do you need to run the experiment in order to be sure of your prediction? Why or why not?

## ACCELERATION OF A BALL WITH AN INITIAL VELOCITY

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Effect of initial velocity in constant acceleration motion:

5. Open *PracticeFit*. What is the instantaneous velocity vs. time relationship with constant acceleration? What is the instantaneous position vs. time relationship with constant acceleration? Function 1 is a simple linear relationship with only integers for the coefficients. Fit the function that is given and then decide with your group how to have the same acceleration but change the initial velocity. You are comparing zero and non-zero initial velocities. You can do this no matter what the mystery fit function is. Discuss with your group how to accomplish this.
6. For the instantaneous velocity vs. time graph, what is the difference between the zero and non-zero initial velocity if the acceleration is the same? What happens to the function if the initial velocity and the acceleration have opposite signs?
7. Function 3 is a simple quadratic function that always has integers as the coefficients, and the linear coefficient is always zero (i.e.  $B=0$ ). What is the physical significance of this? Fit the function given and then change the initial velocity parameter to have a non-zero initial velocity. Compare the zero and non-zero initial velocity functions. What effect does initial velocity have on the instantaneous position of the projectile? Make the signs of the initial velocity and the acceleration opposite of each other. What effect does that have on the instantaneous position?

<b>PREDICTION</b>
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Sketch a graph of a ball's acceleration as a function of time after it is launched downward with an initial velocity. Sketch a graph of the ball's acceleration as a function of time after it is dropped from rest. Compare the two and state how your graph will change if the object's initial velocity increases or decreases.

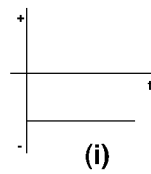
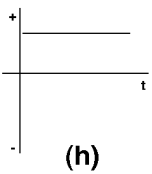
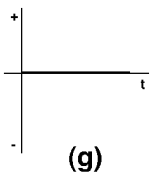
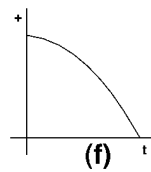
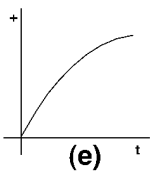
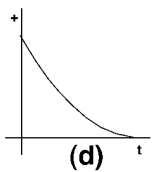
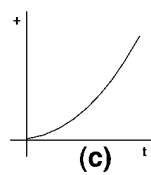
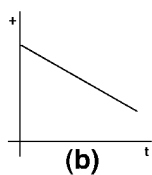
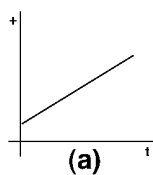
Do you think that the acceleration **increases**, **decreases**, or **stays the same** as the initial velocity of the object increases? Explain your reasoning.

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## ☑ CHECK YOUR UNDERSTANDING

### 1101 LAB 1: DESCRIPTION OF MOTION IN ONE DIMENSION

1. Suppose you are looking down from a helicopter at three cars traveling in the same direction along the freeway. The positions of the three cars every 2 seconds are represented by dots on the diagram below.
  - a. At what clock reading (or time interval) do Car A and Car B have very nearly the same speed? Explain your reasoning.
  - b. At approximately what clock reading (or readings) does one car pass another car? In each instance you cite, indicate which car, A, B or C, is doing the overtaking. Explain your reasoning.
  - c. Suppose you calculated the average velocity for Car B between  $t_1$  and  $t_5$ . Where was the car when its instantaneous velocity was equal to its average velocity? Explain your reasoning.
  - d. Which graph below best represents the position-versus-time graph of Car A? Of Car B? Of car C? Explain your reasoning.
  - e. Which graph below best represents the *instantaneous velocity vs. time* graph of Car A? Of Car B? Of car C? Explain your reasoning. (HINT: Examine the distances traveled in successive time intervals.)
  - f. Which graph below best represents the *instantaneous acceleration vs. time* graph of Car A? Of Car B? Of car C? Explain your reasoning.



**☑ CHECK YOUR UNDERSTANDING**  
**LAB 1: DESCRIPTION OF MOTION IN ONE DIMENSION**

2. A mining cart starts from rest at the top of a hill, rolls down the hill, over a short flat section, then back up another hill, as shown in the diagram above. Assume that the friction between the wheels and the rails is negligible.
- a. Which graph below best represents the position-versus-time graph? Explain your reasoning.
  - b. Which graph below best represents the instantaneous velocity-versus-time graph? Explain your reasoning.
  - c. Which graph below best represents the instantaneous acceleration-versus-time graph? Explain your reasoning.

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## 1101 LAB 2: DESCRIPTION OF MOTION IN TWO DIMENSIONS

In this laboratory you continue the study of accelerated motion in more situations. The carts you used in Laboratory I moved in only one dimension. However, as you know, objects don't *always* move in a straight line! However, motion in two and three dimensions can be decomposed into one-dimensional motions; what you learned in the first lab can be applied to this lab.

You will study the motions of an object in free fall and an object tossed into the air. In these labs, you will need to consider the effects of *air resistance* on the motion of the objects. Can it always be neglected? As always, if you have any questions, talk with your fellow students or your instructor.

### OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Determine the motion of an object in free-fall by considering what quantities and initial conditions affect the motion.
- Determine the motion of a projectile from its horizontal and vertical components by considering what quantities and initial conditions affect the motion.

### PREPARATION:

Read Knight, Jones & Field Chapter 2 section 2.7 and Chapter 3. Review your results and procedures from Laboratory I. Before coming to the lab you should be able to:

- Determine instantaneous and average velocities and accelerations from video images.
- Analyze a vector in terms of its components along a set of perpendicular axes.
- Add and subtract vectors.

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## 1101 Lab 2 Problem 1: MOTION UP AND DOWN AN INCLINE

A proposed ride at the Valley Fair amusement park launches a roller coaster car up an inclined track. Near the top of the track, the car reverses direction and rolls backwards into the station. As a member of the safety committee, you have been asked to compute the acceleration of the car throughout the ride and determine if the acceleration of an object moving up a ramp is different from that of an object moving down the same ramp. To check your results, you decide to build a laboratory model of the ride.

Read: Knight, Jones & Field Chapter 3, Sections 3.3 to 3.4

### EQUIPMENT

You have a stopwatch, meterstick, track endstop, wood block, video camera and a computer with analysis software. You will also have a cart to roll up an inclined track.



### WARM UP

1. Draw a picture of the cart rolling **up** the ramp. Draw arrows above the cart to show the direction of the velocity and the direction of the acceleration. Choose a coordinate system and include this in your picture.
2. Draw a new picture of the cart rolling **down** the ramp. Draw arrows above the cart to show the direction of the velocity and the direction of the acceleration. Label your coordinate system.
3. Sketch a graph of the *instantaneous acceleration vs. time* for the entire motion of the cart as it rolls up and then back down the track after an initial push. Label the instant where the cart reverses its motion near the top of the track. Explain your reasoning. Write down the equation(s) that best represents this graph. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
4. From your acceleration vs. time graph, answer Warm-up question 3 for *instantaneous velocity vs. time* instead. *Hint: Be sure to consider both the **direction** and the **magnitude** of the velocity as the cart rolls up and down the track.* Use the same scale for your time axes. Can any of the constants in the velocity equation(s) be determined from the constants in the acceleration equation(s)?
5. Now do the same for *position vs. time*.

### PREDICTION

Based on your results for the earlier problem **Motion Down an Incline**, make a rough sketch of the *acceleration vs. time* graph for the cart moving down the inclined track. On the same

## MOTION UP AND DOWN AN INCLINE

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graph, sketch how you think the *acceleration vs. time graph* will look for the cart moving **up** the track at the same angle.

Do you think the *magnitude* of the cart's acceleration as it moves **up** an inclined track will increase, decrease, or stay the same? What about the *magnitude* of the cart's acceleration as it moves **down** a track inclined at the same angle? Explain your reasoning. Does the *direction* of the cart's acceleration change throughout its motion, or stay the same? Remember, for a direct comparison to the problem **Motion Down an Incline**, you should use the same coordinate system.

### EXPLORATION

What is the best way to change the angle of the inclined track in a reproducible way? How are you going to measure this angle with respect to the table? *Hint: Think about trigonometry.* How steep of an incline do you want to use?

Start the cart up the track with a gentle push. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP ON ITS WAY DOWN!** Observe the cart as it moves up the inclined track. At the instant the cart reverses direction, what is its velocity? What is its acceleration? Observe the cart as it moves down the inclined track. Do your observations agree with your prediction? If not, this is a good time to discuss with your group and modify your prediction.

When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

Try several different angles. If the angle is too large, the cart may not go up very far and give you too few video frames for the measurement. If the angle is too small it will be difficult to measure the acceleration. Determine the useful range of angles for your track. Take a few practice videos and play them back to make sure you have captured the motion you want.

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking. Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan from the Exploration section to make a video of the cart moving up and then down the track at your chosen angle. Make sure you get enough points for each part of the motion to determine the behavior of the acceleration. Record the time the cart travels, and the distance traveled. *Don't forget to measure and record the angle (with estimated uncertainty).*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case?

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and total time to determine the maximum and minimum value for each axis before taking data.

### ANALYSIS

From the time given by the stopwatch (or the time stamp on the video) and the distance traveled by the cart, calculate the average acceleration. Estimate the uncertainty.

Using MotionLab, determine the fit functions that best represent the *position vs. time graphs* in the x and y directions. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Can you tell from your graph where the cart reaches its highest point?

Do the same for the *velocity vs. time graphs* in the x and y directions. Compare these functions with the position vs. time functions. What was the velocity when the cart reached its maximum height on the track? How do you know?

Determine the acceleration as a function of time as the cart goes up and then down the ramp. Make a graph of the *acceleration vs. time*. Can you tell from your graph where the cart reaches its highest point? Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer.*

Compare the acceleration function you just graphed with the average acceleration you calculated from the time and the distance the cart traveled.

### CONCLUSIONS

How do your position vs. time and velocity vs. time graphs compare with your answers to the warm-up and the prediction? What are the limitations on the accuracy of your measurements and analysis?

How did the acceleration of the cart **up** the track compare to the acceleration **down** the track? Did the acceleration change magnitude or direction at any time during its motion? Was the acceleration zero, or nonzero at the maximum height of its motion? Explain how you reached your conclusions about the cart's motion.



## 1101 Lab 2 Problem 2: MASS AND MOTION DOWN AN INCLINE

Before the end of summer arrives, you and some friends drive to a local amusement park to ride the new roller coaster. During the busy afternoon, the roller coaster is always full of people. But as the day comes to an end and the park is less crowded, you want to go down the roller coaster once more. However, your friends say that the ride down the first hill won't be as fast as it was earlier, because there is less mass in the roller coaster, so they don't want to go. What do you think? To determine how the acceleration of an object down a ramp depends on its mass, you decide to model the situation using a cart moving down an inclined track.

Read: Knight, Jones & Field Chapter 3, Sections 3.3 to 3.4

### EQUIPMENT

You have a stopwatch, meterstick, track endstop, wood block, camera and a computer with video analysis software. You will also have a cart to roll down an inclined track and additional cart masses to add to the cart.



### WARM UP

Answer the following questions using your experiences from earlier problems and your personal opinion.

1. Sketch a graph of how you would expect an instantaneous acceleration vs. time graph to look for a cart released from rest on an inclined track. Next to this graph, sketch a new graph of the acceleration vs. time for a cart with a much larger mass. Explain your reasoning. Write down the equation(s) that best represent each of these graphs. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
2. Sketch a graph of instantaneous velocity vs. time for each case. Use the same scale for the time axes as the acceleration graphs. Write down the equation(s) for each graph. If there are constants in your equations, what kinematic quantities do they represent? How would you determine these constants from your graph?
3. Now do the same for position vs. time. Can any of the constants in your functions be determined from the equations representing the acceleration vs. time or velocity vs. time graphs?

### PREDICTION

Make a sketch of how you think the *acceleration vs. mass* graph will look for carts with different masses released from rest from the top of an inclined track.

## MASS AND MOTION DOWN AN INCLINE

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Do you think the acceleration of the cart **increases**, **decreases**, or **stays the same** as the mass of the cart increases? Explain your reasoning.

### EXPLORATION

Slant the track at the *same* angle you used in the earlier problem, **Motion Down an Incline**.

Observe the motion of several carts of different mass when released from rest at the top of the track. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!** From your estimate of the size of the effect, determine the range of mass that will give the best results in this problem. Determine the first two masses you should use for the measurement.

How do you determine how many different masses do you need to use to get a conclusive answer? How will you determine the uncertainty in your measurements? How many times should you repeat these measurements? Explain.

When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking. Write down your measurement plan.

*Make sure everyone in your group gets the chance to operate the camera and the computer.*

### MEASUREMENT

Using the measurement plan you devised in the Exploration section, make a video of the cart moving down the track at your chosen angle. Make sure you get enough points to determine the behavior of the acceleration. Record the time duration of the cart's trip, and the distance traveled. *Don't forget to measure and record the angle (with estimated uncertainty).*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case?

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them.

Make several videos with carts of different mass to check your qualitative prediction. If you analyze your data from the first two masses you use *before* you make the next video, you can

determine which mass to use next. You should minimize the number of measurements you need.

### ANALYSIS

From the time given by the stopwatch (or the time stamp on the video) and the distance traveled by the cart, calculate the average acceleration. Estimate the uncertainty.

Using MotionLab, determine the fit functions that best represent the *position vs. time graphs* in the x and y directions. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the *velocity vs. time graphs* in the x and y directions. Compare these functions with the position vs. time functions.

Determine the acceleration as the cart goes down the track for different masses. Make a graph of the *acceleration vs. mass*. Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer.*

Compare the acceleration of the cart you found from the video analysis with the average acceleration you calculated from the time and the distance the cart traveled.

Do you have enough data to convince others of your conclusion about how the acceleration of the cart depends on its mass? If the acceleration does indeed depend on the mass of the cart, what might be causing this difference?

### CONCLUSION

How will you respond to your friend? Does the *acceleration* down a nearly frictionless roller coaster depend on the mass of the people in the coaster? Does the *velocity* of the coaster depend on its mass? (Will the roller coaster be just as fast with fewer people?) State your result in the most general terms supported by your analysis.

Did your measurements of the cart agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

## 1101 Lab 2 Problem 3: PROJECTILE MOTION AND VELOCITY

In medieval warfare, one of the greatest technological advancement was the trebuchet. The trebuchet was used to sling rocks into castles. You are asked to study the motion of such a projectile for a group of local enthusiast planning a medieval war reenactment. Unfortunately an actual trebuchet had not been built yet, so you decide to first look at the motion of a thrown ball as a model of rocks thrown by a trebuchet. Specifically, you are interested in how the horizontal and the vertical components of the velocity for a thrown object change with time.

Read: Knight, Jones & Field Chapter 3, Sections 3.1 to 3.4, 3.6 & 3.7

### EQUIPMENT

For this problem you will have a ball, stopwatch, meterstick, video camera and a computer with video analysis software.



### WARM UP

1. Make a large (about one-half page) rough sketch of the trajectory of the ball after it has been thrown. Draw the ball in at least five different positions; two when the ball is going up, two when it is going down, and one at its maximum height. Label the horizontal and vertical axes of your coordinate system.
2. On your sketch, draw and label the expected acceleration vectors of the ball (relative sizes and directions) for the five different positions. Decompose each acceleration vector into its vertical and horizontal components.
3. On your sketch, draw and label the velocity vectors of the object at the same positions you chose to draw your acceleration vectors. Decomposes each velocity vector into its vertical and horizontal components. Check to see that the changes in the velocity vector are consistent with the acceleration vectors.
4. Looking at your sketch, how do you expect the ball's *horizontal acceleration* to change with time? Write an equation giving the ball's horizontal acceleration as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
5. Looking at your sketch, how do you expect the ball's *horizontal velocity* to change with time? Is it consistent with your statements about the ball's acceleration from the previous question? Write an equation for the ball's horizontal velocity as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
6. Write an equation for the ball's *horizontal position* as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Are any of these constants related to the equations for horizontal velocity or acceleration?

7. Repeat Warm-up questions 4-6 for the *vertical* component of the acceleration, velocity, and position. How are the constants for the acceleration, velocity and position equations related?

<b>PREDICTION</b>
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1. Make a rough sketch of how you expect the graph of the *horizontal velocity vs. time* to look for the thrown object. Do you think the horizontal component of the object's velocity **changes** during its flight? If so, how does it change? Or do you think it is **constant**? Explain your reasoning.
2. Make a rough sketch of how you expect the graph of the *vertical velocity vs. time* to look for the object. Do you think the vertical component of the object's velocity **changes** during its flight? If so, how does it change? Or do you think it is **constant**? Explain your reasoning.

<b>EXPLORATION</b>
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Review your lab journal from earlier problems.

Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice throwing the ball until you can get the ball's motion to fill the video screen (or at least the undistorted part of the video screen) **after** it leaves your hand. Determine how much time it takes for the ball to travel and estimate the number of video points you will get in that time. Is that enough points to make the measurement? Adjust the camera position to give you enough data points.

Although the ball might be the most obvious choice to use to calibrate the video, the image quality due to its motion might make this difficult. Instead, you might hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory, for calibration purposes. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object close to the camera and then further away. What do you notice about the size of the reference object in the video image? Determine the best place to put the reference object for calibration.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see a blurred image due to the camera's setup and design. If you cannot further decrease the exposure time of each frame, devise a plan to measure the position of the same part of the "blur" in each video frame. You should be able to adjust the camera settings to create non-blurred images of objects in motion.

Write down your measurement plan.

**MEASUREMENT**

Measure the total distance the ball travels and total time to determine the maximum and minimum value for each position axis before taking data with the computer.

Make a video of the ball being tossed. Make sure you can see the ball in every frame of the video.

Digitize the position of the ball in enough frames of the video so that you have sufficient data to accomplish your analysis. Set the scale for the axes of your graph so that you can see the data points as you take them.

**ANALYSIS**

Using MotionLab, determine the fit functions that best represent the position vs. time graphs in the x and y directions. How can you estimate the values of the constants of each function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the velocity vs. time graphs in the x and y directions. Compare these functions with the position vs. time functions. How can you calculate the values of the constants of these functions from the functions representing the position vs. time graphs? You can also estimate the value of the constants from the graph. What kinematics quantities do these constants represent?

From the velocity vs. time graph(s) determine the acceleration of the ball independently for each component of the motion as a function of time. What is the acceleration of the ball just after it is thrown, and just before it is caught or lands? What is the magnitude of the ball's acceleration at its highest point? Is this value reasonable?

Determine the launch velocity of the ball from the velocity vs. time graphs in the x and y directions. Is this value reasonable? Determine the velocity of the ball at its highest point. Is this value reasonable?

**CONCLUSION**

Did your measurements agree with your initial predictions? Why or why not? If they do not agree, are there any assumptions that you have made, that might not be correct? What are the limitations on the accuracy of your measurements and analysis?

How does the horizontal velocity component of a launched rock depend on time? How does the vertical velocity component depend on time? State your results in the most general terms supported by your analysis. At what position does the ball have the minimum velocity? Maximum velocity?

## 1101 Lab 2 Problem 4: PROJECTILE MOTION AND MASS

You are again asked to study the motion of a projectile for a group of local enthusiasts planning a medieval war reenactment. Understandably, it was hard to find rocks of the same mass to launch using a trebuchet during medieval times. The second part of your study requires you to determine if the mass of an object would affect your conclusions in earlier problem, **Projectile Motion and Velocity**. Specifically, you want to determine how the horizontal and vertical components of the acceleration of a thrown object depend on its mass.

Read: Knight, Jones & Field Chapter 3, Sections 3.1 to 3.4, 3.6 & 3.7

EQUIPMENT
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For this problem you will have a collection of balls, stopwatch, meterstick, video camera and a computer with video analysis software.



WARM UP
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If you have done the problem **Projectile Motion and Velocity**, you might skip Questions 1 – 7 and review your notes from that problem.

1. Make a large (about one-half page) rough sketch of the trajectory of the ball after it has been thrown. Draw the ball in at least five different positions; two when the ball is going up, two when it is going down, and one at its maximum height. Label the horizontal and vertical axes of your coordinate system.
2. On your sketch, draw and label the expected acceleration vectors of the ball (relative sizes and directions) for the five different positions. Decompose each acceleration vector into its vertical and horizontal components.
3. On your sketch, draw and label the velocity vectors of the object at the same positions you chose to draw your acceleration vectors. Decompose each velocity vector into its vertical and horizontal components. Check to see that the changes in the velocity vector are consistent with the acceleration vectors.
4. Looking at your sketch, how do you expect the ball's *horizontal acceleration* to change with time? Write an equation giving the ball's horizontal acceleration as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?
5. Looking at your sketch, how do you expect the ball's *horizontal velocity* to change with time? Is it consistent with your statements about the ball's acceleration from the previous question? Write an equation for the ball's horizontal velocity as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

6. Write an equation for the ball's *horizontal position* as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph? Are any of these constants related to the equations for horizontal velocity or acceleration?
7. Repeat Warm-up questions 4-6 for the *vertical* component of the acceleration, velocity, and position. How are the constants for the acceleration, velocity and position equations related?
8. How are the components of the acceleration of a projectile related to the mass of the object? How are the components of the velocity related to mass?

### PREDICTIONS

1. Make a rough sketch of how you expect the graph of the horizontal component of *acceleration vs. mass* to look for the object in projectile motion. Do you think the horizontal component of an object's acceleration will **increase**, **decrease**, or **stay the same** as the mass of that object increases? Explain your reasoning.
2. Make a rough sketch of how you expect the graph of the vertical component of *acceleration vs. mass* to look for the object in projectile motion. Do you think the vertical component of an object's acceleration will **increase**, **decrease**, or **stay the same** as the mass of that object increases? Explain your reasoning.

### EXPLORATION

If you have done the earlier problem, **Projectile Motion and Velocity**, the following is a review - you need only do what you feel is necessary.

Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice throwing the ball until you can get the ball's motion to fill the video screen (or at least the undistorted part of the video screen) **after** it leaves your hand. Determine how much time it takes for the ball to travel and estimate the number of video points you will get in that time. Is that enough points to make the measurement? Adjust the camera position to give you enough data points.

Although the ball might be the most obvious choice to use to calibrate the video, the image quality due to its motion might make this difficult. Instead, you might hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory, for calibration purposes. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object close to the camera and then further away. What do you notice about the size of the reference object in the video image? Determine the best place to put the reference object for calibration.



Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see a blurred image due to the camera's setup and design. If you cannot further decrease the exposure time of each frame, devise a plan to measure the position of the same part of the "blur" in each video frame. You should be able to adjust the camera settings to create non-blurred images of objects in motion.

Write down your measurement plan.

### MEASUREMENT

If you have done **Projectile Motion and Velocity**, the following is a review.

A way to save time in this lab is for each group in your class to determine the horizontal and vertical acceleration for **a different** mass and report their findings to the class. You should be able to draw a sketch of the horizontal and vertical components of the acceleration vs. mass of the object from the data collected by the class.

If you are not using data from other groups analyzing different balls, you yourself will have to use several different balls of different masses. Use your experience from earlier problems to determine which balls and how many are needed.

Measure the total distance the ball travels and total time to determine the maximum and minimum value for each position axis before taking data with the computer. Make a video of the ball being tossed. Make sure you can see the ball in every frame of the video.

Acquire the position of the ball in enough frames of the video so that you have sufficient data to accomplish your analysis. Set the scale for the axes of your graph so that you can see the data points as you take them.

### ANALYSIS

Using MotionLab, determine the fit functions that best represent the position vs. time graphs in the x and y directions. How can you estimate the values of the constants of each function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the velocity vs. time graphs in the x and y directions. Compare these functions with the position vs. time functions. How can you calculate the values of the constants of these functions from the functions representing the position vs. time graphs? You can also estimate the value of the constants from the graph. What kinematics quantities do these constants represent?

From the velocity vs. time graph(s) determine the acceleration of the ball independently for each component of the motion as a function of time. What is the acceleration of the ball just after it is thrown, and just before it is caught? What is the magnitude of the ball's acceleration at its highest point? Is this value reasonable?

Determine the launch velocity of the ball from the velocity vs. time graphs in the x and y directions. Is this value reasonable? Determine the velocity of the ball at its highest point. Is this value reasonable?

Report the value of your object's mass (with uncertainty) and its horizontal and vertical accelerations as a function of time to the class. Also report the object's average acceleration for each component (with uncertainties) to the class and record the values from other groups. Make graphs of horizontal and vertical acceleration vs. mass.

<b>CONCLUSION</b>
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How does the horizontal component of a projectile's acceleration depend on its mass? How does the vertical component of the acceleration depend on mass? State your result in the most general terms supported by your analysis. Did your measurements agree with your initial predictions? Why or why not?

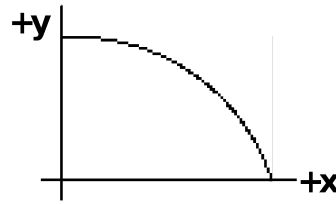
How does your conclusion tie together with your results from previous problems? What are the limitations on the accuracy of your measurements and analysis?

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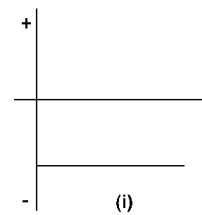
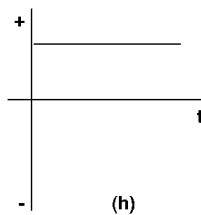
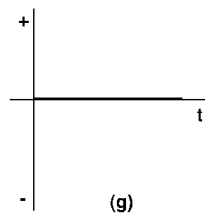
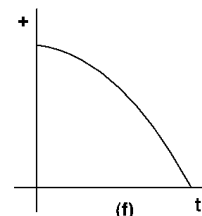
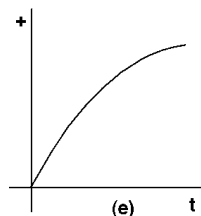
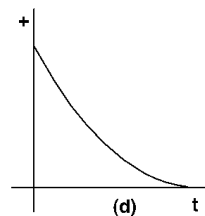
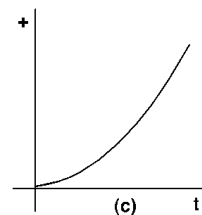
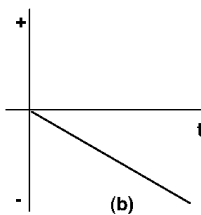
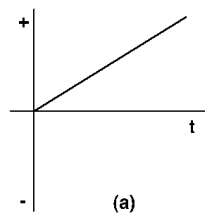
## ☑ CHECK YOUR UNDERSTANDING

### 1101 LAB 2: DESCRIPTION OF MOTION IN TWO DIMENSIONS

1. A baseball is hit horizontally with an initial velocity  $v_0$  at time  $t_0 = 0$  and follows the parabolic arc shown at right.



- a. Which graph below best represents its horizontal position ( $x$ ) versus time graph? Explain your reasoning.
- b. Which graph below best represents the horizontal velocity ( $v_x$ ) versus time graph? Explain your reasoning.
- c. Which graph below best represents the horizontal acceleration ( $a_x$ ) versus time graph? Explain your reasoning.
- d. Which graph below best represents the vertical position ( $y$ ) versus time graph? Explain your reasoning.
- e. Which graph below best represents the vertical velocity ( $v_y$ ) versus time graph? Explain your reasoning.
- f. Which graph below best represents the vertical acceleration ( $a_y$ ) versus time graph? Explain your reasoning.

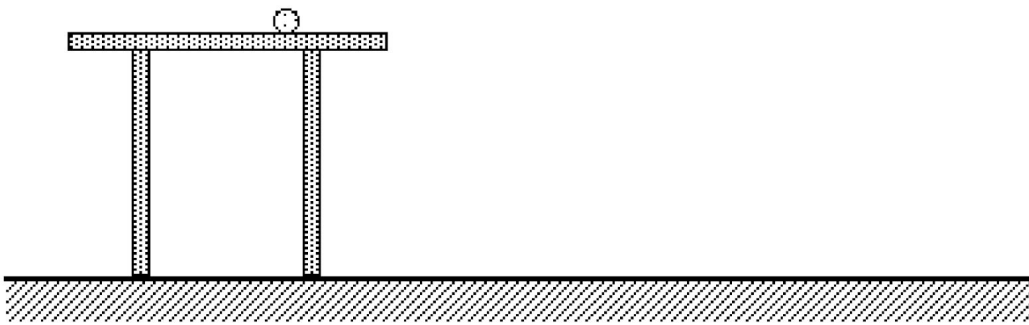


2. Two metal balls are the same size, but one weighs twice as much as the other. The balls are dropped from the top of a two-story building at the same instant of time. Which ball will reach the ground first, or will they reach the ground at the same time? Explain your reasoning.

☒ CHECK YOUR UNDERSTANDING

**1101 LAB 2: DESCRIPTION OF MOTION IN TWO DIMENSIONS**

3. Suppose you throw a ball vertically up into the air with an initial velocity  $v_0$ .
  - a. What is the acceleration of the ball at its maximum height? Explain your reasoning.
  - b. What would the acceleration-versus-time graph look like from the moment the ball leaves your hand to the moment before it returns to your hand?
4. A ball slides off the edge of a table with a horizontal velocity  $v_x$  and lands on the floor.



- a. On the diagram above, sketch a possible trajectory (the path followed by the ball) from the edge of the table to the floor.
- b. On the same diagram sketch another trajectory, that of another ball having a very much larger mass than that of the first ball, but exactly the same initial velocity  $v_x$ . Explain your reasoning.

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## 1101 LAB 3: FORCES

The problems in this laboratory will help you investigate the effect of forces on the motion of objects. In the first problem, you will investigate the effects of forces on a sliding object. In the second problem, you will apply the force concept and the vector nature of forces to a situation in which nothing moves. The third and fourth problems investigate the behavior of the frictional forces.

### OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Make and test quantitative predictions about the relationship of forces on objects and the motion of those objects for real systems.
- Use forces as vector quantities.
- Characterize the behavior of the frictional force.
- Improve your problem solving skills.

### PREPARATION:

Read Knight, Jones & Field Chapter 4 & 5. Review Chapter 3 sections 3.1 to 3.3 regarding the properties of vectors. Review your lab journal notes about the behavior of an object sliding down an inclined track.

Before coming to lab you should be able to:

- Define and use sine, cosine and tangent for a right triangle.
- Recognize the difference between mass and weight.
- Determine the net force on an object from its acceleration.
- Draw and use free-body diagrams.
- Break force vectors into components and determine the total force from the components.
- Explain what is meant by saying a system is in "equilibrium."
- Write down the force law for a frictional force.

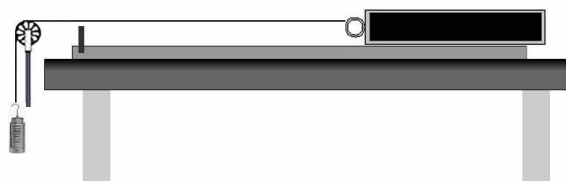
## 1101 Lab 3 Problem 1: HOW SURFACES AFFECT THE KINETIC FRICTIONAL FORCE

You are helping a friend design a new game to use on the Midway at the Minnesota State Fair. The game is similar to shuffleboard -- players use a short stick to push a puck hard enough so that it will travel along a level surface and fall into one of several holes. To construct the pucks your friend wants to use either natural wood, or felt covered wood. He has decided that the sliding surface will be aluminum. He needs to know which block surface (felt or wood) to use for the game. If there is too much friction, no one will ever get the puck into the holes. If there is too little friction, then the game will be too easy. He knows you are taking a physics course, so he asks you to help. To solve this problem, you devise an experiment to measure the kinetic frictional force between the block and the board.

Read: Knight, Jones & Field Chapter 4, Sections 4.6 & 4.7, and Chapter 5, Sections 5.4 & 5.5

### EQUIPMENT

A block is pulled along a level track. You have a friction block that has both a wood and a felt side. You also have a stopwatch, meterstick, string, pulley, table clamp, aluminum track, mass set, video camera and a computer with analysis software.



### WARM UP

1. Make a sketch the setup that is similar to the picture in the Equipment section. Draw and label vectors to indicate the direction of the velocity and the direction of the acceleration for both the hanging object A and the block. Also assign symbols to the “known” quantities in the problem: the mass of object A and the mass of the block.
2. Write down the principles of physics that you will use to solve the problem. What quantities can you measure using the video analysis software?
3. Draw separate free-body diagrams of the forces on the block and the forces on object A after they start accelerating. Assign symbols to all of the forces, and define what they represent next to your diagram. It is useful to draw the acceleration vector for the object next to its free-body diagram. It is also useful to put the force vectors on a separate coordinate system for each object (e.g. explicitly state the positive and negative direction for each object). Remember that on a force diagram, the origin (tail) of all vectors is at the origin of the coordinate system.
4. For *each* force diagram (one for the block and another one for object A), write down Newton's 2<sup>nd</sup> law in both the x and y directions. It is important to make sure that all of your signs are correct. For example, if the acceleration of the block is in the positive direction, is the acceleration of object A positive or negative? Your answer will depend on how you define your coordinate system.

## HOW THE SURFACE AFFECTS THE KINETIC FRICTIONAL FORCE

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5. From your force diagram, write down Newton's 2<sup>nd</sup> law for each mass using the sum of the individual forces (the force of the string, weights of masses and frictional forces) as they relate to quantities you either know (masses) or can measure (the acceleration of the block).
6. Now you have two equations with two unknowns (frictional force and force of the string). Everything else in the equation can be measured. Combine your equations using algebra to write an expression for the kinetic frictional force on the block in terms of the mass of object A, the mass of the block, and the acceleration of the block

<b>PREDICTION</b>
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Write an expression for the frictional force on the sliding block as a function of the mass hanging on the string (object A), the mass of the block, and the acceleration of the block.

<b>EXPLORATION</b>
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For both surfaces under evaluation (felt and wood), slide the block along the track. Make sure it slides smoothly. If it does not, try cleaning the surfaces.

Determine the length of string you should use to connect the block to the mass hanger holding masses (object A). Remember that you will want to take a video of the system while both objects are accelerating (before object A hits the floor). Decide on a position where you will release the block that fits in the frame of the camera, and will give you enough data points for the motion.

Find a range of masses for object A that allows the block to accelerate smoothly across the track. Explore the different accelerations using a large range of masses. Try these masses for the two contact surfaces to be sure the block accelerates uniformly in both cases. Choose a range of masses that will give a smooth acceleration. You should use the same range of block masses for each surface.

Practice releasing the block from the position you determined and one of your chosen masses for object A. Determine how much time it takes for object A to hit the floor and estimate the number of video points you will get in that time. Are there enough points to make the measurement? Adjust the camera position, mass range of object A, or the release position/length of the string to give you enough data points. Be sure to check this for both surfaces of the block.

Write down a plan of how you will take your measurements. What will you use for a reference object to calibrate your video? Make sure that the plan will adequately check your prediction.

<b>MEASUREMENT</b>
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Carry out the measurement plan you determined in the Exploration section. You can change the mass of the block, the mass of object A, or the block surface and determine if the frictional force behaves as you predict.

Make sure you measure and record the mass of the block and object A (with uncertainties). Repeat the necessary measurements using a different block surface.

Complete the entire analysis of one case before making videos and measurements of the next case. *Make sure each person in your group gets a chance to operate the computer.*

### ANALYSIS

Using MotionLab, determine the fit functions that best represent the position vs. time graphs for the sliding block in the x and y directions. How can you estimate the values of the constants of each function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the velocity vs. time graphs in the x and y directions. Compare these functions with the position vs. time functions.

From each video, determine the acceleration of the sliding block (with uncertainty). Is the average acceleration different for the beginning of the video (when the object is moving slowly) and the end of the video (when the object is moving fast)? Before you begin any time consuming analysis, determine if the acceleration of the block is constant. If it is, you can use kinematic relationships to simplify your task. Decide on the minimum number of data points that you need to analyze in order to determine the acceleration accurately and reliably. Remember that it is not the purpose of this problem to find accelerations!

For each contact surface, use your predicted expression from the Warm-up and Prediction to calculate the kinetic frictional force with the appropriate units. Have you measured all of the quantities that you need for this expression? If not, make sure you measure them before you leave the lab.

### CONCLUSION

What does your data show about the effect of the contact surfaces on the kinetic frictional force? Did your results agree with your initial prediction? Why or why not?

Which surface (wood or felt) will you recommend to your friend? Why? Will one surface be more useful to the game at the State Fair? What are the limitations on the accuracy of your measurements and analysis?



## 1101 Lab 3 Problem 2: FORCES IN EQUILIBRIUM

You have a summer job with a research group studying the ecology of a rainforest in South America. To avoid walking on the delicate rainforest floor, the team members walk along a rope walkway that the local inhabitants have strung from tree to tree through the forest canopy. Your supervisor is concerned about the maximum amount of equipment each team member should carry to safely walk from tree to tree. If the walkway sags too much, the team member could be in danger, not to mention possible damage to the rainforest floor! You are assigned to set the load standards.

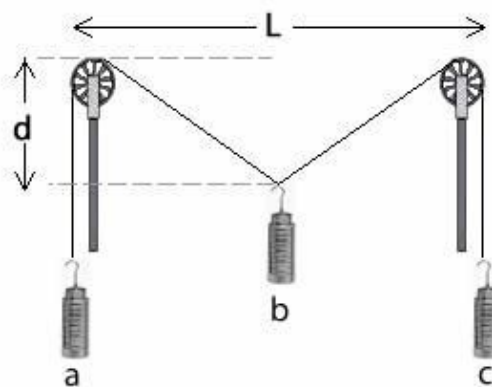
Each end of the rope supporting the walkway goes over a branch and then is attached to a large weight hanging down. When the team member is at the center of the walkway between two trees, you need to determine how the sag of the walkway is related to the mass of the counterweights and the total mass of the team member and their equipment. To check your calculation, you decide to model the situation.

Read: Knight, Jones & Field Chapter 5 Sections 5.1 & 5.3. See Chapter 3 Sections 3.3 for vector review.

### EQUIPMENT

You have a meterstick, two pulleys, two table clamps, string and three mass sets.

The system consists of a central object, B, suspended halfway between two pulleys by a string. The whole system is in equilibrium. The counterweight objects A and C, which have the same mass, allow you to determine the force exerted on the central object by the string.



### WARM UP

1. Draw a picture of the setup similar to the one in the Equipment section. Be sure to include symbols for the horizontal distance  $L$ , and the mass of object B and objects A and C. Label the angle that the string sags below the horizontal as  $\theta$  and the displacement of point P as " $d$ ". Use trigonometry to show how the vertical displacement ( $d$ ) of object B is related to the angle  $\theta$  and the horizontal distance  $L$ .
2. Draw separate free-body diagrams of the forces on objects A, B, and C. Is there a difference between the force on mass B and at point P? Assign symbols to all of the forces, and define what they represent next to your diagram. It is also useful to put the force vectors on a separate coordinate system for each object (force diagram). Remember that on a force diagram, the origin (tail) of all vectors is at the origin of the coordinate system.

3. Since this is a static situation, what is the acceleration for each object? For *each* force diagram break all forces into their x and y components and write down Newton's 2nd law along each coordinate axis.
4. Solve your equations for the vertical displacement ( $d$ ) of object B in terms of the mass ( $M$ ) of object B, the mass ( $m$ ) of objects A and C, and the horizontal distance ( $L$ ) between the pulleys. Your final equation **should not** depend on angles. *Hint: Rewrite the angle in terms of lengths and distances specified in your diagram using trigonometric function and Pythagorean Theorem. This will take SEVERAL algebraic steps.*
5. Use your equation to sketch the shape of the graph of the vertical displacement ( $d$ ) versus mass of object B.

<b>PREDICTION</b>
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Write an equation for the change in the vertical displacement ( $d$ ) of the central object B in terms of the horizontal distance between the two pulleys ( $L$ ), the mass of object B ( $M$ ), and the mass ( $m$ ) of objects A and C.

Use your equation to sketch the expected graph of the vertical displacement of object B versus its mass ( $M$ ). When you are making your graph, consider what happens when  $M = 2m$ , and when  $M > 2m$ .

<b>EXPLORATION</b>
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Build the system of pulleys and masses without the central object so that the string looks horizontal. Make sure to use an appropriate length of string; if it is too short, the mass hangers from objects A and C will interfere with the pulleys when object B is lowered. Attach a central object and observe how the string sags. Decide on the origin from which you will measure the vertical position of object B.

Try changing the mass of A and C (keep them equal for the measurements, but you will want to explore the case where they are not equal). Are you able to create a stable system with unequal masses for A and C? Choose a set of masses for A and C that will allow you to get enough data to determine the vertical displacement as it depends on the mass of object B.

For the entire range of weights you will use, determine if the pulleys turn freely. How can you determine if the assumption that these pulleys are frictionless is good?

With the system in equilibrium, move the pulleys closer to one another and observe what happens to the vertical displacement of object B. Does the result make sense? Observe what happens when you move the pulleys farther apart. Decide on a separation distance between the two pulleys for your measurements.

Determine the range of masses for object B so that your system can be in equilibrium. Decide on the number of measurements that you will need to determine if your prediction agrees with the results. You may need to refer to your prediction to determine the proper range of masses.

<b>MEASUREMENT</b>
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Using your plan from the exploration section, measure the vertical position of the central object as you increase its mass. Make a table and record your measurements. Also record the masses of objects A and C, and the horizontal separation of the pulleys. What units should you use? Don't forget to record your uncertainties.

<b>ANALYSIS</b>
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Make a graph of the *measured* vertical displacement of the central object as a function of its mass based on your data. On the same graph, plot your *predicted* equation for vertical displacement versus mass of the central object.

Where do the two curves match? Where do the two curves start to diverge from one another? What are the assumptions that you made about the system? What accounts for the discrepancy between the predicted and the measured displacement?

<b>CONCLUSION</b>
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What will you report to your supervisor? How does the vertical displacement of this object depend on its mass? Did your measurements of the vertical displacement of object B agree with your initial predictions? If not, why? What are the limits on the accuracy of your measurements and analysis?

What information would you need to apply your calculation to the walkway through the rainforest?

Estimate reasonable values for the information you need, and solve the problem for the walkway over the rainforest.

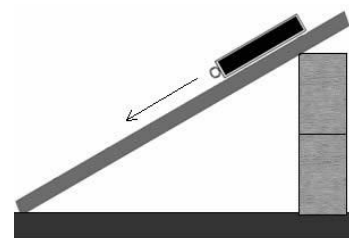
## 1101 Lab 3 Problem 3: NORMAL FORCE AND THE KINETIC FRICTIONAL FORCE (PART A)

You have taken a job with a theater company and you are in charge of setting up the props. The props are transported in crates by a truck. The crates are unloaded by being pushed down a ramp. You realize that the frictional force is making your job difficult, so you decide to investigate how to reduce the frictional force. You are interested in determining how the kinetic frictional force depends on the normal force acting on an object. *As a first step, you decide to vary the normal force by changing the angle of the ramp.*

Read: Knight, Jones & Field, Chapter 5, sections 5.4 & 5.5

### EQUIPMENT

You have an aluminum track, wood & felt friction block, stopwatch, meter stick, balance, wood blocks, mass set, video camera and computer for analysis.



### WARM UP

1. Make a sketch of the friction block sliding down the inclined track. Draw and label vectors to indicate the direction of the velocity and the direction of the acceleration. Also assign a symbol to the mass of the block and label it on the drawing.
2. Draw a free-body diagram of the forces on the block as it slides down the ramp. Draw the acceleration vector for the block near the free-body diagram. Choose a coordinate system, and draw the force vectors on your coordinate system (a force diagram). What angles between your force vectors and your coordinate axes are the same as the angle between the ramp and the table? Determine all of the angles between the force vectors and the coordinate axes.
3. Write down Newton's 2<sup>nd</sup> law in both the x and y directions. For any forces that are at an angle to your coordinate system, be sure to consider the *components* along the x and y axes. Make sure that all of your signs are consistent. Your answer will depend on how you define your coordinate system.
4. Using the equations in step 3, determine an equation for the *normal force* in terms of quantities you know or can measure (the mass of the friction block, the angle of the track, and g).
5. Using the equations in step 3, determine an equation for the magnitude of the *kinetic frictional force* on the block in terms of quantities you know or can measure (the mass of the block, the angle of the track, g, and the acceleration of the block). How will you obtain the value of the acceleration from the video analysis software?
6. In this problem, you will change the normal force on the block by changing the angle of the track (keeping the mass of the block constant). If you increase the angle of the track, does

## NORMAL FORCE AND THE KINETIC FRICTIONAL FORCE (PART A)

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the normal force on the block increase or decrease? Use your equation for the normal force from question 4 to explain your reasoning. What happens to the kinetic frictional force?

7. The normal force and the kinetic frictional force can also be related using a coefficient of kinetic friction ( $\mu_k$ ). What is this relationship? Use the equation to sketch a graph of the magnitude of the kinetic frictional force on the block as a function of the magnitude of the normal force. How could you determine the value of  $\mu_k$  from this graph?

### PREDICTIONS

Sketch a graph of the magnitude of the kinetic frictional force on the sliding block as a function of the magnitude of the normal force.

Does the kinetic frictional force on the block **increase**, **decrease**, or **stay the same** as the normal force on the block increases? Is the relationship linear, or curved? Explain your reasoning.

### EXPLORATION

Find an angle where the block accelerates smoothly down the ramp. Try this when the block has different masses on top of it. If the block sticks, try using more mass or tilting your ramp from the table to the floor instead of just using the wooden blocks. Find a mass that allows the block to accelerate smoothly down the track for a range of angles. What measurements could you make with a meter stick to determine the angle of incline?

Decide on a position where you will release the block that fits in the frame of the camera, and will give you enough data points for the motion. Practice releasing the block from this position with your chosen mass for the block. Determine how much time it takes for the block to slide down the track and estimate the number of video points you will get in that time. Are there enough points to make the measurement? Adjust the camera position, mass of the block, or the release position to give enough data points. What will you use for a calibration object in your video?

Select a series of angles and a block mass that will make your measurements most reliable.

Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan from the Exploration section to select a block mass and series of angles that will make your measurements the most reliable. When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

Take a video of the block's motion for one angle. *Make sure you measure and record the angle of the track with uncertainty.* Analyze your data before making the next video so you can determine how many videos you need to make, and what the angle should be for each video.

Repeat this procedure with the same mass but for different angles. Make sure each new angle allows the block to move freely down the incline. Be sure to measure and record your angles with the uncertainty. Collect enough data to convince yourself and others of your conclusion about how the kinetic frictional force on the block depends on the normal force on the block.

### ANALYSIS

Using MotionLab, determine the fit functions that best represent the *position vs. time graphs* in the x and y directions. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the *velocity vs. time graphs* in the x and y directions. Compare these functions with the position vs. time functions.

Determine the acceleration as the block slides down the track for a given angle. From your answers to the Warm-up questions, calculate the magnitude of the kinetic frictional force and the normal force on the block.

Graph the magnitude of the kinetic frictional force versus the magnitude of the normal force for one block mass and the different angles you used. On the same graph, show your predicted relationship. What physical quantity does the slope of the line represent?

What are the limitations on the accuracy of your measurements and analysis? Over what range of values does the measured graph best match the predicted graph? Do the two curves ever start to diverge from one another? What does this tell you about the system and the limitations on its accuracy?

### CONCLUSION

How does the magnitude of the kinetic frictional force on an object depend on the normal force on the object? Did your measurements agree with your initial prediction? If they did not, explain why.

From your graph, determine the value of the coefficient of kinetic friction. Compare your value (with uncertainty) with values obtained by the other teams. Are they consistent? How does your value compare to the values in the table at the end of the lab?

What role does the kinetic frictional force play as the crates go sliding down the ramp? How could you change the angle of the ramp to your advantage as you unload the props?

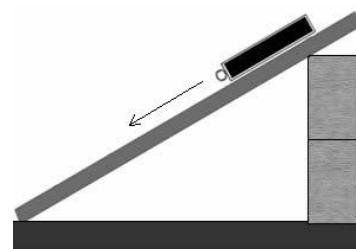
## PROBLEM #4: NORMAL FORCE AND THE KINETIC FRICTIONAL FORCE (PART B)

You have taken a job with a theater company and you are in charge of setting up the props. The props are transported in crates by a truck. The crates are unloaded by being pushed down a ramp. You realize that the frictional force is making your job difficult, so you decide to investigate how to reduce the frictional force. You are interested in determining how the kinetic frictional force depends on the normal force acting on an object. As a first step (Part A), you varied the normal force by changing the angle of the ramp. *As a second step, you decide to vary the normal force by changing the mass of the object.*

Read: Knight, Jones & Field, Chapter 5, sections 5.4 & 5.5

### EQUIPMENT

You have an aluminum track, wood & felt friction block, stopwatch, meter stick, balance, wood blocks, mass set, video camera and computer for analysis.



### WARM UP

*Note: If you have completed the earlier problem, Normal Force and the Kinetic Frictional Force (Part A), refer to your previous answers to the Warm-up for questions 1-5 and question 7.*

1. Make a sketch of the wood block sliding down the inclined track. Draw and label vectors to indicate the direction of the velocity and the direction of the acceleration. Also assign a symbol to the mass of the block and label it on the drawing.
2. Draw a free-body diagram of the forces on the block as it slides down the ramp. Draw the acceleration vector for the block near the free-body diagram. Choose a coordinate system, and draw the force vectors on your coordinate system (a force diagram). What angles between your force vectors and your coordinate axes are the same as the angle between the ramp and the table? Determine all of the angles between the force vectors and the coordinate axes.
3. Write down Newton's 2nd law in both the x and y directions. For any forces that are at an angle to your coordinate system, be sure to consider the *components* along the x and y axes. It is also important to make sure that all of your signs are correct. For example, is the acceleration of the block positive or negative? Your answer will depend on how you define your coordinate system.
4. Using the equations in step 3, determine an equation for the *normal force* in terms of quantities you know or can measure (the mass of the block, the angle of the track, and  $g$ ).
5. Using the equations in step 3, determine an equation for the magnitude of the *kinetic frictional force* on the block in terms of quantities you know or can measure (the mass of the

## NORMAL FORCE AND THE KINETIC FRICTIONAL FORCE (PART B)

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block, the angle of the track,  $g$ , and the acceleration of the block). How will you obtain the value of the acceleration from the video analysis software?

6. In this problem, you will change the normal force on the block by changing the mass of the block (keeping the angle of the track constant). If you increase the mass of the block, does the normal force on the block increase or decrease? Use your equation for the normal force from question 4 to explain your reasoning. What happens to the kinetic frictional force?
7. The normal force and the kinetic frictional force can also be related using a coefficient of kinetic friction ( $\mu_k$ ). What is this relationship? Use the equation to sketch a graph of the magnitude of the kinetic frictional force on the block as a function of the magnitude of the normal force. How could you determine the value of  $\mu_k$  from this graph?

<b>PREDICTION</b>
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Sketch a graph of the magnitude of the kinetic frictional force on the sliding block as a function of the magnitude of the normal force.

Does the kinetic frictional force on the block **increase**, **decrease**, or **stay the same** as the normal force on the block increases? Is the relationship linear, or curved? Explain your reasoning.

<b>EXPLORATION</b>
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Find an angle where the block accelerates smoothly down the ramp. Try this when the block has different masses on top of it. If the block sticks, try using more mass or tilting your ramp from table to floor instead of just using the wooden blocks. Find an angle that allows the block to accelerate smoothly down the track for a range of masses. What measurements could you make with a meter stick to determine the angle of incline?

Decide on a position where you will release the block that fits in the frame of the camera, and will give you enough data points for the motion. Practice releasing the block from this position with your chosen angle for the track. Determine how much time it takes for the block to slide down the track and estimate the number of video points you will get in that time. Are there enough points to make the measurement? Adjust the camera position, angle of the track, or the release position to give you enough data points. What will you use for a calibration object in your video?

Select a series of block masses and a single track angle that will make your measurements most reliable.

Write down your measurement plan.



## MEASUREMENT

Follow your measurement plan from the Exploration section to select a track angle and series of block masses that will make your measurements the most reliable. When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

Take a video of the block's motion for one block mass. *Make sure you measure and record the angle of the track and the block mass with uncertainty.* Analyze your data before making the next video so you can determine how many videos you need to make, and what the block mass should be for each video.

Repeat this procedure with the same angle but for different masses. Make sure each new mass allows the block to move freely down the incline. Be sure to measure and record your angle and masses with the uncertainty. Collect enough data to convince yourself and others of your conclusion about how the kinetic frictional force on the block depends on the normal force on the block.

## ANALYSIS

Using MotionLab, determine the fit functions that best represent the *position vs. time graphs* in the x and y directions. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Do the same for the *velocity vs. time graphs* in the x and y directions. Compare these functions with the position vs. time functions.

Determine the acceleration as the block slides down the track for a given angle. From your answers to the Warm-up questions, calculate the magnitude of the kinetic frictional force and the normal force on the block.

Graph the magnitude of the kinetic frictional force versus the magnitude of the normal force for one track angle and the different block masses you used. On the same graph, show your predicted relationship. What physical quantity does the slope of the line represent?

What are the limitations on the accuracy of your measurements and analysis? Over what range of values does the measured graph best match the predicted graph? Do the two curves ever start to diverge from one another? What does this tell you about the system and the limitations on its accuracy?

CONCLUSION
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Explain how the magnitude of the kinetic frictional force on an object depends on the normal force on the object. Did your measurements agree with your initial prediction? If not, explain why.

From your graph, determine the value of the coefficient of kinetic friction. Compare your value (with uncertainty) with values obtained by the other teams. Are they consistent? How does your value compare to the values in the table at the end of the lab?

What role does the kinetic frictional force play as the crates go sliding down the ramp? How could you change the angle of the ramp to your advantage as you unload the props?

If you also did the problem *Normal Force and the Kinetic Frictional Force (Part A)*, compare the results from Part A and Part B. Do you think it is better to vary the normal force by changing the angle or by changing the mass of the object? Why?

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## Table of Coefficients of Friction

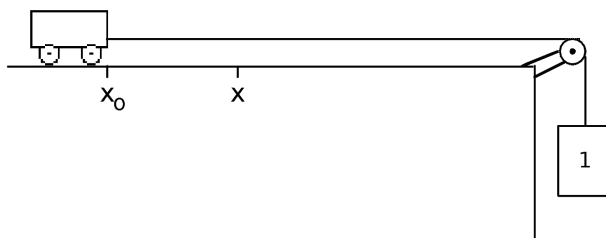
Surfaces	$\mu_s^{**}$	$\mu_k^{**}$
steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
copper on steel	0.53	0.36
steel on lead	0.9	0.9
copper on cast iron	1.1	0.3
copper on glass	0.7	0.5
wood on wood	0.25 - 0.5	0.2
glass on glass	0.94	0.4
metal on metal (lubricated)	0.15	0.07
Teflon on Teflon	0.04	0.04
rubber on concrete	1.0	0.8
ice on ice	0.1	0.03

\*\* All values are approximate.

## ☑ CHECK YOUR UNDERSTANDING

### 1101 LAB 3: FORCES

1. A cart and Block 1 are connected by a massless string that passes over a frictionless pulley, as shown in the diagram below.



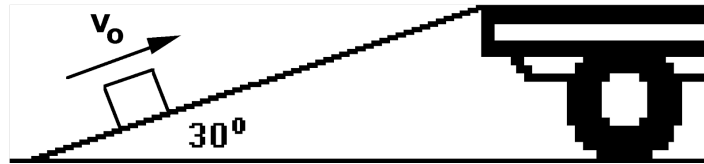
When Block 1 is released, the string pulls the cart toward the right along a horizontal table. For each question below, explain the reason for your choice.

- a. The *speed* of the cart is:
- (a) constant.
  - (b) continuously increasing.
  - (c) continuously decreasing.
  - (d) increasing for a while, and constant thereafter.
  - (e) constant for a while, and decreasing thereafter.
- b. The *force* of the string on Block 1 is
- (a) zero.
  - (b) greater than zero but less than the weight of Block 1.
  - (c) equal to the weight of Block 1.
  - (d) greater than the weight of Block 1.
  - (e) It is impossible to tell without knowing the mass of Block 1.
- c. When the cart traveling on the table reaches position  $x$ , the string breaks. The cart then
- (a) moves on at a constant speed.
  - (b) speeds up.
  - (c) slows down.
  - (d) speeds up, then slows down.
  - (e) stops at  $x$ .
- d. Block 1 is now replaced by a larger block (Block 2) that exerts *twice the pull* as was exerted previously. The cart is again reset at starting position  $x_0$  and released. The string again breaks at position  $x$ . Now, what is the *speed* of the cart at position  $x$  *compared to* its speed at that point when pulled by the smaller Block 1?
- (a) Half the speed it reached before.
  - (b) Smaller than the speed it reached before, but not half of it.
  - (c) Equal to the speed it reached before.
  - (d) Double the speed it reached before.
  - (e) Greater than the speed it reached before, but not twice as great.

## ☑ CHECK YOUR UNDERSTANDING

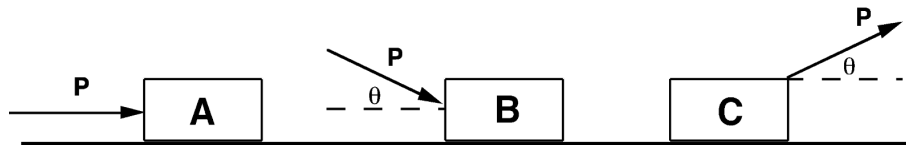
### 1101 LAB 3: FORCES

2. A crate is given an initial push up the ramp of a truck. It starts sliding up the ramp with an initial velocity  $v_0$ , as shown in the diagram below. The coefficient of kinetic friction between the box and the ramp is  $\mu_k$ .



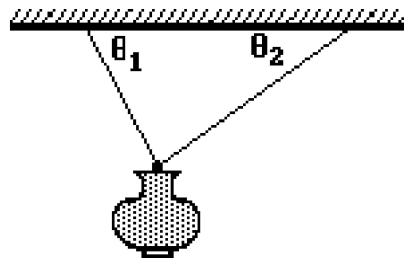
Will the magnitude of the acceleration of the sliding crate be greater on the way up or on the way back down the ramp? Or will the accelerations be the same? Explain using appropriate free-body diagrams.

3. The same constant force ( $P$ ) is applied to three identical boxes that are sliding across the floor. The forces are in different directions, as shown in the diagram below.



On which of the three boxes is the frictional force the largest? The smallest? Or is the frictional force on each box the same? Explain using appropriate free-body diagrams and Newton's second law.

4. A lamp is hanging from two light cords. The cords make unequal angles with the ceiling, as shown in the diagram at right.



- a. Draw the free-body diagram of the *lamp*. Clearly describe each force drawn.
- b. Is the horizontal component of the pull of the left cord on the lamp greater than, less than, or equal to the horizontal component of the pull of the right cord on the lamp? Explain your reasoning.
- c. Is the vertical component of the pull of the left cord on the lamp greater than, less than, or equal to the vertical component of the pull of the right cord on the lamp? Explain your reasoning.
- d. Is the vertical component of the pull of the left cord on the lamp greater than, less than, or equal to half the weight of the lamp? Explain your reasoning.

## 1101 LAB 4: CIRCULAR MOTION

The problems in this laboratory will help you investigate objects moving in uniform circular motion. This is the same motion that describes satellites in orbit around the earth, or objects whirled around on a rope.

Circular motion can be explained with the same concepts as those used in explaining projectile motion: position, velocity, acceleration, and time. Unlike projectile motion, which always has an acceleration of  $g$ , the acceleration of an object undergoing circular motion depends on its position and velocity with respect to the center of the motion.

In problems one and two, you will determine the magnitude and direction of acceleration for a rotating platform with uniform circular motion. In problems three and four, you will use acceleration and net force required for circular motion to determine the period of an object whirled horizontally by a rope. In problem five, you will explore torque and equilibrium.

### OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Determine the acceleration of an object undergoing uniform circular motion.
- Use position, velocity, acceleration, and force as vector quantities.
- Use forces to make quantitative predictions for objects in circular motion

### PREPARATION:

Read Knight, Jones & Field Chapter 3 Sections 3.8, and Chapter 6. Review your results and procedures from Laboratories I, II, and III. Before coming to the lab you should be able to:

- Determine an object's instantaneous and average velocity and acceleration from video images.
- Analyze a vector in terms of its components.
- Add and subtract vectors graphically.

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## 1101 Lab 4 Problem 1: CIRCULAR MOTION AND ACCELERATION (PART A)

You have been appointed to an amusement ride safety committee for the Mall of America's Nickelodeon Universe, which is reviewing the safety of a ride that consists of seats mounted on each end of a rotating steel beam. For most of the ride, the beam rotates about its center in a horizontal circle at a constant speed. One committee member insists that a person moving in a circle at constant speed is not accelerating, so there is no need to be concerned about the ride's safety. Another thinks that the person has a constant acceleration when moving at a constant speed. Yet a third argues that the person's acceleration depends on the rate of change of their velocity, not their speed. Since each component of the person's velocity changes with time, their acceleration must change with time. You decide to settle the issue by making a model of the ride and measuring the **magnitude** of the acceleration of different positions on the model when it spins at a constant speed.

Read: Knight, Jones & Field Chapter 3 Sections 3.8 (derivation), and Chapter 6 Section 6.1 to 6.2.

### EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown to the right. You also have a stopwatch, meterstick and the video analysis equipment.



### WARM UP

1. Make a drawing of the path of an object in circular motion at constant speed. On that path, use a dot to represent the object's position at time  $t_1$ . Label this point as O, and draw a vector at O to represent the magnitude and direction of the object's velocity at time  $t_1$ . Draw another dot to represent the object's position at a later time  $t_2$ , shortly after  $t_1$ , and label this point P. Draw a vector at P to show the magnitude and direction of the object's velocity at time  $t_2$ .
2. Redraw the velocity vectors with the tail of one vector (point P) at the tail of the other vector (point O). Keep the same size and direction as in the previous drawing. To find the acceleration of the object, you are interested in the change in velocity ( $\Delta v$ ). The change  $\Delta v$  is the increment that must be added to the velocity at time  $t_1$  so that the resultant velocity has the new direction after the elapsed time  $\Delta t = t_2 - t_1$ . Add the change in velocity  $\Delta v$  to your drawing of the velocity vectors; it should be a straight line connecting the heads of the vectors.
3. On your drawing from question 1, label the distance  $r$  from the center of the circle to points O and P. In the limit that the time interval is very small, the arc length distance traveled by the object can be approximated as a straight line. Use this approximation to label the

## CIRCULAR MOTION AND ACCELERATION (PART A)

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distance traveled by the object along the circle from point O to P in terms of the object's velocity and the elapsed time.

4. The triangle drawn in question 2 (with  $v$  and  $\Delta v$ ) is similar to the triangle drawn in question 3 (with  $r$  and the straight line distance traveled by the object) because they have the same apex angle. Use the relationship of similar triangles to write an equation that connects the sides and the bases of the two triangles.
5. Solve your equation for  $\Delta v/\Delta t$  to get an expression for the acceleration in terms of the object's uniform velocity and the distance  $r$ .
6. From your equation, is the acceleration of an object in circular motion ever zero? Does the magnitude of the acceleration change with time?

### PREDICTION

Does an object moving in a circle accelerate? If so, does the *magnitude* of the acceleration change with time? Explain your reasoning. Use the acceleration equation you derived in the Warm-up to support your claim.

### EXPLORATION

Attach the metal platform to the A-frame base and practice spinning it at different speeds. How many rotations does the platform make before it slows down appreciably? Use the stopwatch to measure the total time. Determine which spin gives the closest approximation to constant speed. At that speed, how many video frames will you get for one rotation? Will this be enough to calculate the acceleration as a function of time?

Check to see if the rotating platform is level. Place the apparatus on the floor and adjust the camera tripod so that the camera is directly above the center of the rotating platform.

Practice taking some videos. Choose a position on the platform to represent a person on the spinning ride. How will you make sure that you always click on this same position on the platform when acquiring data?

Decide how to calibrate your video. Where would you put your origin?

### MEASUREMENT

Obtain position and velocity data for a specific point on the platform as it spins. Your video should consist of more than two complete rotations. Does the initial position of the rotating platform in your video affect your data? Measure the distance from the center of the platform to rider position with a ruler.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of the total distance the object travels and the total time to determine the maximum and minimum values for each axis.



ANALYSIS
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Choose a function to represent the graph of horizontal position versus time and another for the graph of vertical position versus time. Can you determine any of the constants from your graph? You can waste a lot of time if you just try to guess the constants in your equations. How can you tell when a complete rotation occurred from each graph? *Hint: Think about what functions might match the general shape of your graph. Are the data linear, or curved? Try some of the menu options. If you still have trouble choosing a function, ask your TA for more hints.*

Similarly, choose a function to represent the graph of horizontal velocity versus time and another for the graph of vertical velocity versus time.

**Export your data to a spreadsheet. MotionLab will have the data available for export inside the .txt file generated once you save your session.**

The exported data should include horizontal and vertical positions you acquired, and the time stamp. What is a relationship between velocity and position? Make two new columns in your spreadsheet, and use this relationship to calculate the x and y components of the velocity for each pair of successive position measurements.

How can you determine the magnitude of the velocity from the x and y components of the velocity? Make a new column in your spreadsheet of the data that includes the magnitude of the velocity for each point.

Use your equation from the Warm-up to calculate the magnitude of the acceleration of the object in circular motion for each point. Include this in the data table. Is the acceleration zero, or nonzero? Do the values change with time, or remain relatively constant?

Make sure you save a copy of your data, because you might need it for your lab report or the next lab problem.

CONCLUSION
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Does the magnitude of the acceleration agree with your prediction? Is it constant, or does it change with time? What will you tell the committee? State your result in the most general terms supported by your analysis. What are the limitations on the accuracy of your measurements and analysis?

## 1101 Lab 4 Problem 2: CIRCULAR MOTION AND ACCELERATION (PART B)

You have finally convinced the safety committee that a person on the spinning ride at Camp Snoopy accelerates even though the ride moves at a constant speed. The next step for the Committee is to determine the **direction** of the acceleration and thus the direction of the net force on a person so that they can complete their safety proposal.

Read: Knight, Jones & Field Chapter 3 Sections 3.8 (derivation), and Chapter 6 Section 6.1 to 6.2.

### EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown to the right. You also have a stopwatch, meterstick and the video analysis equipment.



### WARM UP

1. Make a drawing of the path of an object in circular motion at constant speed. On that path, use a dot to represent the object's position at time  $t_1$ . Label this point as O, and draw a vector at O to represent the magnitude and direction of the object's velocity at time  $t_1$ . Draw another dot to represent the object's position at a later time  $t_2$ , shortly after  $t_1$ , and label this point P. Draw a vector at P to show the magnitude and direction of the object's velocity at time  $t_2$ .
2. Redraw the velocity vectors with the tail of one vector (point P) at the tail of the other vector (point O). Keep the same size and direction as in the previous drawing. The change  $\Delta v$  is the increment that must be added to the velocity at time  $t_1$  so that the resultant velocity has the new direction after the elapsed time  $\Delta t = t_2 - t_1$ . Add the change in velocity  $\Delta v$  to your drawing of the velocity vectors; it should be a straight line connecting the heads of the vectors.
3. Recalling the relationship between change in velocity and acceleration, construct a vector that represents the direction and magnitude of the *average acceleration* between the pair of velocities. Would the direction of the acceleration be different for very close points on the object's path?
4. Repeat steps 1-3 for two different neighboring positions on the object's circular path. Is the direction of the acceleration for this pair of velocities the same, or different as before? What can you conclude (in general) about the direction of acceleration?

### PREDICTION

Determine the direction of the acceleration for an object rotating in a circle at a constant speed. Explain your reasoning.

### EXPLORATION

*If you have already done the problem **Circular Motion and Acceleration (PartA)**, you can use that video and data and move on to the analysis.*

Attach the metal platform to the A-frame base and practice spinning it at different speeds. How many rotations does the platform make before it slows down appreciably? Use the stopwatch to measure the total time. Determine which spin gives the closest approximation to constant speed. At that speed, how many video frames will you get for one rotation? Will this be enough to calculate the acceleration as a function of time?

Check to see if the rotating platform is level. Place the apparatus on the floor and adjust the camera tripod so that the camera is directly above the center of the rotating platform.

Practice taking some videos. Choose a position on the platform to represent a person on the spinning ride. How will you make sure that you always click on this same position on the platform when acquiring data?

Decide how to calibrate your video. Where would you put your origin?

### MEASUREMENT

*If you have already done the problem **Circular Motion and Acceleration (PartA)**, you can use those measurements and move on to the analysis.*

Obtain position and velocity data for a specific point on the platform as it spins. Your video should consist of more than two complete rotations. Does the initial position of the rotating platform in your video affect your data? Measure the distance from the center of the platform to rider position with a ruler.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of the total distance the object travels and the total time to determine the maximum and minimum values for each axis.

### ANALYSIS

Use the spreadsheet data values you exported to determine the approximate velocity vector components at each position of the object's motion. Why are the velocity vectors approximate? Make a large drawing of the motion of the object, labeling the position and velocity components for a few of the consecutive values. Show the direction of the velocity at these

points. You should use as many dots as needed to convince yourself and others of the direction of the velocity of the object.

Use the velocity vectors (and the change in the velocity vectors) to determine the approximate direction of the acceleration vectors at each position. Why are the acceleration vectors approximate? You should use as many dots as needed to convince yourself and others of the direction of the acceleration of the object.

<b>CONCLUSION</b>
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What is the direction of the acceleration for an object rotating with a constant speed? What will you tell your committee members? How fast can the ride spin before harming the riders? Is this answer consistent with your prediction? Why or why not?

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## 1101 Lab 4 Problem 3: ROTATIONAL PERIOD AND FORCE (PART A)

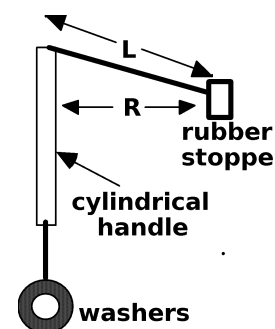
Another popular amusement ride consists of seats attached to ropes which are then whirled in a circle. As a member of the safety committee you are asked to determine the relationship between the force exerted by the rope to keep the riders rotating approximately horizontal to the ground and the period of rotation. Specifically, you must determine how the force required keeping an object rotating at a constant speed changes depending on the object's rotational period. This is an essential study because it will help determine how fast the rotation can be without snapping the ropes.

Read: Knight, Jones & Field Chapter 6 Section 6.2 & 6.3.

### EQUIPMENT

You have a string that passes through a cylindrical handle. One end of the string is attached to a rubber stopper and the other end is attached to a hanging washer. By gently rotating the vertical handle, you can make the rubber stopper move with a constant speed in a horizontal circle around the handle. You also have a stopwatch, a meter stick, and a triple-beam balance.

In this lab problem you will have several washers available to vary the hanging mass on the string.



### WARM-UP

1. Make a sketch of the problem situation similar to the one in the Equipment section. Indicate the path taken by the rubber stopper. In this case you may want to make two pictures: a top view and a side view. Label the length of the string between the top of the cylinder and the rotating stopper, the mass of the rubber stopper and hanging washer(s), and the velocity and acceleration vectors of the stopper.
2. Because gravity pulls downward on the stopper, the string slopes slightly downward in the picture. For simplicity, in this problem you can assume the string is approximately parallel to the ground. (The vertical forces on the stopper are small enough in comparison to the horizontal force(s) to be neglected.) Draw a new side view picture with the stopper moving purely horizontally.
3. Draw separate free-body diagrams of the forces on the stopper and the forces on the hanging washer(s) while the stopper is moving horizontally. What assumptions, if any, are you making? Assign symbols to all of the forces, and define what they represent next to your diagrams. For easy reference, it is useful to draw the acceleration vector for the object next to its free-body diagram. It is also useful to put the force vectors on a separate coordinate system for each object (force diagram). Remember that on a force diagram, the origin (tail) of all vectors is at the origin of the coordinate system.
4. For *each* force diagram (one for the stopper and another one for the washers), write down Newton's 2nd law in both the x and y directions. What is the direction of the acceleration of the stopper? Your answer will depend on how you define your coordinate system.

## ROTATION PERIOD AND FORCE (PART A)

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5. Write down a relationship between the weight of the hanging washer(s) and the force acting on the stopper by the string. What is the force acting on the *string*?
6. How can you determine the stopper's centripetal acceleration from its speed? How can you determine the stopper's speed from its period? Combine these relationships with the ones in questions 4 and 5 to write an equation for the stopper's period in terms of the mass of the hanging washers ( $M$ ), the mass of the stopper ( $m$ ), and the length of the string from the handle to the stopper ( $L$ ).
7. Use the relationship from question 5 to write an equation for the force on the *string* in terms of the stopper's period of rotation ( $T$ ), the mass of the stopper ( $m$ ), and the length  $L$ . Use this equation to sketch a graph of the force on the string versus the period of rotation.

### PREDICTION

Write an equation for the period of rotation for the stopper moving at a constant speed in a nearly horizontal circle. The equation should be in terms of the mass of the washer ( $M$ ), mass of the stopper ( $m$ ), and the length of the string from handle to stopper ( $L$ ).

Determine how the force exerted on the string holding the stopper depends on the period of rotation. Use this equation to sketch a graph of the force on the string versus the period of rotation.

### EXPLORATION



**TRY NOT TO HIT YOURSELF, YOUR CLASSMATES, OR YOUR LAB INSTRUCTOR!** The rubber stopper could give someone a serious injury. Wear the safety goggles provided to protect your eyes.

Assemble the apparatus as shown in the Equipment section. While rotating the rubber stopper, the length of the string between the top of the cylinder and the rotating stopper should be held constant. Mark the string with a pen or tape to ensure this.

Hang a different number of washers from the string to see how it feels when you rotate the rubber stopper. Decide on the range of washer masses that you need to use to determine the relationship between the period of rotation and the mass of hanging washers. You may need to refer to your predicted relationship to determine the range of masses to use.

Can you measure one period of rotation accurately with a stopwatch? If not, how many rotations are necessary to accurately measure the period? For very fast rotations, you might need to use many rotations to minimize uncertainty. Try it.

### MEASUREMENT

Record the length of string between the top of the cylinder and the rotating stopper, and the mass of the rubber stopper. Include measurement uncertainties.

For a range of different hanging washers, measure the period of the rubber stopper with a stopwatch. Record your measurements of the period associated with each hanging mass in an organized way.

<b>ANALYSIS</b>
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Using your prediction equation, calculate the predicted period for each hanging mass you used.

What is the relationship between the hanging washer mass and the tension force on the string? Calculate the force on the string for each of your measured periods.

Make a graph of the force on the string versus the *measured* period of rotation for your data. On the same graph, plot the force on the string versus the *predicted* period of rotation.

<b>CONCLUSION</b>
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What are the limitations on the accuracy of your measurements?

How does the force required to keep an object rotating at a constant speed change depending on the object's rotational period? Explain your answer.

## 1101 Lab 4 Problem 4: ROTATIONAL PERIOD AND FORCE (PART B)

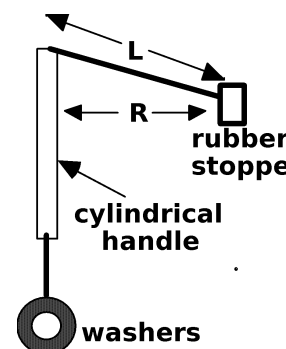
As an extension of your study in the problem, **Rotational Period and Force (Part A)**, you are now asked to determine how the period of rotation of a rider depends on the rider's mass when the radius of rotation is kept the same. This is important since obviously not all theme park visitors weigh the same.

Read: Knight, Jones & Field Chapter 6 Section 6.2 & 6.3.

### EQUIPMENT

You have a string that passes through a cylindrical handle. One end of the string is attached to a rubber stopper and the other end is attached to a hanging washer. By gently rotating the vertical handle, you can make the rubber stopper move with a constant speed in a horizontal circle around the handle. You also have a stopwatch, a meter stick, and a triple-beam balance.

In this lab problem you will have several rubber stoppers of different masses.



### WARM UP

If you have completed part A, refer to your answers to Warm-up questions 1 – 6 from the problem **Rotational Period and force (PART A)**.

1. Make a sketch of the problem situation similar to the one in the Equipment section. Indicate the path taken by the rubber stopper. In this case you may want to make two pictures: a top view and a side view. Label the length of the string between the top of the cylinder and the rotating stopper, the mass of the rubber stopper and hanging washer(s), and the velocity and acceleration vectors of the stopper.
2. Because gravity pulls downward on the stopper, the string slopes slightly downward in the picture. For simplicity, in this problem you can assume the string is approximately parallel to the ground. (The vertical forces on the stopper are small enough in comparison to the horizontal force(s) to be neglected.) Draw a new side view picture with the stopper moving purely horizontally.
3. Draw separate free-body diagrams of the forces on the stopper and the forces on the hanging washer(s) while the stopper is moving horizontally. What assumptions, if any, are you making? Assign symbols to all of the forces, and define what they represent next to your diagrams. For easy reference, it is useful to draw the acceleration vector for the object next to its free-body diagram. It is also useful to put the force vectors on a separate coordinate system for each object (force diagram). Remember that on a force diagram, the origin (tail) of all vectors is at the origin of the coordinate system.
4. For *each* force diagram (one for the stopper and another one for the washers), write down Newton's 2nd law in both the x and y directions. What is the direction of the acceleration of the stopper? Your answer will depend on how you define your coordinate system.



5. Write down a relationship between the weight of the hanging washer(s) and the force acting on the stopper by the string. What is the force acting on the *string*?
6. How can you determine the stopper's centripetal acceleration from its speed? How can you determine the stopper's speed from its period? Combine these relationships with the ones in questions 4 and 5 to write an equation for the stopper's period in terms of the mass of the hanging washers ( $M$ ), the mass of the stopper ( $m$ ), and the length of the string from the handle to the stopper ( $L$ ).
7. Use this equation to sketch a graph of the period of rotation versus the mass of the stopper.

### PREDICTION

Write an equation for the period ( $T$ ) of rotation for the stopper moving at a constant speed in a nearly horizontal circle. The equation should be in terms of the mass of the washer ( $M$ ), mass of the stopper ( $m$ ), and the length of the string from handle to stopper ( $L$ ).

Use this equation to sketch a graph of the period of rotation vs. mass of the stopper.

### EXPLORATION



**TRY NOT TO HIT YOURSELF, YOUR CLASSMATES, OR YOUR LAB INSTRUCTOR!** The rubber stopper could give someone a serious injury. Wear the safety goggles provided to protect your eyes.

Assemble the apparatus as shown in the Equipment section. While rotating the rubber stopper, the length of the string between the top of the cylinder and the rotating stopper should be held constant. Mark the string with a pen or tape to ensure this.

Decide how many washers you want to hang on the string. Make sure this number of washers enables you to produce good results for all of the different stopper masses that you will use.

Can you measure one period of rotation accurately with a stopwatch? If not, how many rotations are necessary to accurately measure the period? Try it.

### MEASUREMENT

Record the length of string between the top of the cylinder and the rotating rubber stopper you will use, and the mass of the rubber stopper. Include uncertainties.

For a range of different masses of rubber stoppers, measure the period of the rubber stopper with a stopwatch. Record your measurements of the period associated with each stopper mass in an organized way.

<b>ANALYSIS</b>
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Using your prediction equation from the Warm-up questions, calculate the *predicted* period for each stopper mass you used.

Make a graph of the *measured* period of the system vs. the mass of the rubber stopper. On the same graph, plot the *predicted* period vs. the mass of the rubber stopper.

<b>CONCLUSION</b>
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How does your predicted graph compare to the graph you found from your measurements? Explain any differences.

What is the limitation on the accuracy of your measurements? How does the period of rotation of the rubber stopper depend on its mass?

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## 1101 Lab 4 Problem 5: TORQUE AND EQUILIBRIUM

You are in a class titled Introduction to Artistic Expression, and your final project must be completed in class. You know that you will be given three objects and two boards to work with to create two separate displays. You also know that you must arrange the objects on the boards so they balance on a tiny pedestal. Once completed, one display will be a single object resting on a board balanced on top of a pedestal; the other will be two objects resting on a board balanced.

You do not know precisely what objects you will have to work with, but you decide to do the calculation for balancing the displays before class so that you have more time to think about the artistic presentation. You decide to make two models using a mass set and a meter stick in order to test where the objects should be placed in order to balance them for each setup. You will use a few masses stacked for the small pedestal.

Read: Knight, Jones & Field Chapter 7 Section 7.2 and Chapter 8 Section 8.1

### EQUIPMENT

You have a meter stick and a mass set to test out your models.

### WARM UP

#### One mass system:

1. Draw a picture for the single mass model. Select a coordinate system. Identify and label the quantities you can measure in this problem, such as masses and lengths. The unknown quantity in this problem is the locations of the mass in relation to the balance point.
2. On your picture, identify each force on your system. Draw a free-body diagram of the board that includes distances from the balance point for each force. For now, identify an arbitrary balance point for the system. (*This is OK, you will use the condition of equilibrium to make the correct choice in a bit.*)
3. Write down an expression for the net torque on the system. What is the net torque when the system is in equilibrium?
4. How many unknowns are there in your torque equation? Since we have too many unknowns to solve this equation, we need to add an additional constraint. Make the object mass twice the meter stick mass.
5. Solve the equation to find the equilibrium location of the mass.

#### Two mass system:

1. Draw a picture for the two mass system, assuming that there are two different masses involved. Select a coordinate system. Identify and label the quantities you can measure in this problem, such as masses and lengths. The unknown quantities in the problem are the locations of the masses in relation to the balance point.
2. On your picture, identify each force on your system. Draw a free-body diagram of the board that includes the distances from the balance point. For now, identify an arbitrary

## TORQUE AND EQUILIBRIUM

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balance point for the system. (*This is OK, you will use the condition of equilibrium to find how you need to correct your choice.*)

3. Write down an expression for the net torque on the system. What is the net torque when the system is in equilibrium?
4. How many unknowns are there in your torque equation? Since we have too many unknowns to solve this equation, we need to add additional constraints. For the two mass system, we decide that the masses must be placed equal distances from the 50cm mark on the meter stick. Also, select your masses so there is at least 30g difference between them. You will need to discuss with your group what these constraints will actually be so you can just consider the quantities to be 'known' as you solve the system of equations.
5. Solve these equations to find the equilibrium location of the two masses.

<b>PREDICTION</b>
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Write a formula for the equilibrium position of each mass balancing on each of the two meter sticks, in terms of masses and their distances from the balance point. Assume that all the masses are different. Identify the variables that are set by the group and the variable that is being measured.

<b>EXPLORATION</b>
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Find the balance point of just the meter stick by itself to make sure that it is in the location you expect it to be. What are you balancing the stick on? How close are you able to determine the balance point?

Experimentally find the balance point for several different configurations. Vary where the masses are located and see if you can predict where the balance point should be. For the two mass setup, remember to always place the masses equidistant from the meter stick's balance point (e.g. equal distances from the 50cm mark). What happens to the balance point when you lengthen or shorten the distance from the 50cm point?

Swap the masses on the ends and experimentally find the balance point. Does this agree with the calculated balance point? Is there a limiting factor with where you can balance the system?

With your group, decide on the two constraints in each of the situations. Work out the algebra in order to predict the three theoretical locations of the masses.

Decide how you will place the masses on the meter stick to ensure that they are at the correct location. What is your estimated uncertainty for this measurement?

<b>MEASUREMENT</b>
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Set the meter stick fully on the table and place the masses at the theoretical distances. Find the balance point of the meter stick by balancing it on top of a few stacked masses. How close is the balance point to where you predicted it to be? What is the best way to adjust the distances to experimentally find the balance point? Should you move all of the masses or just one at a time?

<b>ANALYSIS</b>
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Compare the theoretical balance point to the experimental balance point. Do they fall within the estimated uncertainty of your measurement?

<b>CONCLUSION</b>
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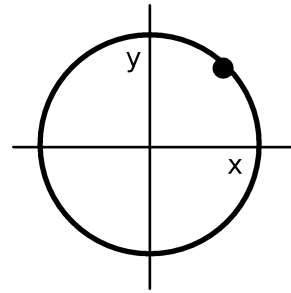
Did your model balance as designed? What corrections did you need to make to get it to balance? Were these corrections a result of some systematic error, or was there a mistake in your prediction? Justify your answer.

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## ☑ CHECK YOUR UNDERSTANDING

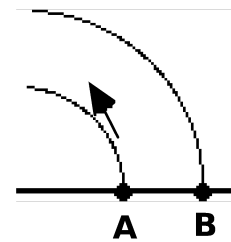
### 1101 LAB 4: CIRCULAR MOTION

1. A ball on the end of a string travels in a clockwise circle at constant speed. On the figure at right, draw the vectors requested below, label them clearly, and explain your choices.



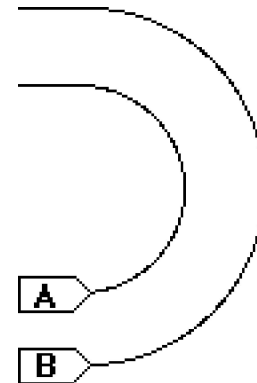
- a. The position vector for the ball.
- b. The velocity vector for the ball.
- c. The acceleration vector for the ball.

2. Two beads are fixed to a rod rotating at constant speed about a pivot at its left end, as shown in the drawing at right.



- a. Which bead has the greater speed? Explain your reasoning.
- b. Which bead has an acceleration of greater magnitude? Explain your reasoning.

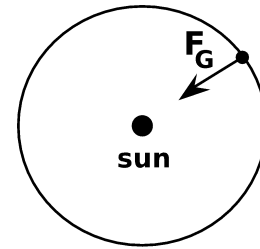
3. Two racing boats go around a semicircular turn in a race course. The boats have the same speed, but boat A is on the inside while boat B is on the outside, as shown in the drawing.



- a. Which boat gets around the turn in less time? Explain your reasoning.
- b. Which boat undergoes the greater change in velocity while in the turn? Explain your reasoning.
- c. Based on the definition of acceleration, which boat has the greater acceleration while in the turn? Explain your reasoning.
- d. Based on the equation for centripetal acceleration, which boat has the greater acceleration while in the turn? Compare your answer to part c. Explain your reasoning.

**☑ CHECK YOUR UNDERSTANDING**  
**1101 LAB 4: CIRCULAR MOTION**

4. A planet moves in a uniform circular orbit around the sun, which exerts a gravitational force  $F_G$  on the planet. What additional *force(s)* act on the planet?
- (a) A force of motion in the direction of the circular orbit.
  - (b) A centrifugal force acting outward (away from the sun).
  - (c) A centripetal force acting inward (toward the sun).
  - (d) A normal force.
  - (e) The gravitational force ( $F_G$ ) is the only force.



Explain the reason for your choice.

5. Centripetal force is simply a special name that we give to the net force that produces a centripetal acceleration. In each case listed below, identify the force, force component, or combination of forces that provides the centripetal force. Draw a force diagram for each case and discuss it.
- a. A child on a swing travels in a circular arc. Analyze the situation at the bottom of the swing.
  - b. A car travels around a circular, flat, horizontal curve.
  - c. A person stands on the equator of the earth, traveling in an earth-sized circle as the earth rotates.
  - d. A car travels in a circular curve that is banked inward.
  - e. A ball rolls inside a circular hoop that is placed on a horizontal table.
  - f. A car drives over the top of a circular hill.
  - g. A tennis ball rolls without slipping over the top of a basketball

## 1101 LAB 5: SPRING FORCES

Most of the laboratory problems so far have involved constant force being independent of both time and position. In this laboratory, you will explore force that is dependent on its position. While there are various mathematical approaches to deal with this situation, this laboratory will only address static situations.

### OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Use a static approach to determine the spring constant of a spring

### PREPARATION:

Read Knight, Jones & Field Chapter 8 Section 8.3.

Before coming to lab you should be able to:

- Determine the force on an object exerted by a spring using the concept of a spring constant.

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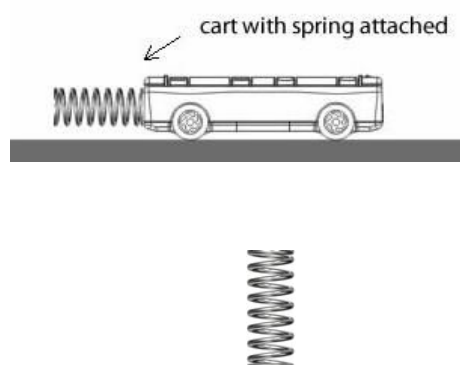
## 1101 Lab 5 Problem 1: MEASURING SPRING CONSTANTS

You are selecting replacement springs for use in a large antique clock. Two kinds of springs are needed to make it work, both compression springs and stretching springs. In order to determine the force that they exert when stretched (or compressed), you need to know their spring constants. A book recommends a static approach, in which objects of different weights either stretch or compress the spring and the displacement from equilibrium is measured. You wish to determine if this static approach yields the same kind of relationship for the spring constant for both types of springs. You decide to take both compression and stretching measurements on the respective springs over a range of weights and then compare.

Read Knight, Jones & Field Chapter 8 Section 8.3.

### EQUIPMENT

You have a stand-alone compression spring, a compression spring affixed to a cart and extension springs available. You also have a table clamp, metal rod, mass set, and meterstick.



**DO NOT STRETCH THE COMPRESSION SPRINGS, THEY EASILY DEFORM. DO NOT STRETCH THE STRETCHING SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 40 CM) OR YOU WILL DAMAGE THEM.**

### WARM UP

1. Make two pictures for each spring, which will give four pictures in all. For the extension spring make one before you suspend an object, and one after an object is suspended and the spring is at rest. For the compression spring make one before you place an object on it, and one after the spring is at rest supporting an object. Draw a coordinate system. On each picture, label the position where the spring is unstretched or uncompressed, the distance from the unstretched or uncompressed position to the stretched or compressed position, the mass of the object, and the spring constant.
2. Draw two force diagrams: one for an object hanging from a spring at rest and one for an object compressing the spring at rest. Label the forces acting on the object. Use Newton's second law to write the equation of equilibrium for the object.
3. Solve the equation for the spring constant in terms of the other values in the equation. What does this tell you about the slope of a displacement (from the unstretched or uncompressed position) versus weight of the object graph?

### PREDICTION

Write an expression for the relationship between the spring constant and the displacement of an object hanging from a spring. Do the same for the displacement of the spring when the

## MEASURING SPRING CONSTANTS

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object is compressing the spring. What is the expected relationship between the applied force and the displacement of the spring?

### EXPLORATION

Select a series of masses that give a usable range of displacements for each spring type. The largest mass should not push or pull the spring past its elastic limit, for two reasons: (1) beyond the elastic limit there is no well-defined spring constant, and (2) a spring stretched beyond the elastic limit will be damaged.

For the extension spring, clamp the metal rod to the table, and hang the spring from the rod. Decide on a procedure that allows you to measure the distance a spring stretches when an object hangs from it in a consistent manner. Decide how many measurements you will need to make a reliable determination of the spring constant.

For the compressing spring, decide how you will stabilize the spring and balance the objects on the spring. You may consider using the cart/spring combination on an inclined plane to do this one, but you need to change your force diagram to know what force is being applied in the axis of the spring.

### MEASUREMENT

For both springs, make the measurements needed to determine the spring constant. Analyze your data as you go along so you can decide how many measurements you need to make to determine the spring constant accurately and reliably in each case.

### ANALYSIS

Graph displacement versus weight for the object-spring system for both springs. From the slope of this graph, calculate the value of the spring constant. Estimate the uncertainty in this measurement of the spring constant. Do both springs have the same relationship between weight and displacement? Do you see limitations using this method?

### CONCLUSION

For each spring, does the graph have the characteristics you predicted? Do both the compression and stretching springs exhibit the relationship between the applied force and the displacement that you expect?

## 1101 Lab 5 Problem 2: EFFECTIVE SPRING CONSTANT

Your company has bought the prototype for a new flow regulator from a local inventor. Your job is to prepare the prototype for mass-production. While studying the prototype, you notice the inventor used some rather innovative spring configurations to supply the tension needed for the regulator valve. In one location the inventor had fastened two different springs side-by-side, as in Figure A below. In another location the inventor attached two different springs end-to-end, as in Figure B below. To decrease the cost and increase the reliability of the flow regulator for mass production, you need to replace each spring configuration with a single spring. These replacement springs must exert the same forces when stretched the same amount as the original spring configurations.

Read Knight, Jones & Field Chapter 8 Sections 8.3

### EQUIPMENT

You have two different springs with the same unstretched length, but different spring constants  $k_1$  and  $k_2$ . These springs can be hung vertically side-by-side (Figure A) or end-to-end (Figure B). You will also have a meterstick, stopwatch, rod, wooden dowel, table clamp and mass set.

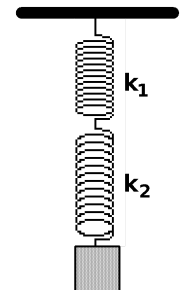
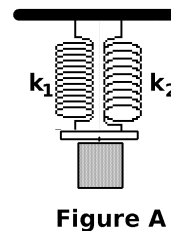


Figure B

### WARM UP

*Apply the following warm-up to the side-by-side configuration, and then repeat for the end-to-end configuration:*

1. Make a picture of the spring configuration similar to each of the drawings in the Equipment section (Figure A and Figure B). Draw a coordinate system. Label the positions of each unstretched spring, the final stretched position of each spring, the two spring constants, and the mass of the object suspended. Put arrows on your picture to represent any forces on the object. Assume that the springs are massless.

For the side-by-side configuration, assume that the light bar attached to the springs remains horizontal (i.e. it does not twist).

For each two spring configurations make a second picture of a single (massless) spring with spring constant  $k'$  that has the same object suspended from it and the same total stretch as the combined springs. Be sure to label this picture in the same manner as the first.

2. Draw force diagrams of both spring systems and the equivalent single spring system. Label the forces. For the end-to-end configuration, draw an additional force diagram of a point at the connection of the two springs.
3. Apply Newton's laws to the object suspended from the combined springs and the object suspended from the single replacement spring. Consider carefully which forces and displacements will be equal to each other

## EFFECTIVE SPRING CONSTANT

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For the end-to-end configuration: Draw an additional force diagram for the connection point between the springs. At the connection point, what is the force of the top spring on the bottom spring? What is the force of the bottom spring on the top spring?

4. Solve your equations for the effective spring constant ( $k'$ ) for the single replacement spring in terms of the two spring constants.

### PREDICTION

The spring constant for a single spring that replaces a configuration of springs is called its *effective spring constant*.

1. Write an expression for the effective spring constant for a side-by-side spring configuration (Figure A) in terms of the two spring constants  $k_1$  and  $k_2$ .
2. Write an expression for the effective spring constant for an end-to-end spring configuration (Figure B) in terms of the two spring constants  $k_1$  and  $k_2$ .

Is the effective spring constant larger when the two springs are connected side-by-side or end-to-end? Explain your reasoning.

### EXPLORATION

To test your predictions, you must decide how to measure each spring constant individually and the effective spring constants of the side-by-side and end-to-end configurations.

Perform an exploration consistent with your selected method from the earlier problem **Measuring Spring Constants**. Remember that the smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 40 CM) OR YOU WILL DAMAGE THEM.**

Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan to take the necessary data. What are the uncertainties in your measurements?

### ANALYSIS

Determine the effective spring constants (with uncertainties) of the side-by-side spring configuration and the end-to-end spring configuration.

Determine the spring constants of the two springs. Calculate the effective spring constants (with uncertainties) of the two configurations using your Prediction equations.

CONCLUSION
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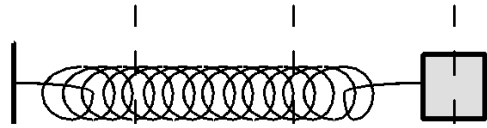
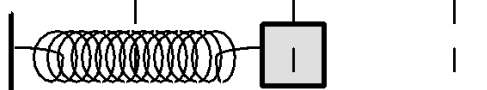


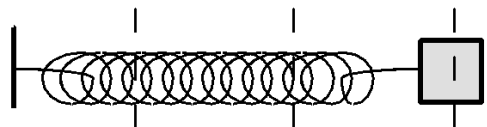
How do the measured values and predicted values of the effective spring constant for the configurations compare?

What are the effective spring constants of a side-by-side spring configuration and an end-to-end spring configuration? Which is larger? Did your measured values agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis? Can you apply what you learned to find the spring constant of a complex system of springs in the flow regulator?

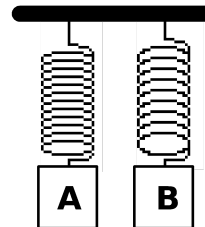
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**✓ CHECK YOUR UNDERSTANDING**  
**1101 LAB 5: SPRING FORCES**

1. The diagram below shows an oscillating mass/spring system at times 0,  $T/4$ ,  $T/2$ ,  $3T/4$ , and  $T$ , where  $T$  is the period of oscillation. For each of these times, write an expression for the displacement ( $x$ ), the velocity ( $v$ ), the acceleration ( $a$ ), the kinetic energy (KE), and the potential energy (PE) *in terms of the amplitude of the oscillations ( $A$ ), the angular velocity ( $\omega$ ), and the spring constant ( $k$ )*.

						
						
						
						
						
	$-x_{\min}$ equilibrium position $+x_{\max}$					
$t$	$x$	$v$	$a$	KE	PE	
0						
$\frac{T}{4}$						
$\frac{T}{2}$						
$\frac{3T}{4}$						
$T$						

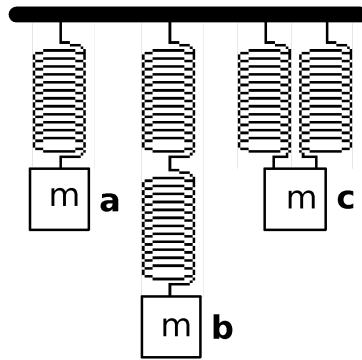
2. Two different masses are attached to different springs that hang vertically. Mass A is larger, but the period of simple harmonic motion is the same for both systems. They are pulled the same distance below their equilibrium positions and released from rest.



- Which spring has the greater spring constant? Explain.
- Which spring is stretched more at its equilibrium position? Explain.
- The instant after release, which mass has the greater acceleration? Explain.
- If potential energy is defined to be zero at the equilibrium position for each mass, which system has the greater total energy of motion? Explain.
- Which mass will have the greater kinetic energy as it passes through its equilibrium position? Explain.
- Which mass will have the greater speed as it passes through equilibrium? Explain.

✓ **CHECK YOUR UNDERSTANDING**  
**1101 LAB 5: SPRING FORCES**

4. Five identical springs and three identical masses are arranged as shown at right.
- Compare the stretches of the springs at equilibrium in the three cases. Explain.
  - Which case, a, b, or c, has the greatest effective spring constant? The smallest effective spring constant? Explain.



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## 1101 LAB 6: IMPULSE AND MOMENTUM

In this lab you will use the *conservation of momentum* to predict the motion of objects resulting from collisions. While it is often difficult or impossible to completely analyze a collision in terms of the forces. However, conservation principles can be used to relate the motion of objects before and after a collision, without a detailed knowledge of the collision process. Both the conservation of momentum and energy are usually required to do this.

### OBJECTIVES:

Successfully completing this laboratory should enable you to:

- Use the conservation of momentum to predict the outcome of interactions between objects.
- Choose a useful reference system when using conservation of momentum.
- Identify the momentum transfer (impulse) when applying energy and momentum conservation to real systems.
- Use the principles of conservation of energy and momentum together to describe the behavior of systems.

### PREPARATION:

Read Knight, Jones & Field Chapter 9. You should also be able to:

- Analyze the motion of an object using video analysis tools.
- Calculate the kinetic energy of a moving object.
- Calculate the total energy and total momentum of a system of objects.

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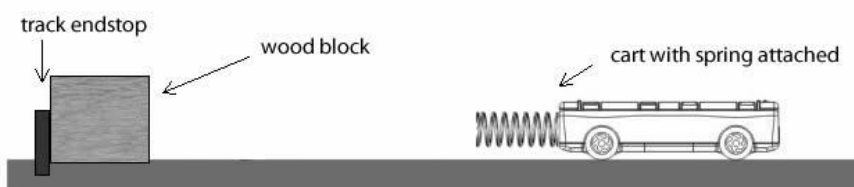
## 1101 Lab 6 Problem 1: IMPULSE AND CHANGING MOMENTUM

Your 15 year old is about to get their driving permit, and you are concerned about the bumpers on your cars as they are expensive to fix, even after low speed impacts. You decide you want to engineer a true “5 mph” bumper that will slow a collision with a fixed object using a spring attachment, which will avoid damaging the car’s actual bumper. You find out that springs with high spring constant values are very expensive so you decide to first investigate using a smaller and much cheaper setup. You decide to model the situation using a cart on a track with a spring attached to one end and a fixed endstop. You will use this simple setup to first determine the best method to pick the expensive springs for the cars.

Read Knight, Jones & Field Chapter 9 Sections 9.1 to 9.3 (review Section 8.3 for Hooke’s law)

### EQUIPMENT

You have a cart with a spring attached, a track with end stop, wood block and a computer with video analysis software.



### WARM UP

*Note: In this lab, you are assuming that two values are equal and you measure them to see if they really are equal.*

The first question deals with the changing momentum of the system.

1. Draw two pictures of the cart: one before the cart hits the end stop and another one after it has bounced off and is no longer in contact with the spring. Label all kinematic quantities and constants in the system. Use the conservation of momentum to write relationship between the motion before and after the collision. What variables in this relationship are measurable with the equipment you have access to?

The remainder of the questions deals with the impulse of the collision.

2. Draw at least four pictures of the cart during the collision with the spring and the end stop including two pictures when the spring is being compressed and two as it expands. Include in each picture the amount of compression in the spring and the direction of the force from the spring on the cart.
3. Write down the relationship of how the force of the spring and the compression of the spring are related in each case. *Be sure to name each force something unique ( $F_1$ ,  $F_2$ , etc.).* What, in this relationship, are measurable quantities with the equipment you have access to?

## IMPULSE AND CHANGING MOMENTUM

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4. Using the four pictures, assume that the time between each picture is equal and that the force in the picture is constant until the next picture. Graph the force of the spring versus time for the duration of the collision.
5. Find the total impulse, by adding together all of the individual areas under the curve in the force versus time graph. Do you expect this estimation to be greater or less than the actual impulse?
6. What are the assumptions made for this model?

<b>PREDICTION</b>
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Using the changing momentum as your 'theoretical' calculation, how do you expect the impulse to compare to the changing momentum?

Do you expect the time over which the collision occurs to affect how well the impulse agrees with the change in momentum?

<b>EXPLORATION</b>
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**Be very careful with the springs attached to the ends of the carts! Do not pull on them or bend them side to side as they cannot be reattached.**

Try varying the mass of the cart to see how that affects the length of the collision time. Does varying the mass increase or decrease the collision time? Does varying the incoming speed of the cart affect the collision time? Which one has a greater effect? Decide if you would like to minimize or maximize the collision time. Given the assumptions of the problem, which do you think would give more accurate results? *Hint: it is best to maximize the number of data points with the spring in contact with the endstop.*

Think about what quantities you need to obtain from the video and what resolution you will need in the video. Be sure that you will be able to see all the interactions necessary in the video.

Once you have found an acceptable speed and mass of the cart, record a video.

Write down your measurement plan for finding the impulse of the cart as it relates to 1) the changing momentum of the cart and 2) the force over time from the spring. Be sure to include your procedure for finding the spring constant.

If you are unable to complete the procedure for finding the spring constant for your cart during the time allotted, you should assume a value of 355 N/m.

<b>MEASUREMENT</b>
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Carry out your measurement plan. Make sure that your video is clear enough to get both the initial and final velocities of the cart and the compression of the spring in each frame of contact with the endstop.

Think about the quantities that you need to measure and the most efficient way to make these measurements. You will be able to skip many of the 'typical' analysis steps in the MotionLab program as you are only using it to acquire data and not predict behavior.

Discuss how to use the analysis software to find the impulse as it relates to the change in momentum of the cart.

Discuss how to use the procedure from Warm-up questions 4 & 5 and the video of the collision to find the impulse as it relates to the force over time.

<b>ANALYSIS</b>
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How do the two different methods of finding impulse compare? Which method gives a larger value? Is this what you were expecting? Were the assumptions made for this model reasonable or unreasonable for the situation? Do you see a difference between your collision time measurements and another group's collision time measurement?

<b>CONCLUSION</b>
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Did this model provide a sufficient answer to the kind of spring you should purchase? Which impulse calculation would you be doing for this scenario- the force over time or the change in momentum? Which do you think is a better estimate of the actual impulse?

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## 1101 Lab 6 Problem 2: PERFECTLY INELASTIC COLLISIONS

You have a summer job at NASA with a group designing a docking mechanism that would allow two space shuttles to connect with each other. The mechanism is designed for one shuttle to move carefully into position and dock with a stationary shuttle. Since the shuttles may be carrying different payloads and have consumed different amounts of fuel, their masses may not be identical: the shuttles could be equally massive, the moving shuttle could be more massive, or the stationary shuttle could have a larger mass. Your supervisor wants you to calculate the magnitude and direction of the velocity of the pair of docked shuttles as a function of the initial velocity of the moving shuttle and the mass of each shuttle. You may assume that the total mass of the two shuttles is constant. You decide to model the problem in the lab using carts to check your predictions.

Read: Knight, Jones & Field Chapter 9 Sections 9.2 to 9.5.

### EQUIPMENT

You have a track, set of carts, cart masses, meterstick, stopwatch, two endstops and video analysis equipment. For this problem, cart A is given an initial velocity towards a stationary cart B. Velcro at the end of each cart allow the carts to stick together after the collision.



### WARM UP

1. Draw two pictures, one showing the situation before the collision and the other one after the collision. Is it reasonable to neglect friction? Label the mass of each cart, and draw velocity vectors on each sketch. Define your system. If the carts stick together after the collision, what must be true about their final velocities?
2. Write a momentum conservation equation for this situation and identify all of the terms in the equation. Are there any of these terms that you cannot measure with the equipment at hand?
3. Solve your conservation equation for the magnitude of the final velocity of the carts in terms of the cart masses and the initial velocity of cart A. What direction is the final velocity of the carts when  $m_A = m_B$ ? When  $m_A > m_B$ ? When  $m_A < m_B$ ?

### PREDICTION

Write an equation for the final velocity of the stuck-together carts in terms of the cart masses and the initial velocity of cart A. What will be the direction of the final velocity?

Consider the following three cases in which the total mass of the carts is the same ( $m_A + m_B = \text{constant}$ ), where  $m_A$  is the moving cart, and  $m_B$  is the stationary cart:

(a)  $m_A = m_B$

(b)  $m_A > m_B$

(c)  $m_A < m_B$

## PERFECTLY INELASTIC COLLISIONS

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In which case will the final velocity of the carts be the largest? The smallest? Explain your reasoning. Does your answer depend on the initial velocity of cart A?

### EXPLORATION

Practice setting the cart into motion so the carts stick together with Velcro after the collision. Try various initial velocities and observe the motion of the carts.

Vary the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Be sure the carts still move freely over the track.

Select the cart masses you will use for  $m_A = m_B$ ,  $m_A > m_B$ , and  $m_A < m_B$  so each situation uses the same total mass. Determine what initial velocity you will give cart A for each case. Use a stopwatch and meter stick to practice giving cart A these initial velocities.

Set up the camera and tripod to give you the best video of the collision immediately before and after the carts collide. What will you use for a calibration object in your videos? What quantities in your prediction equations do you need to measure with the video analysis software? Is it possible to obtain information before and after the collision with one video analysis, or will you need to analyze each video more than once?

Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan from the Exploration section. Record a video of one collision situation. Use a stopwatch and the distance traveled by the cart before impact with the bumper to estimate the initial velocity of the cart.

Open one your video in MotionLab and follow the instructions to acquire data. As a lab group, decide how you will acquire data and analyze the collision. (Will you acquire data for the cart A's motion before the impact and repeat the process for cart A and B after the collision, or will you acquire data for the entire motion of the carts in a single analysis?) Repeat this process for the remaining two collision situations.

Measure and record the masses of the two carts for each situation. Analyze your data as you go along (before making the next video), so you can determine if your initial choice of masses and speeds is sufficient. Collect enough data to convince yourself and others of your conclusions about the efficiency of the collision.

### ANALYSIS

From your videos, determine the velocities of the carts before and after the collision for each situation. Calculate the momentum of the carts before and after the collision. Use your

equation from the Warm-up and Prediction questions to calculate the *predicted* final velocity of the stuck-together carts.

Record the measured and calculated values in an organized data table in your lab journal.

<b>CONCLUSION</b>
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How do your measured and predicted values of the final velocities compare? Compare both magnitude and direction. What are the limitations on the accuracy of your measurements and analysis?

When a moving shuttle collides with a stationary shuttle and they dock (stick together), how does the final velocity depend on the initial velocity of the moving shuttle and the masses of the shuttles? State your results in the most general terms supported by the data.

What conditions must be met for a system's *total momentum* to be conserved? Describe how these conditions were or were not met for the system you defined in this experiment.

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## 1101 Lab 6 Problem 3: EXPLOSIONS

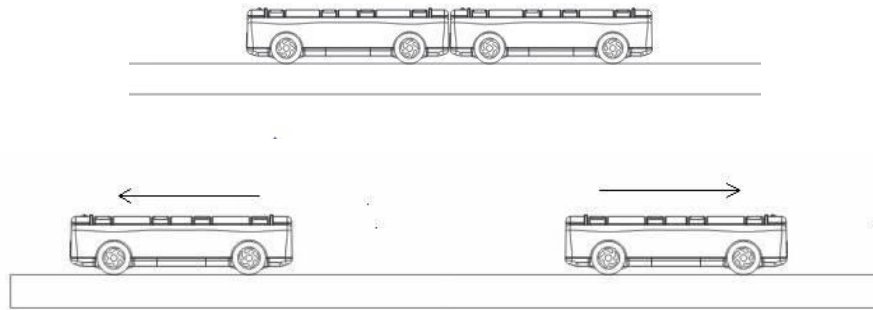
You have a summer job helping a local ice-dancing group prepare for their season. One routine begins with two skaters standing stationary next to each other. They push away from each other and glide to opposite ends of the ice rink. The choreographer wants them to reach the ends simultaneously. Your assignment is to determine where the couple should stand. To test your ideas you build a model of the situation. Your initial calculation assumes that the frictional force between the ice and the skates can be neglected so you decide to use a metal track and two carts of different mass to test it. Specifically, when the carts push each other apart, you want to find where they should start if they are to reach the ends of the track simultaneously.

Read: Knight, Jones & Field Chapter 9 Sections 9.1 to 9.5; also review motion in one dimension in Chapter 2.

### EQUIPMENT

You have a meterstick, stopwatch, an aluminum track, endstops, carts, and a variety of masses to add to the carts.

A small button near one end of a cart releases a plastic arm that can provide the initial push between the two carts.



### WARM UP

1. Draw a picture that shows the position of the carts when they are stationary. Label the cart masses, the cart lengths, the total length of the track between the end stops, and the distances that each cart must travel to hit the ends of the track from their starting position.
2. Draw another picture that shows the situation just after the carts have pushed off from one another. Label the velocities of the carts.
3. Define your system. Write down the conservation of energy equation for this situation. Identify all of the terms in the equation. Where does the energy come from that “explodes” the carts apart? Are there any terms that you do not know and cannot directly measure?
4. Write down the conservation of momentum equation for this situation. Identify all of the terms in the equation. Are there terms that you do not know and cannot directly measure?

5. Decide which conservation principle will be most useful in this situation. Write down your reason for this decision. Are both useful? Use your conservation equation(s) to determine the relationship between the speeds of the carts.
6. What is an equation that relates horizontal distance, velocity, and time? Write down an expression for the time cart A takes to reach the end of the track in terms of the distance it travels and its velocity. (*Hint: to make the calculation easier, you can treat the car as a point mass, ignoring the length of the cart*) Write down an expression for the time cart B takes to reach the end of the track in terms of the distance it travels and its velocity. What must be true of the time for cart A and cart B, if they reach the track ends simultaneously?
7. Write down a relationship between the distance traveled by cart A, the distance traveled by cart B, and the total length of the track.
8. Use your equations from questions 5, 6, and 7 to solve for the initial position of the carts before the explosion in terms of their masses and the total length of the track. How does neglecting the length of the carts affect your answer?

<b>PREDICTION</b>
-------------------

Calculate a formula for the starting position of the two stationary carts as a function of their masses, and the total distance between the ends of the track if they are to reach the ends of the track simultaneously.

*Hint: To make the math easier, you can treat the carts like point masses in your calculation. How does ignoring the cart length affect your result?*

<b>EXPLORATION</b>
--------------------

Position the carts next to each other on the track and let the side with the tip button of one cart close to the other cart, so that when the button is pushed, the pop up arm can provide the initial push for the two carts. Position the end stops on the aluminum track. How will you tell if the carts hit the end stops at the same time?

Practice pushing the tip button so that your finger will not prevent the cart with the button from moving freely right after you press the button. What is the best way to push the button? Will the contact between your finger and the button affect the motion of the carts? Try it. Make sure the carts move along the track smoothly after you push the button.

Try varying the masses of the carts while keeping the total mass of the carts the same. Be sure the carts still move freely over the track. What masses will you use in your final measurement? Determine a range of cart masses that will give you reliable results.

Write down your measurement plan.



---

**MEASUREMENT**

Position the carts on the track so that when the tip button on one cart is pushed, both carts hit the ends at the same time. Record this position and collect enough data so that you can convince yourself and others that you can predict where the carts should start for any reasonable set of masses.

**ANALYSIS**

Where did you need to place the carts so they hit the end at the same time for each case? What is the uncertainty in this measurement?

Use your equation from the Prediction and Warm-up questions to calculate the *predicted* starting position from the cart masses and the total distance between the end stops for each trial.

**CONCLUSION**

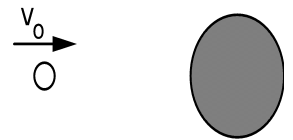
How did your measured starting position for the carts compare to your predicted position? What are the limitations on the accuracy of your measurement and analysis?

What will you tell the choreographer? Can you predict where the skaters should be standing for their push off? How does their starting position depend on the mass of the dancers and the length of the ice rink? Does it depend on how hard they push off? State your results in the most general terms supported by your data.

If you have time, modify your prediction equation to include the lengths of the carts. Recalculate the predicted starting position of the carts for each trial. Are the position values closer to your measured values than when you treated the carts like point masses?

**☑ CHECK YOUR UNDERSTANDING**  
**1101 LAB 6: IMPULSE AND MOMENTUM**

1. If a runner speeds up from 2 m/s to 8 m/s, the runner's *momentum* increases by a factor of
  - (a) 64.
  - (b) 16.
  - (c) 8.
  - (d) 4.
  - (e) 2.
  
2. A piece of clay slams into and sticks to an identical piece of clay that is initially at rest. Ignoring friction, what percentage of the initial kinetic energy goes into changing the internal energy of the clay balls?
  - (a) 0%
  - (b) 25%
  - (c) 50%
  - (d) 75%
  - (e) There is not enough information to tell.
  
3. A tennis ball and a lump of clay of equal mass are thrown with equal speeds directly against a brick wall. The lump of clay sticks to the wall and the tennis ball bounces back with one-half its original speed. Which of the following statements is (are) true about the collisions?
  - (a) During the collision, the clay ball exerts a larger average force on the wall than the tennis ball.
  - (b) The tennis ball experiences the largest change in momentum.
  - (c) The clay ball experiences the largest change in momentum.
  - (d) The tennis ball transfers the most energy to the wall.
  - (e) The clay ball transfers the most energy to the wall.
  
4. A golf ball is thrown at a bowling ball so that it hits head on and bounces back. Ignore frictional effects.
  - a. Just after the collision, which ball has the largest momentum, or are the momenta the same? Explain using vector diagrams of the momentum before and after the collisions.
  - b. Just after the collision, which ball has the largest kinetic energy, or are their kinetic energies the same? Explain your reasoning.



**☑ CHECK YOUR UNDERSTANDING**  
**1101 LAB 6: IMPULSE AND MOMENTUM**

5. A 10 kg sled moves at 10 m/s. A 20 kg sled moving at 2.5 m/s has
- (a) 1/4 as much momentum.
  - (b) 1/2 as much momentum.
  - (c) twice as much momentum.
  - (d) four times the momentum.
  - (e) None of the above.
6. Two cars of equal mass travel in opposite directions with equal speeds on an icy patch of road. They lose control on the essentially frictionless surface, have a head-on collision, and bounce apart.



- a. Just after the collision, the velocities of the cars are
- (a) zero.
  - (b) equal to their original velocities.
  - (c) equal in magnitude and opposite in direction to their original velocities.
  - (d) less in magnitude and in the same direction as their original velocities.
  - (e) less in magnitude and opposite in direction to their original velocities.
- b. In the type of collision described above, consider the system to consist of both cars. Which of the following can be said about the collision?
- (a) The kinetic energy of the system does not change.
  - (b) The momentum of the system does not change.
  - (c) Both momentum and kinetic energy of the system do not change.
  - (d) Neither momentum nor kinetic energy of the system change.
  - (e) The extent to which momentum and kinetic energy of the system do not change depends on the coefficient of restitution.
7. Ignoring friction and other external forces, which of the following statements is (are) true just after an arrow is shot from a bow?
- (a) The forward momentum of the arrow equals that backward momentum of the bow.
  - (b) The total momentum of the bow and arrow is zero.
  - (c) The forward speed of the arrow equals the backward speed of the bow.
  - (d) The total velocity of the bow and arrow is zero.
  - (e) The kinetic energy of the bow is the same as the kinetic energy of the arrow.

## 1101 LAB 7: ENERGY

In this lab, you will begin to use the principle of *conservation of energy* to determine the motion resulting from interactions that are difficult to analyze using force concepts alone. Keep in mind that **energy is always conserved**, it but it is sometimes difficult to calculate the value of all of the energy terms for an interaction.

Not all of the initial energy of a system ends up as visible energy of motion. Some energy is transferred into or out of the system, and some may be transformed to internal energy of the system. Since this energy is not observable in the macroscopic motion of objects, we sometimes say that the energy is "dissipated" in the interaction.

The first three problems explore the application of conservation of energy to collisions. The fourth problem deals with conservation of energy, power output, and the human body.

### OBJECTIVES:

Successfully completing this laboratory should enable you to:

- Use the conservation of energy to predict the outcome of interactions between objects.
- Choose a useful system when using conservation of energy.
- Identify different types of energy when applying energy conservation to real systems.
- Decide when conservation of energy is not useful to predict the outcome of interactions between objects.

### PREPARATION:

Read Knight, Jones & Field Chapter 10. You should also be able to:

- Analyze the motion of an object from videos.
- Calculate the kinetic energy of a moving object.
- Calculate the work done on a system by an external force.
- Calculate the gravitational potential energy of an object with respect to the earth.
- Calculate the elastic potential energy stored in a spring.

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## 1101 Lab 7 Problem 1: KINETIC ENERGY AND WORK

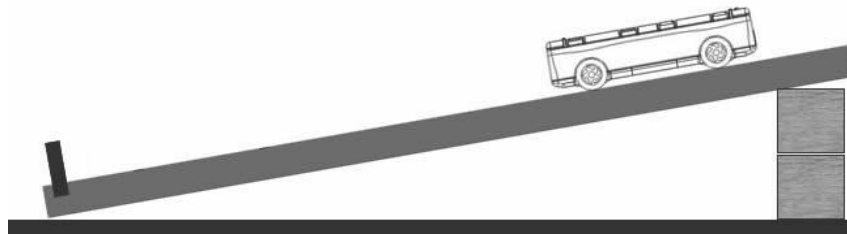
You have been hired as a technical adviser for an upcoming western film. In the script, a wagon containing boxes of gold has been cut loose from the horses by an outlaw. The wagon starts from rest near the top of a hill. The outlaw plans to have the wagon roll down the hill, across a flat section of ground, and over a cliff face into a canyon. The outlaw stations some of his gang in the canyon to collect the gold from the demolished wagon. Little do they know, the Lone Ranger sees the outlaw's action from his lookout post near the base of the hill, and quickly races on horseback to intercept the wagon before it plummets into the canyon. The Lone Ranger must match the speed of the wagon at the base of the hill to hook a strong cord onto the wagon, and then lasso the other end to a large rock.

The director asks you to determine how the velocity of the stagecoach near the bottom of the hill depends on its initial release height up the hill, to coordinate a reasonable required speed for the Lone Ranger's interception. You decide to model the situation using a cart released from rest on an inclined track.

Read: Knight, Jones & Field Chapter 10 Sections 10.1 to 10.4.

### EQUIPMENT

You have a meterstick, stopwatch, wood blocks, an aluminum track, cart, a video camera and a computer with analysis software.



### WARM UP

1. Draw two pictures, one showing the cart at rest at the top of the incline, and another when it is rolling at the bottom of the incline. Draw velocity vectors on your sketch. Define your system. Label the distances, mass of the cart, and the kinetic energy of all objects in your system for both pictures.
2. What is the work done by gravity on the cart from its initial position to when it reaches the bottom of the hill? Hint: remember that to calculate work, you need to multiply the magnitude of the force and the displacement *in the same direction* as the force. You can choose to use either the vertical displacement of the cart, or the distance traveled along the incline.
3. Use the work-kinetic energy theorem to write an equation that relates the work done by gravity on the cart to the change in kinetic energy between its initial release and when it reaches the base of the hill. Assume energy dissipation is small enough to be neglected. Solve your equation for the final velocity of the cart in terms of the vertical release height. (If your equation is in terms of the distance traveled along the incline, use trigonometry to relate this distance to the vertical height of the hill.)

4. Does your equation depend on the steepness of the hill, as measured by the angle of the incline? If you released a car from the same height on hills with different slope steepness, will that effect how fast the cart is traveling at the bottom?

### PREDICTION

For a cart rolling down an inclined track, write an expression for the final velocity in terms of the initial (vertical) release height. Does the final velocity depend on the steepness (angle) of the incline?

Use your expression to sketch a graph of the final velocity versus the initial release height.

### EXPLORATION

Practice releasing the cart from rest on the inclined aluminum track. Try a variety of different track angles, release heights, and cart masses. Record your observations of the cart's motion for each practice run. Do you observe a difference in the final velocity of the cart if you release it from the same height, but with a steeper incline? **BE SURE TO CATCH THE CART AT THE BOTTOM OF THE TRACK!**

Choose a single angle of incline for the aluminum track, the cart mass you will use, and several release heights for the cart. Set up the camera and tripod to give you the best video of the cart's motion down the incline. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

What will you use for a calibration object in your videos? What quantities in your prediction equation do you need to measure with the video analysis software, and what quantities can be measured without the video?

Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan from the Exploration section. Record a video of the cart's motion down the incline for the first release height you have chosen. What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking.

Open your video in MotionLab and follow the instructions to acquire data. As a lab group, decide how you will acquire the value for the final velocity of the cart when it is at the bottom of the hill.

Repeat the data acquisition and analysis for different cart release heights. How many different heights do you need to adequately verify your prediction?

If you have time, try acquiring data to compare two videos with the same vertical release height for the cart, but a steeper incline (different track angle).

### ANALYSIS

Determine the fit functions that best represent the position vs. time graphs for the cart in the x and y directions. (If you are having trouble, review your notes from Lab I Problem 2: Motion Down an Incline.) How can you estimate the values of the constants of each function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Be sure to record all of the fit equations into your lab journal in an organized manner.

Do the same for the velocity vs. time graphs in the x and y directions. Compare these functions with the position vs. time functions.

What quantity or quantities are you interested in acquiring from the graphs of position and velocity in MotionLab? Are the fit functions helpful in this case, or do you need to look at the raw data?

For each cart release height, use your predicted expression from the Warm-up and Prediction to calculate the predicted final velocity of the cart. Have you measured all of the quantities that you need for this expression? If not, be sure to take the measurements before you leave the lab. Make a data table in your lab journal that lists the predicted (calculated) final velocity and the measured final velocity from MotionLab for each release height.

Make a graph of the final velocity versus the release height for your predicted and measured data (plot the measured and predicted velocities on the same graph, but with different colors or symbols.)

### CONCLUSION

How did your measurements compare to your prediction? What are the limitations on the accuracy of your measurements and analysis? What were the sources of your uncertainties?

For a cart rolling down an inclined track, how does the final velocity depend on the initial release height? Does the final velocity depend on the steepness (angle) of the incline? (If you did not try different angles for measurements or in the Exploration, compare notes with another lab group.)

## 1101 Lab 7 Problem 2: GRAVITATIONAL POTENTIAL ENERGY TO ELASTIC POTENTIAL ENERGY

You work in a company that manufactures cords for bungee jumping. You are asked to test out a new kind of cord so that you can write specifications for the weight of the jumper in relation to the height of the jump. You must write a detailed experiment plan with preliminary research results before your team is able to begin testing the new cord. You decide to model the situation using a simple spring and mass to demonstrate the experimental procedure.

Read: Knight, Jones & Field Chapter 10 Section 10.4

### EQUIPMENT

You have springs, a table clamp, rod, meterstick, stopwatch, mass set and the video analysis equipment. You can hang the spring from a rod that is extended from a table clamp.



### WARM UP

1. Draw three stages of the motion of the mass on the spring: one with the mass held at the top at rest before it is dropped (spring un-stretched), one with the mass somewhere in the middle of the fall, and one with the mass at rest at the bottom of the fall. Label the relevant physical quantities on your pictures (height, speed, mass, and stretch of the spring) Label the kinds of energy at each stage of the motion of the mass.
2. For the three pictures, write the energy conservation equation for the system that relates its initial energy to its energy at the point in its motion.
3. To solve for the maximum extension of the spring, use the energy conservation equation for the system that relates its initial energy to its energy when the mass is at its lowest point. What are the constants of the equation and what are the variables?
4. Compare the height that the mass drops and the amount of stretch in the spring. Can you write one in terms of the other in order to simplify the equation?
5. Compare the stretch of the spring when the mass is hanging in equilibrium and when the mass is at its lowest point after being dropped. When the natural length of the spring and the length of the mass have been accounted for, how much longer should the dropped stretch be than the hanging stretch?

### PREDICTION

Write an expression that relates the maximum stretch of the spring to the mass on the spring. What in the expression can be measured and what is constant? Solve the expression for the term that represents the stretch of the spring.



### EXPLORATION

Choose a range of masses that have enough incremental values for you to find the relationship between the maximum stretch of the spring and the mass attached to it. **The stretch of the spring must NOT EXCEED 40 cm** or else the spring constant will change. Carefully test the spring's response to the masses. Devise a way to ensure the spring cannot be over-stretched during the exploration.

Do the smallest mass increments give enough of a difference for you to measure? Does your eye provide sufficient resolution for the stretch length or will you need to use the video?

Decide how you will measure the spring constant (refer to section 8.3 in Knight). What kind of uncertainty can you expect with your method?

Decide how you will measure the maximum displacement of the spring at the bottom of the bounce. What kind of uncertainty does your measurement method introduce? How can you minimize it?

Write down your measurement plan for finding the spring constant of the spring and the maximum stretch of the spring with various masses.

### MEASUREMENT

Carry out measurement plan. Check your measurements along the way to make sure they give the expected result. If not, discuss as a group why. *Hint: you should do the hanging stretch and dropping stretch for each mass concurrently. This will save time and help you keep track of the different stretches more easily.*

### ANALYSIS

Calculate the theoretical stretch of the spring for each mass and include it with the experimental value. Graph the theoretical and experimental data on the same graph. Remember to include the error bars on the experimental data.

What kind of relationship were you expecting? Do you see the same relationship? If you were to keep increasing the mass, would you likely see the pattern continue?

### CONCLUSION

Does your experimental data support the prediction within the expected uncertainty? What are the sources of error that come from your experiment?

## 1101 Lab 7 Problem 3: COLLISIONS REVISITED: ENERGY AND MOMENTUM WHEN OBJECTS BOUNCE APART

You have a summer job with the Minnesota Traffic Safety Board investigating the damage done to vehicles in different kinds of traffic accidents. Your boss wants you to investigate the damage done in low speed collisions when a moving vehicle hits a stationary vehicle and they bounce apart. Your boss believes that, given the same initial energy, the damage to the vehicles in a collision when cars bounce apart will be less when the moving vehicle has a smaller mass than the stationary vehicle (e.g., a compact car hits a van).

To resolve the issue, you decide to model the collision with carts of different masses and measure the energy efficiency of three different cart collisions: one in which the moving cart is more massive, one in which the stationary cart is more massive, and one in which the moving and stationary carts are equally massive. You define efficiency as the ratio of the final kinetic energy of the system to the initial kinetic energy.

Read: Knight, Jones & Field Chapter 10 Sections 10.1 to 10.9.

### EQUIPMENT

You have a track, set of carts, cart masses, meterstick, stopwatch, two endstops and video analysis equipment.

For this problem, cart A is given an initial velocity towards a stationary cart B. Magnets at the end of each cart are used as bumpers to ensure that the carts bounce apart after the collision.



### WARM UP

1. Draw two pictures, one showing the situation before the collision and the other one after the collision. Is it reasonable to neglect friction? Label the mass of each cart, and draw velocity vectors on each sketch. Define your system. Write down expressions for the kinetic energy of the system before and after the collision.
2. Write down an energy conservation equation for this collision. (Remember to take into account the energy dissipated.)
3. Write an equation for the efficiency of the collision in terms of the final and initial kinetic energy of the carts, and then in terms of the cart masses and their initial and final speeds. How do you expect the final velocities for carts A and B to compare for each of the three situations? If the initial kinetic energy of cart A remains the same for all three situations, which situation is most efficient? Least efficient? Or do you expect them to be equally efficient? Explain your reasoning.

## ENERGY AND MOMENTUM WHEN OBJECTS BOUNCE APART

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4. Solve your equation from question 2 for the energy dissipated in the collision. Solve your equation from question 3 for the final kinetic energy in terms of the efficiency and initial kinetic energy, and substitute this into the equation for energy dissipated. Your equation should now be only in terms of the efficiency and the initial kinetic energy of cart A.
5. How is the energy dissipated related to efficiency? For a constant initial kinetic energy, does a collision with a low efficiency have high or low energy dissipation? Based on your rankings of efficiency from question 3, which collision will cause the most damage (have the most energy dissipated)?

<b>PREDICTION</b>
-------------------

Consider the following three cases in which the total mass of the carts is the same ( $m_A + m_B = \text{constant}$ ), where  $m_A$  is the moving cart, and  $m_B$  is the stationary cart:

(a)  $m_A = m_B$

(b)  $m_A > m_B$

(c)  $m_A < m_B$

Write an expression for the efficiency (the ratio of the final kinetic energy of the system to the initial kinetic energy of the system) of the collision between moving cart A and stationary cart B. Rank the collision situations a, b, and c from most efficient to least efficient. (Make an educated guess and explain your reasoning.)

Write an expression for the energy dissipated in a collision in which the carts bounce apart, as a function of the mass of each cart, the initial kinetic energy of the system, and the energy efficiency of the collision. If you assume the kinetic energy of an incoming vehicle is the same in the three cases, which situation will cause the most damage?

<b>EXPLORATION</b>
--------------------

Practice setting the cart into motion so the carts bounce apart from the magnetic bumpers. Try various initial velocities and observe the motion of the carts.

Vary the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Be sure the carts still move freely over the track.

To keep the initial kinetic energy approximately the same for different masses of cart A, how should you change the initial velocity of the moving cart? Try it out.

Select the cart masses you will use for  $m_A = m_B$ ,  $m_A > m_B$ , and  $m_A < m_B$  for the same total mass. Determine an initial velocity for each case that will give you approximately the same initial kinetic energy for cart A. Use a stopwatch and meter stick to practice giving cart A these initial velocities.

Set up the camera and tripod to give you the best video of the collision immediately before and after the carts collide. What will you use for a calibration object in your videos? What

quantities in your prediction equations do you need to measure with the video analysis software? Is it possible to obtain information before and after the collision with one video analysis, or will you need to analyze each video more than once? Write down your measurement plan.

### MEASUREMENT

Follow your measurement plan from the Exploration section. Record a video of one collision situation. Use a stopwatch and the distance traveled by the cart before impact with the bumper to estimate the initial velocity of the cart.

Open one your video in MotionLab and follow the instructions to acquire data. As a lab group, decide how you will acquire data and analyze the collision. (Will you acquire data for the cart A's motion before the impact and repeat the process for cart A and B after the collision, or will you acquire data for the entire motion of the carts in a single analysis?) Change the masses of the carts and repeat this process for the remaining two collision situations.

Measure and record the masses of the two carts for each situation. Analyze your data as you go along (before making the next video), so you can determine if your initial choice of masses and speeds is sufficient. Collect enough data to convince yourself and others of your conclusions about the efficiency of the collision.

### ANALYSIS

From your videos, determine the velocities of the carts before and after the collision. Use your equations from the Warm-up and Prediction questions to calculate the initial and final kinetic energy, efficiency, and energy dissipated for each case.

Record the measured and calculated values in an organized data table in your lab journal.

### CONCLUSION

Given the same initial energy, in which case(s) ( $m_A = m_B$ ,  $m_A > m_B$ , or  $m_A < m_B$ ) was the energy efficiency the largest? The smallest? Was it ever the same? Could the collisions you measured be considered essentially elastic collisions? Why or why not? (The energy efficiency for an elastic collision is 1.)

Was a significant portion of the energy dissipated? Was it the same for each collision situation? How does it compare to the case where the carts stick together after the collision? Into what other forms of energy do you think the cart's initial kinetic energy is most likely to transform?

Was your boss right? Is the damage done to vehicles when a car hits a stationary truck and they bounce apart less than when a truck hits a stationary car (given the same initial kinetic energies)? State your results that support this conclusion.

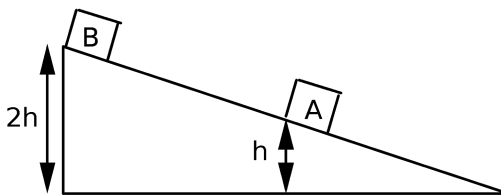
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# ☑ CHECK YOUR UNDERSTANDING

## 1101 LAB 7: ENERGY

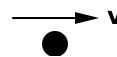
1. A 1-kg ball dropped from a height of 2 meters rebounds only 1.5 meters after hitting the floor. The amount of energy dissipated during the collision with the floor is
  - (a) 5 joules.
  - (b) 10 joules.
  - (c) 15 joules.
  - (d) 20 joules.
  - (e) More than 20 joules.

2. Two boxes start from rest and slide down a *frictionless* ramp that makes an angle of  $30^\circ$  to the horizontal. Block A starts at height  $h$ ; while Block B starts at a height of  $2h$ .



- a. Suppose the two boxes have the same mass. At the bottom of the ramp,
  - (a) Box A is moving twice as fast as box B.
  - (b) Box B is moving twice as fast as box A.
  - (c) Box A is moving faster than box B, but not twice as fast.
  - (d) Box B is moving faster than box A, but not twice as fast.
  - (e) Box A has the same speed as box B.
- b. Suppose box B has a larger mass than box A. At the bottom of the ramp,
  - (a) Box A is moving twice as fast as box B.
  - (b) Box B is moving twice as fast as box A.
  - (c) Box A is moving faster than box B, but not twice as fast.
  - (d) Box B is moving faster than box A, but not twice as fast.
  - (e) Box A has the same speed as box B.

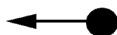
3. A hockey puck is moving at a constant velocity to the right, as shown in the diagram. Which of the following forces will do *positive* work on the puck (i.e., cause an *input* of energy)?



**(a)**



**(b)**



**(c)**



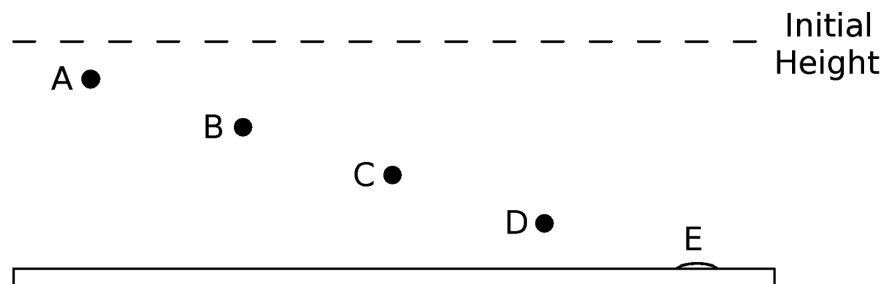
**(d)**



**(e)**

**☑ CHECK YOUR UNDERSTANDING**  
**1101 LAB 7: ENERGY**

4. Five balls made of different substances are dropped from the same height onto a board. Four of the balls bounce up to the maximum height shown on the diagram below. Ball E sticks to the board.



- a. For which ball was the most energy dissipated in the collision?
- (a) Ball A
  - (b) Ball B
  - (c) Ball C
  - (d) Ball D
  - (e) Ball E
- b. Which ball has the largest energy efficiency?
- (a) Ball A
  - (b) Ball B
  - (c) Ball C
  - (d) Ball D
  - (e) Ball E
5. Two carts initially at rest on flat tracks are pushed by the same constant force. Cart 1 has twice the mass of cart 2. They are pushed through the same distance.
- a. Which cart has the largest kinetic energy at the end and why?
- b. Which cart takes the most time to travel the distance?

### Recording Video and using MotionLab - Video Analysis of Motion

Analyzing pictures (movies or videos) is a powerful tool for understanding how objects move. This appendix will guide a person in the use of the app ProCam on an iPod to record videos and MotionLab to analyze motion. LabVIEW™ is a general-purpose data acquisition programming system. It is widely used in academic research and industry. Later you will use LabVIEW™ to acquire data from other instruments.

Using video to analyze motion is a two-step process. The first step is to record a video on the iPod and import it to the computer. The second step is to analyze the video to get a kinematic description of the recorded motion.

#### MAKING VIDEOS – USING PROCAM

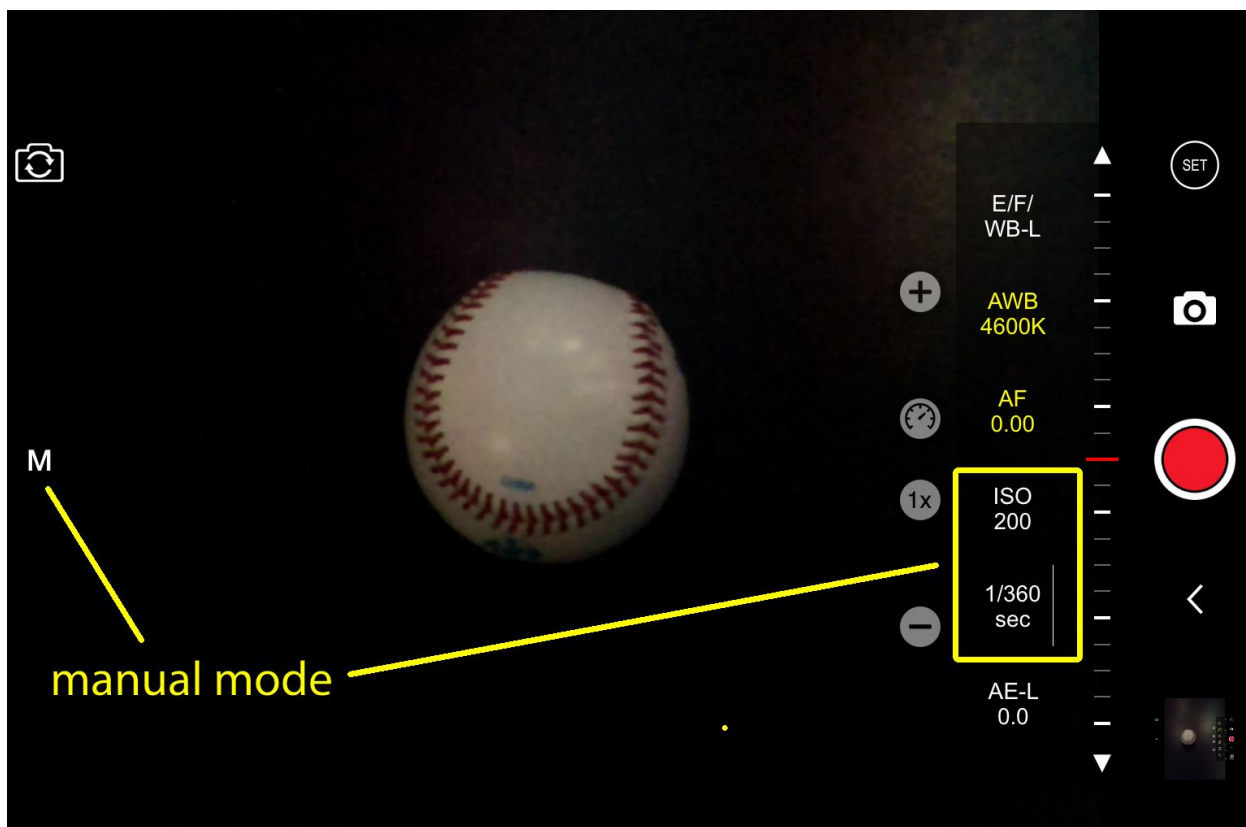
Press the home button (the circle button on the front) in order to unlock the iPod. The application to use is called ProCam. Press the home button to reach the home screen where all the apps are displayed; the ProCam icon is at the bottom of the screen. After you have opened ProCam, you should see a "live" video image of whatever is in front of the camera. You can open the setup menu on the right side of the screen using the "<" button located below the record button. If the menu is already open, the button will appear as ">". When the menu is open, your screen should look like the screenshot below.



Make sure that the program is set to "video mode" (and not "camera mode") by selecting the "video" button. It should be highlighted yellow. The desired format is 60 fps or 120 fps - 720p; you can scroll vertically to find the appropriate format if it is not already selected.

Once in video mode, use the "M" button on the left side of the screen to manually set the exposure.

The screenshot below shows the options for the manual settings. The settings you will adjust are the ISO and the shutter speed; click on the setting and adjust its value using the vertical scroll bar. The ISO determines how sensitive the camera is to light (so higher ISO means more noise), and the shutter speed controls how long the shutter is open. An ISO of 200 and shutter speed around 1/360 should work for most purposes, but you can adjust the values as necessary to get a clear image where the moving object is discrete in each frame. It is okay if the video appears dark because you can adjust the contrast later, after you import the video.



To import your videos to the computer, connect the iPod via the white cable. Click "allow" (or "trust") to the message asking for access to the device. A pop-up window will appear on the computer. Select "Open device to view files" and then browse to your videos. The path is Internal Storage > DCIM > 100APPLE.

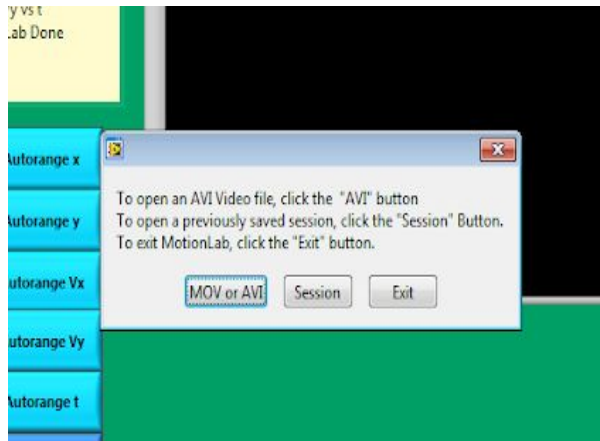
Your videos need to be stored within the **C:\LabData** folder. Make sure that you are using **C:\LabData** otherwise an error will occur as you try to open the videos in MotionLab. Within the LabData folder, you can make a folder with your section number for you to store your own files. Your folder name cannot include spaces. Copy the files from the iPod into your folder and rename them so you can identify them. Once you have copied your videos to the computer, you should delete them off the iPod to avoid cluttering. If there are multiple videos on the iPod you can sort them by date created to find yours.



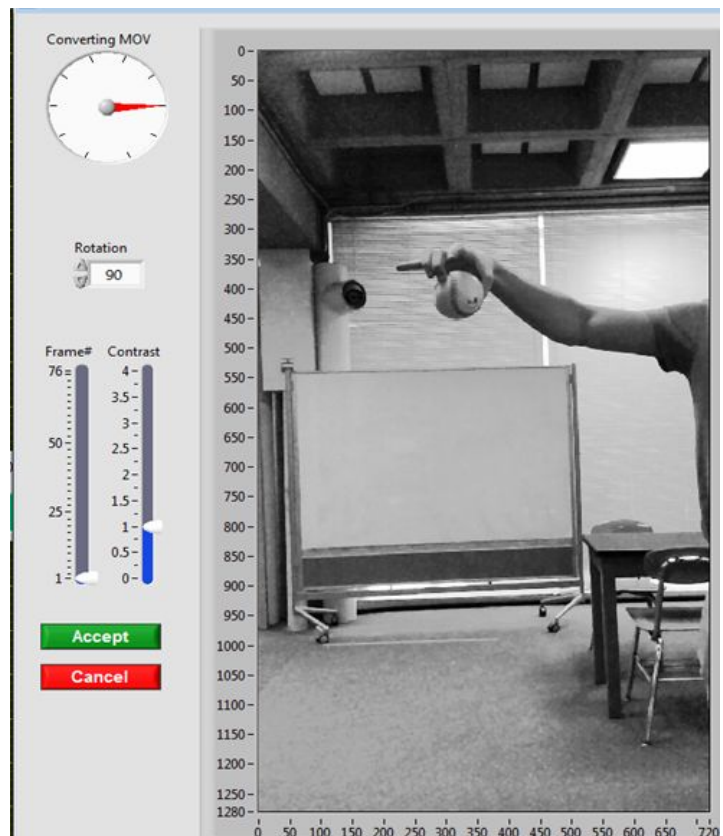
## ANALYSIS BASICS – USING MOTIONLAB

Open the video analysis application *MotionLAB*. You should take a moment to identify several elements of the program. As a whole the application looks complex; once it is broken down it is easy to use.

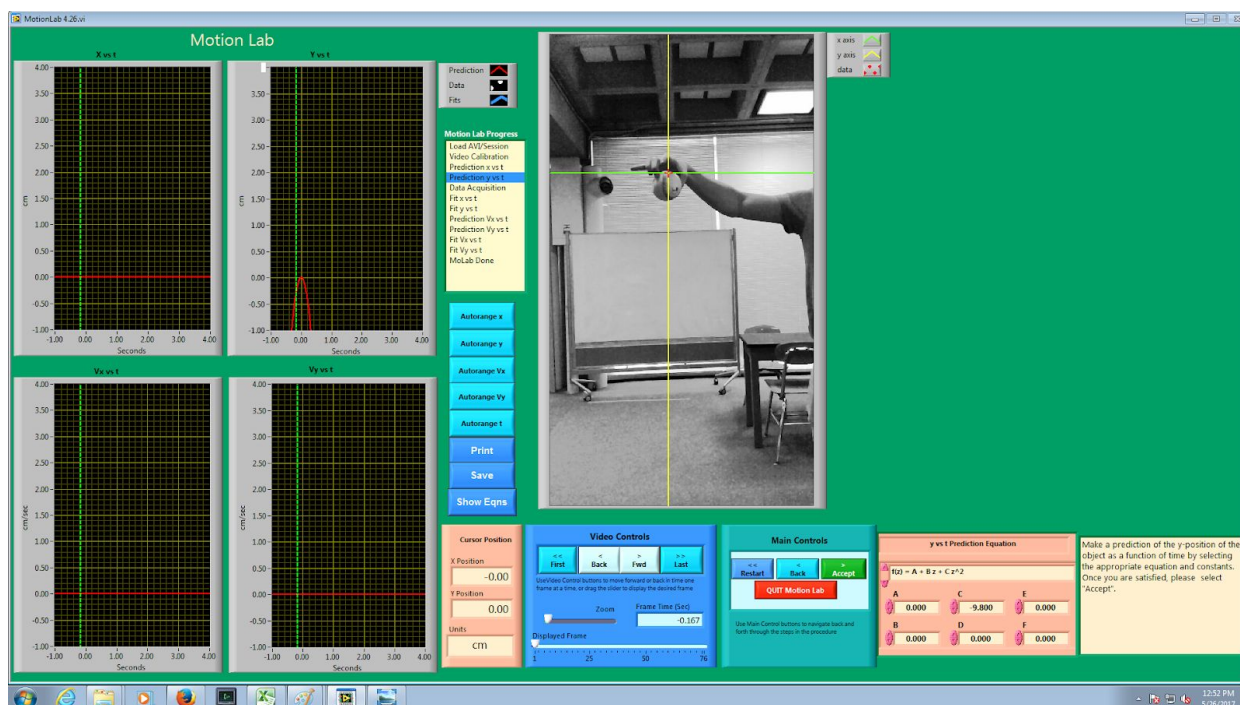
The application will prompt you to open a movie (or previously saved session) as shown here.



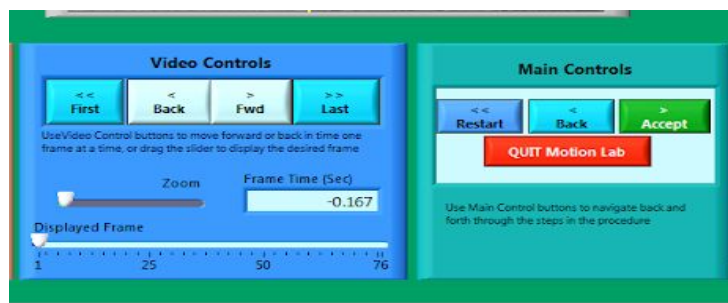
When you open your video, a dialog box will appear where you will be able to rotate the movie and change the contrast before as shown in the screenshot below. Before accepting the movie to be imported use the frame slider to make sure you see no blurring of the motion. If you do ask your instructor if the range of motion is good enough for analysis, if not you may need to change the shutter speed in ProCam.



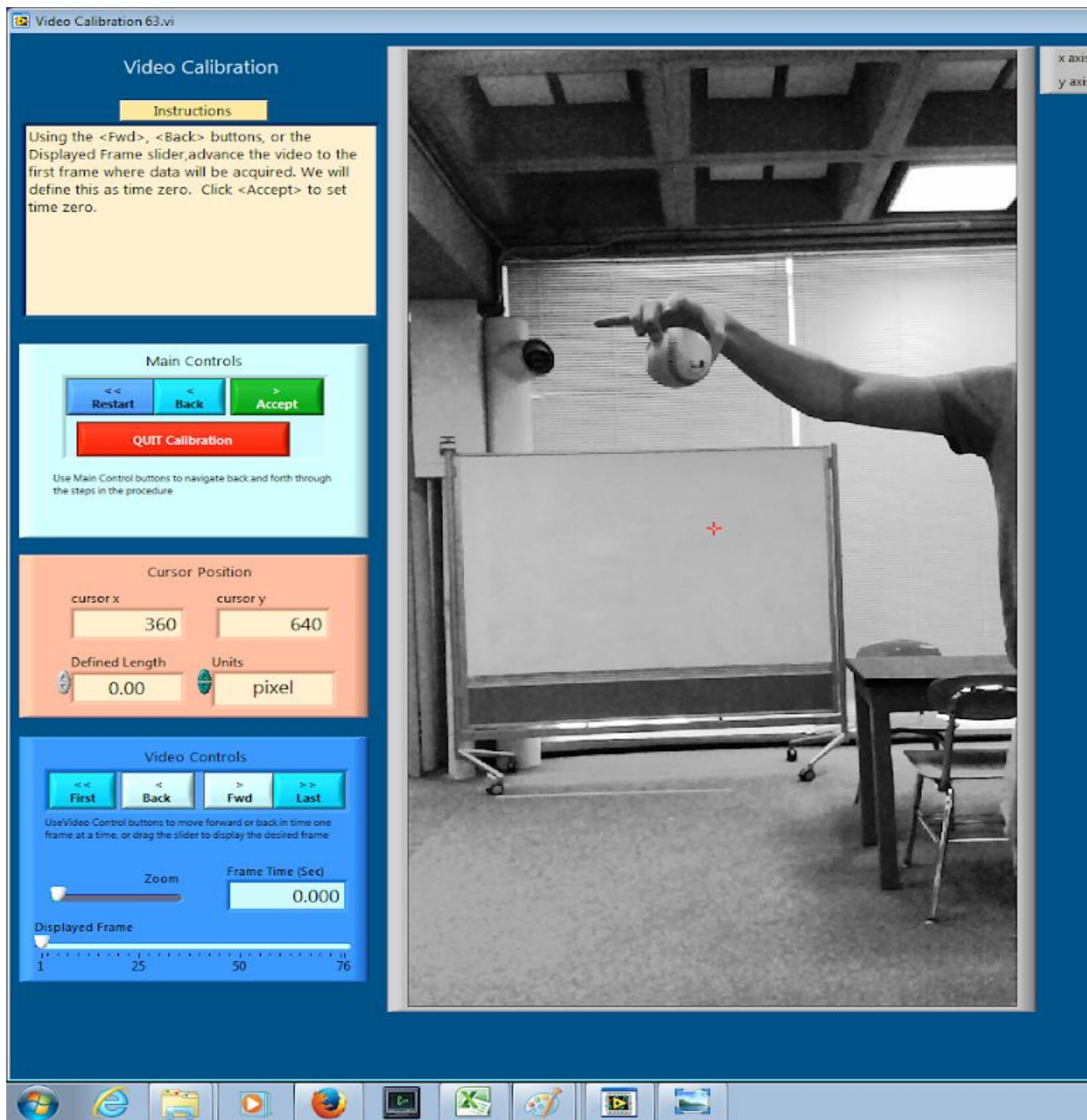
The lower right corner displays a dialog box with instructions for each step during your movie analysis. To the left of the video screen is the progress indicator. It will highlight the step you are currently performing.



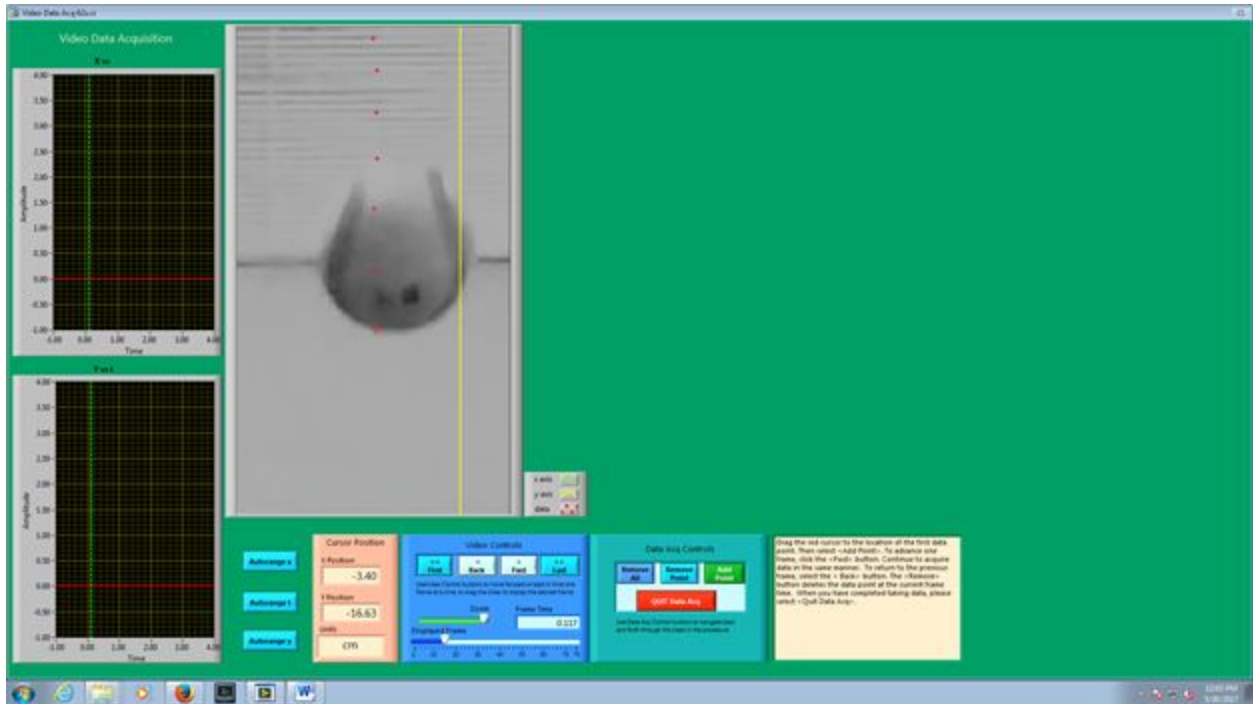
Below the video display is the Video Controls for moving within your AVI movie. The slider bar indicating the displayed frame can also be used to move within the movie. The slider bar for the zoom allows you to zoom in on the red cursor on a frame, which is useful when collecting data (see "Data Collection" section). Directly to the right of the Video Controls is the Main Controls. The Main Control box is your primary session control. Use the Main Control buttons to navigate back and forth through the steps shown in the progress box. The red Quit Motion Lab button closes the program.



During the course of using MotionLAB, larger resolution screens pop up to allow you to calibrate your movie and take data as accurately as possible. The calibration screen has an instructions box to the left of the video with Main Controls and Video Controls directly below. The calibration screen automatically opens once your movie is loaded.



The data acquisition screen appears only after you enter predictions (the progress indicator will display which step you are at.) More will be said about predictions in a bit. The data acquisition screen has the same instructions box and Video Controls, along with a Data Acquisition Control box. The Data Acquisition controls allow you to take and remove data points. The red Quit Data Acq button exits the data collection subroutine and returns to the main screen once your data has been collected. The red cursor will be moved around to take position data from each frame using your mouse. Using the zoom slider the image will center on the red cursor and will re-center on the red cursor for each datapoint added.



**Be careful not to quit without printing and saving your data!** You will have to go back and analyze the data again if you fail to select *Print Results* before selecting *Quit*.

There are just a few more items to point out before getting into calibration, making predictions, taking data and matching your data in more detail. To the right of the main controls shows the equation box for entering predictions and matching data. To the left of the image and below the progress indicator you have controls for automatically setting the range of the graph data based on the data points acquired. You can manually change the range by clicking on the maximum or minimum values on the graphs. Directly below that you have controls for printing and saving. Printing generates a PDF file with the graphs and the equations used for fitting. Saving generates a text file of your data points which can be further analysed using spreadsheet programs. The graphs that display your collected data are shown below. Your predictions are displayed with red lines; fits are displayed with blue lines.

## CALIBRATION

While the computer is a very handy tool, it is not smart enough to identify objects or the sizes of

those objects in the videos that you take and analyze. For this reason, you will need to enter this information into the computer. If you are not careful in the calibration process, your analysis will not make any sense.

After you open the video that you wish to analyze the calibration screen will open automatically. Advance the video to a frame where the first data point will be taken. To advance the video to where you want time  $t=0$  to be, you need to use the video control buttons. This action is equivalent to starting a stopwatch.

When you are ready to continue with the calibration, locate the object you wish to use to calibrate the size of the video. You must do your best to use an object that is in the plane of motion of your object being analyzed. At times the object under motion can be used, but often placing an additional object in the plane of motion is required.

Follow the direction in the *Instructions* box and define the length of an object that you have measured for the computer. Zooming in can help with clearly finding the boundaries. Once this is completed, input the scale length with proper units. Read the directions in the *Instructions* box carefully.

Lastly, decide if you want to rotate your coordinate axes. If you choose not to rotate the axes, the computer will use the first calibration point as the origin with positive  $x$  to the right and positive  $y$  up. If you choose to rotate your axis, follow the directions in the *Instructions* box very carefully. Your chosen axes will appear on the screen once the process is complete. This option may also be used to reposition the origin of the coordinate system, should you require it, however it might be best to start completely over.

Once you have completed this process, select Quit Calibration.

## ANALYSIS PREDICTIONS

This video analysis relies on your graphical skills to interpret the data from the videos. Before doing your analysis, you should be familiar with the **Review of Graphs** and **Accuracy, Precision and Uncertainties** appendices..

Before analyzing the data, enter your prediction of how you expect the data to behave. This pattern of making predictions before obtaining results is the only reliable way to take data. How else can you know if something has gone wrong? This happens so often that it is given a name (Murphy's Law). It is also a good way to make sure you have learned something, but only if you stop to think about the discrepancies or similarities between your prediction and the results.

In order to enter your prediction into the computer, you first need to decide on your coordinate axes, origin, and scale (units) for your motion. Record these in your lab journal.



Next you will need to select the generic equation,  $f(z)$ , which describes the graph you expect for the motion along your x-axis seen in your video. You must choose the appropriate function that matches the predicted curve. The analysis program is equipped with several equations, which are accessible using the pull-down menu on the equation line. The available equations are shown to the right.

✓  $f(z) = A + B z$   
 $f(z) = A + B z + C z^2$   
 $f(z) = A + B z + C z^2 + D z^3$   
 $f(z) = A + B z + C z^2 + D z^3 + E z^4$   
 $f(z) = A + B \sin (C z + D)$   
 $f(z) = A + B \cos (C z + D)$   
 $f(z) = A + B \sin (C + D z + E z^2)$   
 $f(z) = A + B \cos (C + D z + E z^2)$   
 $f(z) = A + B \exp (- C z)$   
 $f(z) = A + B (1 - \exp (- C z))$   
 $f(z) = A + (B z + C) \sin (D + E z + F z^2)$   
 $f(z) = A + (B z + C) \cos (D + E z + F z^2)$

You can change the equation to one you would like to use by clicking on the arrows to the left of the equation

After selecting your generic equation, you next need to enter your best approximation for the parameters A and B and C and D where you need them. Take this time to think of the physical meaning of the parameters and what units these constants are in. Note that in the equations  $z$  stands for time. If you took good notes of these values during the filming of your video, inputting these values should be straightforward.

Once you are satisfied that the equation you selected for your motion and the values of the constants are correct, click "Accept" in the *Main Controls*. Your prediction equation will then show up on the graph on the computer screen. If you wish to change your prediction simply repeat the above procedure. Repeat this procedure for the Y direction.

## DATA COLLECTION

To collect data, you first need to identify a very specific point on the object whose motion you are analyzing. Next, move the cursor over this point and click the green *ADD Data Point* button in Data Acquisition control box. The computer records this position and time. The computer will automatically advance the video to the next frame, leaving a mark on the point you have just selected. Then move the cursor back to the same place on the object and click *ADD Data Point* button again. As long as you always use the same point on the object, you will get reliable data from your analysis. It is helpful to zoom in on the object using the zoom slider under Video Controls, which will make it easier to click on the same point on the object. The data will automatically appear on the graph on your computer screen each time you accept a data point. If you don't see the data on the graph, you will need to change the scale of the axes. If you are satisfied with your data, choose *Quit Data Acq* from the *controls*

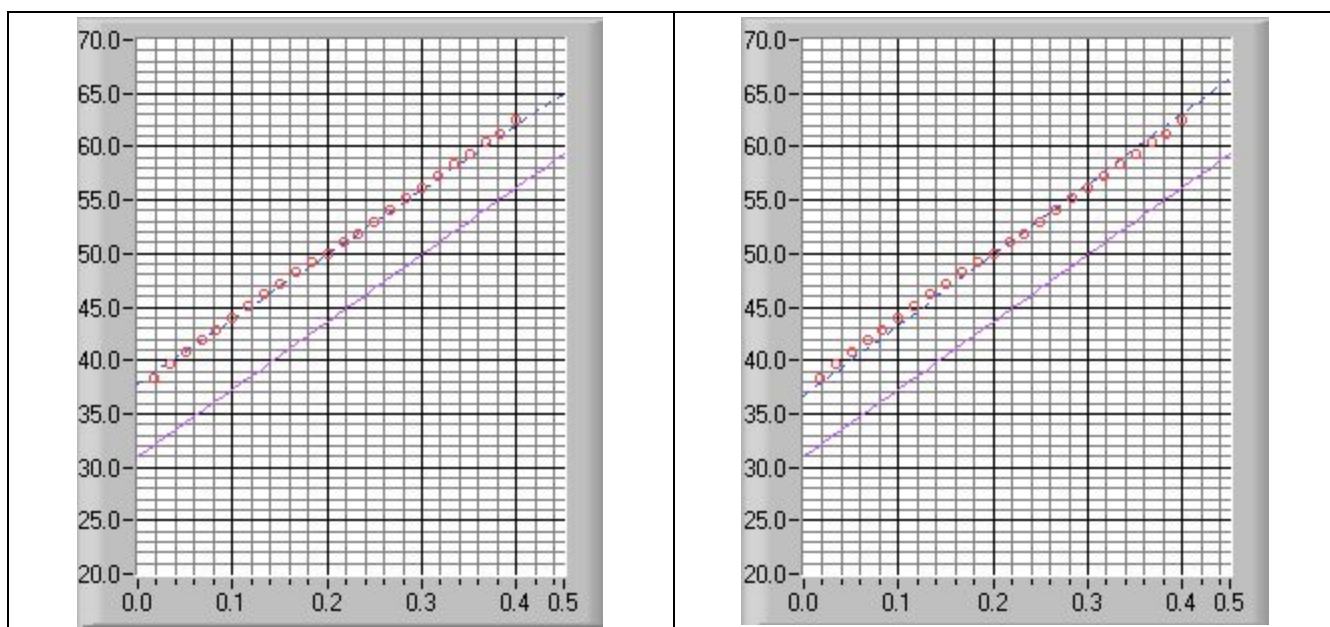
## FITTING YOUR DATA

Deciding which equation best represents your data is the most important part of your data analysis. The actual mechanics of choosing the equation and constants is similar to what you did for your predictions.

First you must find your data on your graphs. Usually, you can find your full data set by using the *Autorange* buttons to the left of the graphs.

Secondly, after you find your data, you need to determine the best possible equation to describe this data. After you have decided on the appropriate equation, you need to determine the constants of this equation so that it best fits the data. Although this can be done by trial and error, it is much more efficient to think of how the behavior of the equation you have chosen depends on each parameter. Calculus can be a great help here.

Lastly, you need to estimate the uncertainty in your fit by deciding the range of other lines that *could* also fit your data. This method of estimating your uncertainty is described in the appendix **Accuracy, Precision and Uncertainty**. Slightly changing the values for each constant in turn will allow you to do this quickly. For example, the X-motion plots below show both the predicted line (down) and two other lines that also fit the data (near the circles).



After you have found the uncertainties in your constants, return to your best-fit line and use it as your fit by selecting *Accept x- (or y-) fit* in the *Program Controls* panel.

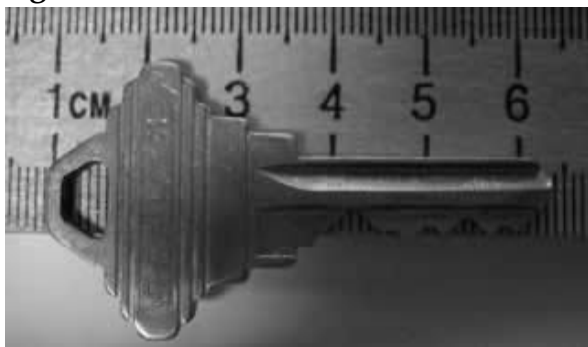
## LAST WORDS

These directions are not meant to be exhaustive. You will discover more features of the video analysis program as you use it. Be sure to record these features in your lab journal.

## Appendix: Significant Figures

Calculators make it possible to get an answer with a huge number of figures. Unfortunately, many of them are meaningless. For instance, if you needed to split \$1.00 among three people, you could never give them each exactly \$0.333333... The same is true for measurements. If you use a meter stick with millimeter markings to measure the length of a key, as in Figure 1, you could not measure more precisely than a quarter or half or a third of a mm. Reporting a number like 5.37142712 cm would not only be meaningless, it would be misleading.

**Figure 1**



In your measurement, you can precisely determine the distance down to the nearest millimeter and then improve your precision by estimating the next figure. It is always assumed that the last figure in the number recorded is uncertain. So, you would report the length of the key as 5.37 cm. Since you estimated the 7, it is the uncertain figure. If you don't like estimating, you might be tempted to just give the number that you know best, namely 5.3 cm, but it is clear that 5.37 cm is a better report of the measurement. An estimate is always necessary to report the most precise measurement. When you quote a measurement, the reader will always assume that the last figure is an estimate. Quantifying that estimate is known as **estimating uncertainties**. Appendix C will illustrate how you might use those estimates

to determine the uncertainties in your measurements.

### What are significant figures?

The number of significant figures tells the reader the precision of a measurement. Table 1 gives some examples.

**Table 1**

Length (centimeters)	Number of Significant Figures
12.74	4
11.5	3
1.50	3
1.5	2
12.25345	7
0.8	1
0.05	1

One of the things that this table illustrates is that not all zeros are significant. For example, the zero in 0.8 is not significant, while the zero in 1.50 is significant. Only the zeros that appear after the first non-zero digit are significant.

A good rule is to always express your values in scientific notation. If you say that your friend lives 143 m from you, you are saying that you are sure of that distance to within a few meters (3 significant figures). What if you really only know the distance to a few tens of meters (2 significant figures)? Then you need to express the distance in scientific notation  $1.4 \times 10^2$  m.

### Is it always better to have more figures?

Consider the measurement of the length of the key shown in Figure 1. If we have a scale with ten etchings to every millimeter, we could use a microscope to measure the spacing to the nearest tenth of a millimeter and guess at the one hundredth millimeter. Our measurement could be 5.814 cm with



the uncertainty in the last figure, four significant figures instead of three. This is because our improved scale allowed our estimate to be more precise. This added precision is shown by more significant figures. The more significant figures a number has, the more precise it is.

### How do I use significant figures in calculations?

When using significant figures in calculations, you need to keep track of how the uncertainty propagates. There are mathematical procedures for doing this estimate in the most precise manner. This type of estimate depends on knowing the statistical distribution of your measurements. With a lot less effort, you can do a cruder estimate of the uncertainties in a calculated result. This crude method gives an overestimate of the uncertainty but it is a good place to start. For this course this simplified uncertainty estimate (described in Appendix C and below) will be good enough.

### Addition and subtraction

When adding or subtracting numbers, the number of decimal places must be taken into account.

*The result should be given to as many decimal places as the term in the sum that is given to the **smallest** number of decimal places.*

#### Examples:

Addition	Subtraction
6.242	5.875
+4.23	<u>-3.34</u>
<u>+0.013</u>	2.535
10.485	
10.49	2.54

The uncertain figures in each number are shown in **bold-faced** type.

### Multiplication and division

When multiplying or dividing numbers, the number of significant figures must be taken into account.

*The result should be given to as many significant figures as the term in the product that is given to the **smallest** number of significant figures.*

The basis behind this rule is that the least accurately known term in the product will dominate the accuracy of the answer.

As shown in the examples, this does not always work, though it is the quickest and best rule to use. When in doubt, you can keep track of the significant figures in the calculation as is done in the examples.

#### Examples:

Multiplication	
15.84	17.27
<u>x 2.5</u>	<u>x 4.0</u>
7920	69.080
<u>3168</u>	
39.600	
40	69

Division	
<u>117</u>	<u>25</u>
23)2691	75)1875
<u>23</u>	<u>150</u>
39	375
<u>23</u>	375
161	
161	
$1.2 \times 10^2$	$2.5 \times 10^1$

**PRACTICE EXERCISES**

1. Determine the number of significant figures of the quantities in the following table:

Length (centimeters)	Number of Significant Figures
17.87	
0.4730	
17.9	
0.473	
18	
0.47	
$1.34 \times 10^2$	
$2.567 \times 10^5$	
$2.0 \times 10^{10}$	
1.001	
1.000	
1	
1000	
1001	

2. Add: 121.3 to  $6.7 \times 10^2$ :
3. Multiply: 34.2 and  $1.5 \times 10^4$

## Appendix: Accuracy, Precision and Uncertainty - ERROR ANALYSIS

How tall are you? How old are you? When you answered these everyday questions, you probably did it in round numbers such as "five foot, six inches" or "nineteen years, three months." But how true are these answers? Are you exactly 5' 6" tall? Probably not. You estimated your height at 5' 6" and just reported two significant figures. Typically, you round your height to the nearest inch, so that your actual height falls somewhere between 5' 5½" and 5' 6½" tall, or 5' 6"  $\pm$  ½". This  $\pm$  ½" is the **uncertainty**, and it informs the reader of the precision of the **value** 5' 6".

### What is uncertainty?

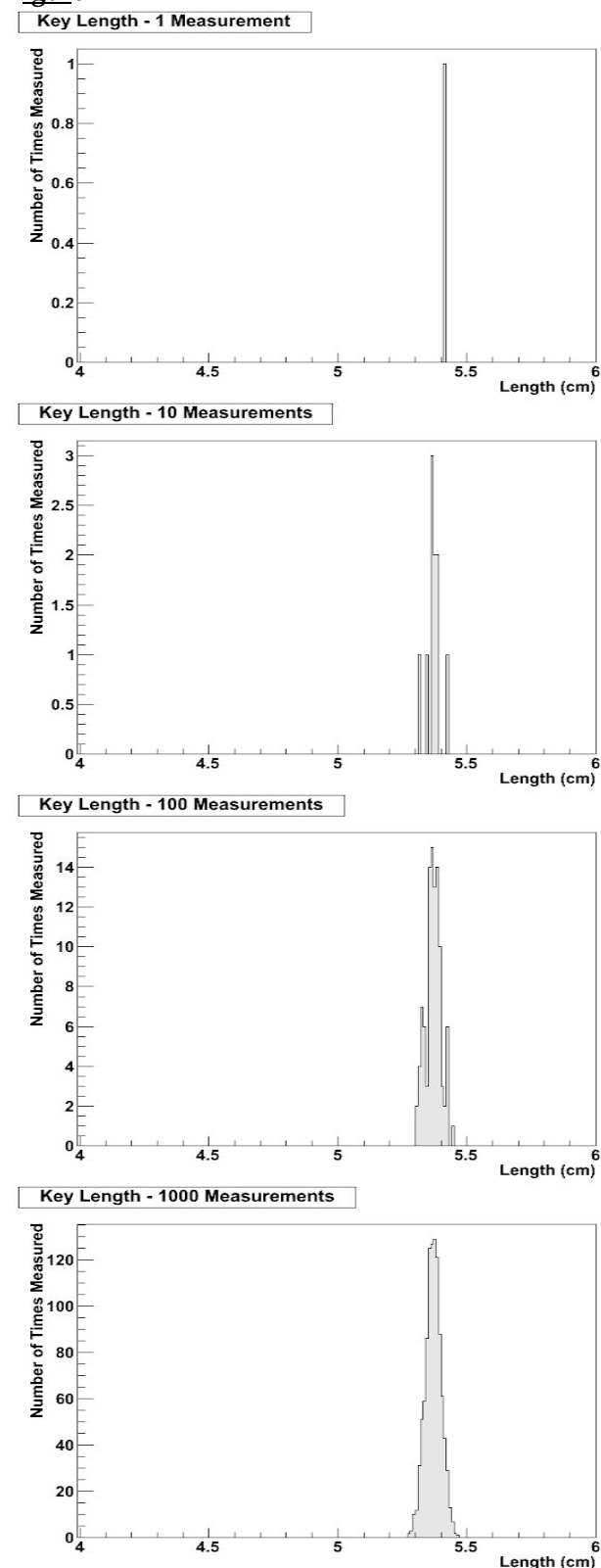
Whenever you measure something, there is always some uncertainty. There are two categories of uncertainty: **systematic** and **random**.

(1) **Systematic uncertainties** are those that consistently cause the value to be too large or too small. Systematic uncertainties include such things as reaction time, inaccurate meter sticks, optical parallax and miscalibrated balances. In principle, systematic uncertainties can be eliminated if you know they exist.

(2) **Random uncertainties** are variations in the measurements that occur without a predictable pattern. If you make precise measurements, these uncertainties arise from the estimated part of the measurement. Random uncertainty can be reduced, but never eliminated. We need a technique to report the contribution of this uncertainty to the measured value.

Uncertainties cause every measurement you make to be **distributed**. For example, the key in Figure 2 is approximately 5.37cm long. For the sake of argument, pretend that it is exactly 5.37cm long. If you measure its length many times, you expect that most of the measurements will be close to, but not exactly, 5.37cm, and that there will be a few measurements much more than or much less than 5.37cm. This effect is due to random uncertainty. You can never know how accurate any single measurement is, but you expect that many measurements will cluster around the real length, so you can take the average as the "real" length, and more measurements will give you a better answer; see Figure 1.

Figure 1



You must be very careful to estimate or eliminate (by other means) systematic uncertainties well because they cannot be eliminated in this way; they would just shift the distributions in Figure 1 left or right.

Roughly speaking, the average or “center” of the distribution is the “measurement,” and the width or “deviation” of the distribution is the random uncertainty.

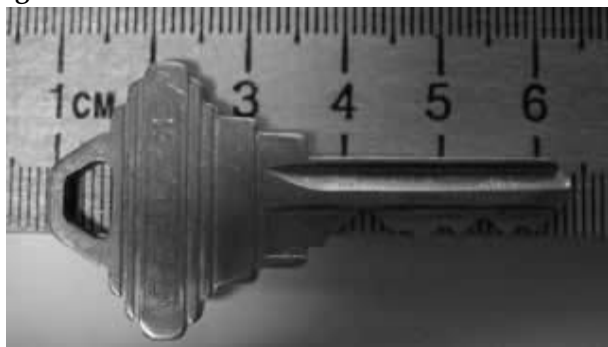
### How do I determine the uncertainty?

This Appendix will discuss three basic techniques for determining the uncertainty: **estimating the uncertainty**, measuring the **average deviation**, and finding the **uncertainty in a linear fit**. Which one you choose will depend on your situation, your available means of measurement, and your need for precision. If you need a precise determination of some value, and you are measuring it directly (e.g., with a ruler or thermometer), the best technique is to measure that value several times and use the average deviation as the uncertainty. Examples of finding the average deviation are given below.

### How do I estimate uncertainties?

If time or experimental constraints make repeated measurements impossible, then you will need to estimate the uncertainty. When you estimate uncertainties you are trying to account for anything that might cause the measured value to be different if you were to take the measurement again. For example, suppose you were trying to measure the length of a key, as in Figure 2.

Figure 2



If the true value were not as important as the magnitude of the value, you could say that the key's length was 5 cm, give or take 1 cm. This is a crude estimate, but it may be acceptable. A better estimate of the key's length, as you saw in Appendix A, would

be 5.37 cm. This tells us that the worst our measurement could be off is a fraction of a mm. To be more precise, we can estimate it to be about a third of a mm, so we can say that the length of the key is  $5.37 \pm 0.03$  cm.

Another time you may need to estimate uncertainty is when you analyze video data. Figures 3 and 4 show a ball rolling off the edge of a table. These are two consecutive frames, separated in time by  $1/30$  of a second.

Figure 3

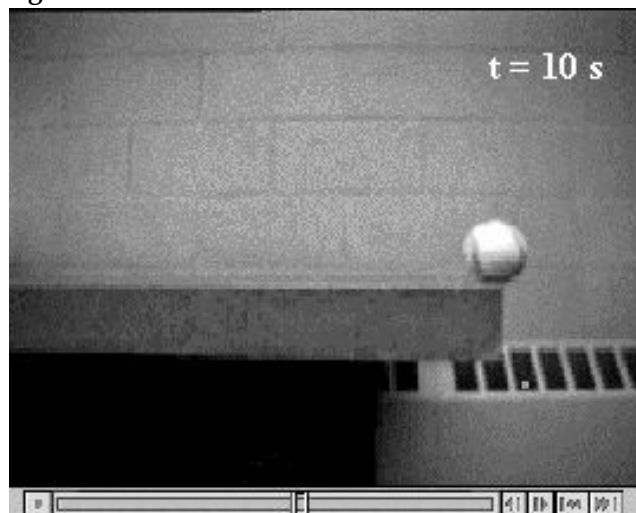
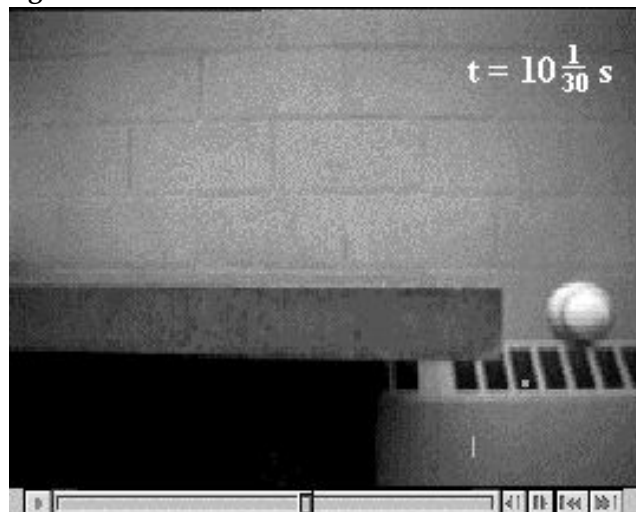


Figure 4



The exact moment the ball left the table lies somewhere between these frames. We can estimate that this moment occurs midway between them ( $t = 10 \frac{1}{60} s$ ). Since it must occur at some point between them, the worst our estimate could be off by

is  $\frac{1}{60} s$ . We can therefore say the time the ball leaves the table is  $t = 10 \frac{1}{60} \pm \frac{1}{60} s$ .

### How do I find the average deviation?

If estimating the uncertainty is not good enough for your situation, you can experimentally determine the uncertainty by making several measurements and calculating the average deviation of those measurements. To find the average deviation: (1) Find the average of all your measurements; (2) Find the absolute value of the difference of each measurement from the average (its deviation); (3) Find the average of all the deviations by adding them up and dividing by the number of measurements. Of course you need to take enough measurements to get a distribution for which the average has some meaning.

In example 1, a class of six students was asked to find the mass of the same penny using the same balance. In example 2, another class measured a different penny using six different balances. Their results are listed below:

#### Class 1: Penny A massed by six different students on the same balance.

Mass (grams)
3.110
3.125
3.120
3.126
3.122
<u>3.120</u>
3.121 average.
The deviations are: 0.011g, 0.004g, 0.001g, 0.005g, 0.001g, 0.001g
Sum of deviations: 0.023g
Average deviation: (0.023g)/6 = 0.004g
Mass of penny A: $3.121 \pm 0.004g$

#### Class 2: Penny B massed by six different students on six different balances

Mass (grams)
3.140
3.133
3.144
3.118
3.126
<u>3.125</u>
3.131 average

The deviations are: 0.009g, 0.002g, 0.013g, 0.013g, 0.005g, 0.006g
Sum of deviations: 0.048g
Average deviation: (0.048g)/6 = 0.008g
Mass of penny B: $3.131 \pm 0.008g$

### Finding the Uncertainty in a Linear Fit

Sometimes, you will need to find the uncertainty in a linear fit to a large number of measurements. The most common situation like this that you will encounter is fitting position or velocity with respect to time from MotionLab.

When you fit a line to a graph, you will be looking for the “best fit” line that “goes through the middle” of the data; see the appendix about graphs for more about this procedure. To find the uncertainty, draw the lines with the greatest and least slopes that still roughly go through the data. These will be the upper and lower limits of the uncertainty in the slope. These lines should also have lesser and greater y-intercepts than the “best fit” line, and they define the lower and upper limits of the uncertainty in the y-intercept.

Note that when you do this, the uncertainties above and below your “best fit” values will, in general, **not** be the same; this is different than the other two methods we have presented.

For example, in Figure 5, the y-intercept is  $4.25 \pm 2.75/-2.00$ , and the slope is  $0.90 \pm 0.20/-0.25$ .

Figure 5a

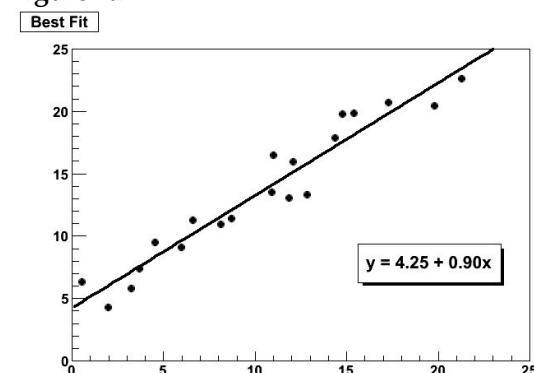


Figure 5b

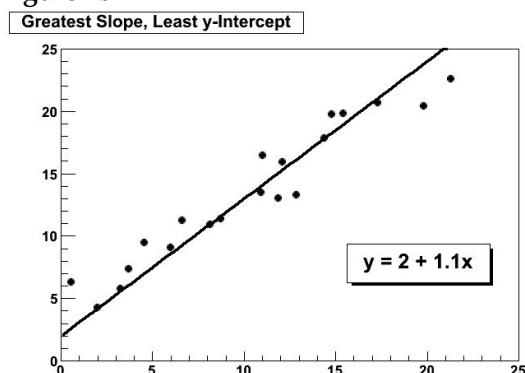
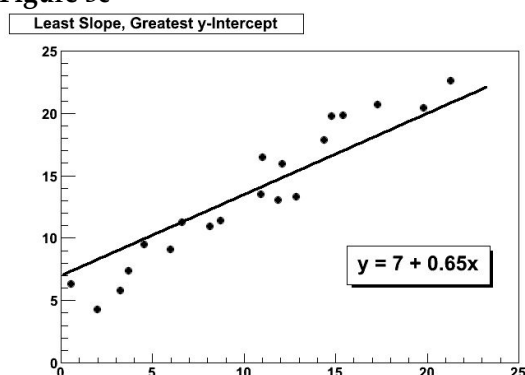


Figure 5c

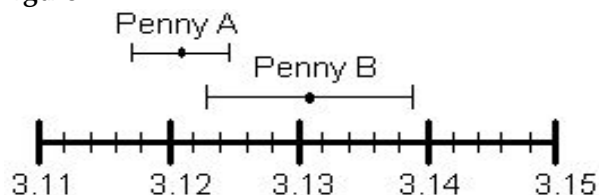


However you choose to determine the uncertainty, you should always state your method clearly in your report.

### How do I know if two values are the same?

Go back to the pennies. If we compare only the average masses of the two pennies we see that they are different. But now include the uncertainty in the masses. For penny A, the most likely mass is somewhere between 3.117g and 3.125g. For penny B, the most likely mass is somewhere between 3.123g and 3.139g. If you compare the ranges of the masses for the two pennies, as shown in Figure 6, they just overlap. Given the uncertainty in the masses, we are able to conclude that the masses of the two pennies could be the same. If the range of the masses did not overlap, then we ought to conclude that the masses are probably different.

Figure 6



Mass of pennies (in grams) with uncertainties

An important application of this is determining agreement between experimental and theoretical values. If you use a formula to generate a theoretical value of some quantity and use the method below to generate the uncertainty in the calculation, and if you generate an experimental value of the same quantity by measuring it and use the method above to generate the uncertainty in the measurement, you can compare the two values in this way. If the ranges overlap, then the theoretical and experimental values agree. If the ranges do not overlap, then the theoretical and experimental values do not agree.

### Which result is more precise?

Suppose you use a meter stick to measure the length of a table and the width of a hair, each with an uncertainty of 1 mm. Clearly you know more about the length of the table than the width of the hair. Your measurement of the table is very precise but your measurement of the width of the hair is rather crude. To express this sense of precision, you need to calculate the percentage uncertainty. To do this, divide the uncertainty in the measurement by the value of the measurement itself, and then multiply by 100%. For example, we can calculate the precision in the measurements made by class 1 and class 2 as follows:

Precision of Class 1's value:

$$(0.004 \text{ g} \div 3.121 \text{ g}) \times 100\% = 0.1 \%$$

Precision of Class 2's value:

$$(0.008 \text{ g} \div 3.131 \text{ g}) \times 100\% = 0.3 \%$$

Class 1's results are more precise. This should not be surprising since class 2 introduced more uncertainty in their results by using six different balances instead of only one.

### Which result is more accurate?

**Accuracy** is a measure of how your measured value compares with the real value. Imagine that class 2 made the measurement again using only one balance. Unfortunately, they chose a balance that was poorly

calibrated. They analyzed their results and found the mass of penny B to be  $3.556 \pm 0.004$  g. This number is more precise than their previous result since the uncertainty is smaller, but the new measured value of mass is very different from their previous value. We might conclude that this new value for the mass of penny B is different, since the range of the new value does not overlap the range of the previous value. However, that conclusion would be **wrong** since our uncertainty has not taken into account the inaccuracy of the balance. To determine the accuracy of the measurement, we should check by measuring something that is known. This procedure is called calibration, and it is absolutely necessary for making accurate measurements.

*Be cautious! It is possible to make measurements that are extremely precise and, at the same time, grossly inaccurate.*

### How can I do calculations with values that have uncertainty?

When you do calculations with values that have uncertainties, you will need to estimate (by calculation) the uncertainty in the result. There are mathematical techniques for doing this, which depend on the statistical properties of your measurements. A very simple way to estimate uncertainties is to find the *largest possible uncertainty* the calculation could yield. **This will always overestimate the uncertainty of your calculation**, but an overestimate is better than no estimate or an underestimate. The method for performing arithmetic operations on quantities with uncertainties is illustrated in the following examples:

#### Addition:

$$(3.131 \pm 0.008 \text{ g}) + (3.121 \pm 0.004 \text{ g}) = ?$$

First, find the sum of the values:

$$3.131 \text{ g} + 3.121 \text{ g} = 6.252 \text{ g}$$

Next, find the largest possible value:

$$3.139 \text{ g} + 3.125 \text{ g} = 6.264 \text{ g}$$

The uncertainty is the difference between the two:

$$6.264 \text{ g} - 6.252 \text{ g} = 0.012 \text{ g}$$

**Answer:  $6.252 \pm 0.012$  g.**

*Note: This uncertainty can be found by simply adding the individual uncertainties:*

$$0.004 \text{ g} + 0.008 \text{ g} = 0.012 \text{ g}$$

#### Multiplication:

$$(3.131 \pm 0.013 \text{ g}) \times (6.1 \pm 0.2 \text{ cm}) = ?$$

First, find the product of the values:

$$3.131 \text{ g} \times 6.1 \text{ cm} = 19.1 \text{ g-cm}$$

Next, find the largest possible value:

$$3.144 \text{ g} \times 6.3 \text{ cm} = 19.8 \text{ g-cm}$$

The uncertainty is the difference between the two:

$$19.8 \text{ g-cm} - 19.1 \text{ g-cm} = 0.7 \text{ g-cm}$$

**Answer:  $19.1 \pm 0.7$ g-cm.**

*Note: The percentage uncertainty in the answer is the sum of the individual percentage uncertainties:*

$$\frac{0.013}{3.131} \times 100\% + \frac{0.2}{6.1} \times 100\% = \frac{0.7}{19.1} \times 100\%$$

<p><b>Subtraction:</b></p> $(3.131 \pm 0.008 \text{ g}) - (3.121 \pm 0.004 \text{ g}) = ?$ <p>First, find the difference of the values:</p> $3.131 \text{ g} - 3.121 \text{ g} = 0.010 \text{ g}$ <p>Next, find the largest possible difference:</p> $3.139 \text{ g} - 3.117 \text{ g} = 0.022 \text{ g}$ <p>The uncertainty is the difference between the two:</p> $0.022 \text{ g} - 0.010 \text{ g} = 0.012 \text{ g}$ <p><b>Answer: <math>0.010 \pm 0.012 \text{ g}</math>.</b></p> <p><i>Note: This <u>uncertainty</u> can be found by simply adding the <u>individual uncertainties</u>:</i></p> $0.004 \text{ g} + 0.008 \text{ g} = 0.012 \text{ g}$ <p><i>Notice also, that zero is included in this range, so it is possible that there is no difference in the masses of the pennies, as we saw before.</i></p>	<p><b>Division:</b></p> $(3.131 \pm 0.008 \text{ g}) \div (3.121 \pm 0.004 \text{ g}) = ?$ <p>First, divide the values:</p> $3.131 \text{ g} \div 3.121 \text{ g} = 1.0032$ <p>Next, find the largest possible value:</p> $3.139 \text{ g} \div 3.117 \text{ g} = 1.0071$ <p>The uncertainty is the difference between the two:</p> $1.0071 - 1.0032 = 0.0039$ <p><b>Answer: <math>1.003 \pm 0.004</math></b></p> <p><i>Note: The <u>percentage uncertainty</u> in the answer is the sum of the <u>individual percentage uncertainties</u>:</i></p> $\frac{0.008}{3.131} \times 100\% + \frac{0.004}{3.121} \times 100\% = \frac{0.0039}{1.0032} \times 100\%$ <p><i>Notice also, the largest possible value for the numerator and the smallest possible value for the denominator gives the largest result.</i></p>
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The same ideas can be carried out with more complicated calculations. Remember this will always give you an overestimate of your uncertainty. There are other calculation techniques, which give better estimates for uncertainties. If you wish to use them, please discuss it with your instructor to see if they are appropriate.

These techniques help you estimate the random uncertainty that always occurs in measurements.

They will not help account for mistakes or poor measurement procedures. There is no substitute for taking data with the utmost of care. A little forethought about the possible sources of uncertainty can go a long way in ensuring precise and accurate data.



**PRACTICE EXERCISES:**

**B-1.** Consider the following results for different experiments. Determine if they agree with the accepted result listed to the right. Also calculate the precision for each result.

- |                                     |                               |
|-------------------------------------|-------------------------------|
| a) $g = 10.4 \pm 1.1 \text{ m/s}^2$ | $g = 9.8 \text{ m/s}^2$       |
| b) $T = 1.5 \pm 0.1 \text{ sec}$    | $T = 1.1 \text{ sec}$         |
| c) $k = 1368 \pm 45 \text{ N/m}$    | $k = 1300 \pm 50 \text{ N/m}$ |

**B-2.** The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be  $5.37 \pm 0.05 \text{ cm}$ . The width was found to be  $3.42 \pm 0.02 \text{ cm}$ . What is the area and the average deviation?

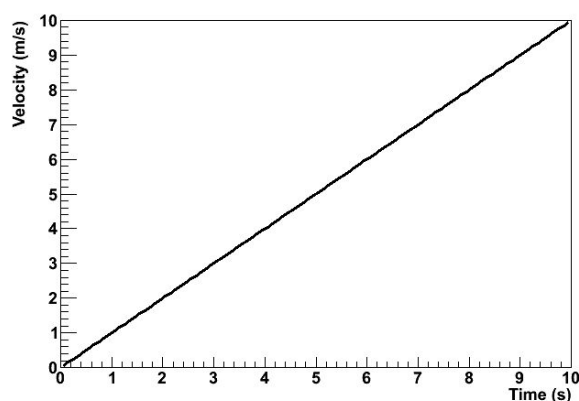
**B-3.** Each member of your lab group weighs the cart and two mass sets twice. The following table shows this data. Calculate the total mass of the cart with each set of masses and for the two sets of masses combined.

Cart (grams)	Mass set 1 (grams)	Mass set 2 (grams)
201.3	98.7	95.6
201.5	98.8	95.3
202.3	96.9	96.4
202.1	97.1	96.2
199.8	98.4	95.8
200.0	98.6	95.6

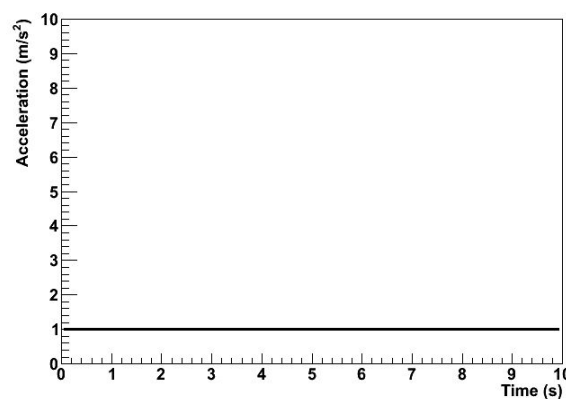
Graphs are visual tools used to represent relationships (or the lack thereof) among numerical quantities in mathematics. In particular, we are interested in the graphs of functions.

## What is a graph?

In this course, we will be dealing almost exclusively with graphs of functions. When we graph a quantity  $A$  with respect to a quantity  $B$ , we mean to put  $B$  on the horizontal axis and  $A$  on the vertical axis of a two-dimensional region and then to draw a set of points or curve showing the relationship between them. We do not mean to graph any other quantity from which  $A$  or  $B$  can be determined. For example, a plot of acceleration versus time has acceleration itself,  $a(t)$ , on the vertical axis, not the corresponding velocity  $v(t)$ ; the time  $t$ , of course, goes on the horizontal axis. See Figure 1.



(a)



(b)

**Figure 1:** Graphs of acceleration  $a$  and velocity  $v$  for an object in 1-dimensional motion with constant acceleration.

Traditionally, we call the vertical axis the “ $y$ -” axis; the horizontal axis, the “ $x$ -” axis. Please note that there is nothing special about these variables. They are not fixed, and they have no special meaning. If we are graphing, say, a velocity function  $v(t)$  with respect to time  $t$ , then we do not bother trying to identify  $v(t)$  with  $y$  or  $t$  with  $x$ ; in that case, we just forget about  $y$  and  $x$ . This can be particularly important when representing position with the variable  $x$ , as we often do in physics. In that case, graphing  $x(t)$  with respect to  $t$  would give us an  $x$  on both the vertical and horizontal axes, which would be extremely confusing. We can even imagine a scenario wherein we should graph a function  $x$  of a variable  $y$  such that  $y$  would be on the horizontal axis and  $x(y)$  would be on the vertical axis. In particular, in MotionLab, the variable  $z$ , not  $x$ , is always used for the horizontal axis; it represents time. Both  $x$  and  $y$

are plotted on vertical axes as functions of the time  $t$ .

There are graphs which are not graphs of functions, e.g. pie graphs. These are not of relevance to this course, but much of what is contained in this document still applies.

## Data, Uncertainties, and Fits

When we plot empirical data, it typically comes as a set of ordered pairs  $(x,y)$ . Instead of plotting a curve, we just draw dots or some other kind of marker at each ordered pair.

Empirical data also typically comes with some uncertainty in the independent and dependent variables of each ordered pair. We need to show these uncertainties on our graph; this helps us to interpret the region of the plane in which the true value represented by a data point might lie. To do this, we attach error bars to our data points. Error bars are line segments passing through a point and representing some confidence interval about it.

After we have plotted data, we often need to try to describe that data with a functional relationship. We call this process “fitting a function to the data” or, more simply, “fitting the data.” There are long, involved statistical algorithms for finding the functions that best fit data, but we won’t go into them here. The basic idea is that we choose a functional form, vary the parameters to make it look like the experimental data, and then see how it turns out. If we can find a set of

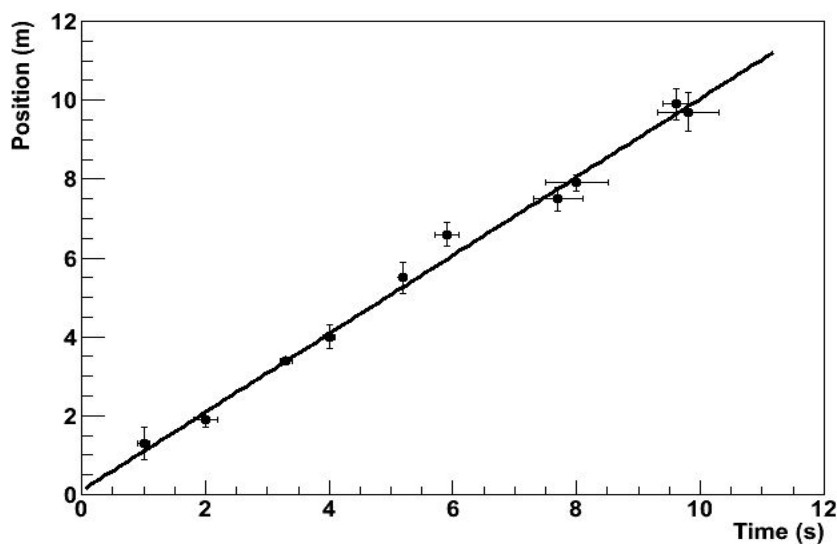


Figure 2: An empirical data set with associated uncertainties and a best-fit line.

parameters that make the function lie very close to most of the data, then we probably chose

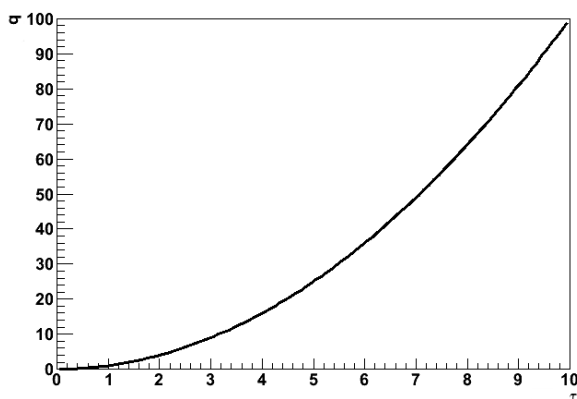
the right functional form. If not, then we go back and try again. In this class, we will be almost exclusively fitting lines because this is easiest kind of fit to perform by eye. Quite simply, we draw the line through the data points that best models the set of data points in question. The line is not a “line graph;” we do not just connect the dots (That would almost never be a line, anyway, but just a series of line segments.). The line does not actually need to pass through any of the data points. It usually has about half of the points above it and half of the points below it, but this is not a strict requirement. It should pass through the confidence intervals around most of the data points, but it does not need to pass through all of them, particularly if the number of data points is large. Many computer programs capable of producing graphs have built-in algorithms to find the best possible fits of lines and other functions to data sets; it is a good idea to learn how to use a high-quality one.

## Making Graphs Say Something

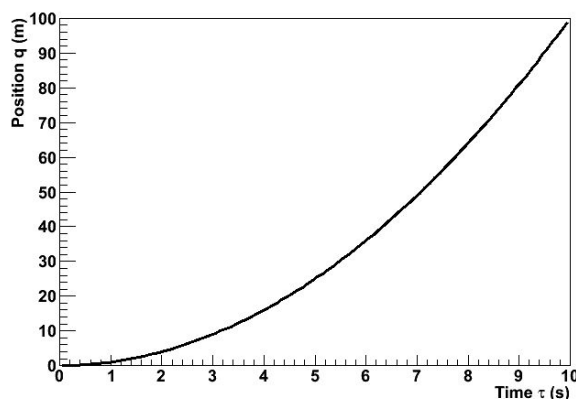
So we now know what a graph is and how to plot it; great. Our graph still doesn’t say much; take the graph in Figure 4(a). What does it mean? Something called  $q$  apparently varies quadratically with something called  $\tau$ , but that is only a mathematical statement, not a physical one. We still need to attach physical meaning to the mathematical relationship that the graph communicates. This is where labels come into play.

Graphs should always have labels on both the horizontal and vertical axes. The labels should be terse but sufficiently descriptive to be unambiguous. Let’s say that  $q$  is position and  $\tau$  is time in Figure 4. If the problem is one-dimensional, then the label “Position” is probably sufficient for the vertical axis ( $q$ ). If the problem is two-dimensional, then we probably need another qualifier. Let’s say that the object in question is moving in a plane and that  $q$  is the vertical component of its position; then “Vertical Position” will probably do the trick. There’s still a problem with our axis labels. Look more closely; where is the object at  $\tau = 6s$ ? Who knows? We don’t know if the ticks represent seconds, minutes, centuries, femtoseconds, or even some nonlinear measure of time, like humans born. Even if we did, the vertical axis has no units, either. We need for the units of each axis to be clearly indicated if our graph is really to say something. We can tell from Figure 4(b) that the object is at  $q = 36m$  at  $\tau = 6s$ . A grain of salt: our prediction graphs will not always need units. For example, if we are asked to draw a graph predicting the relationship of, say, the acceleration due to gravity of an object with respect to its mass, the label “Mass” will do just fine for our horizontal axis. This is because we are not expected to give the precise functional dependence in this situation, only the overall behavior. We don’t know exactly what the acceleration will be at a mass of  $10g$ , and we don’t care. We just need to show whether the variation is increasing, decreasing, constant, linear, quadratic, etc. In this specific case, it might be to our advantage to include

units on the vertical axis, though; we can probably predict a specific value of the acceleration, and that value will be meaningless without them.



(a)

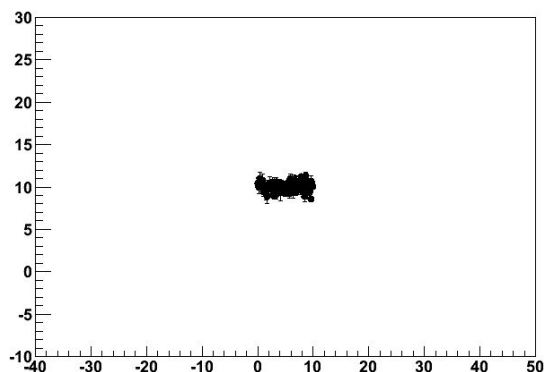
(b) Position  $q$  with respect to time  $\tau$  for a mass of 3kg. The acceleration is constant.

**Figure 4:** Poorly- versus well-labeled and -captioned graphs. The labels and caption make the second graph much easier to interpret.

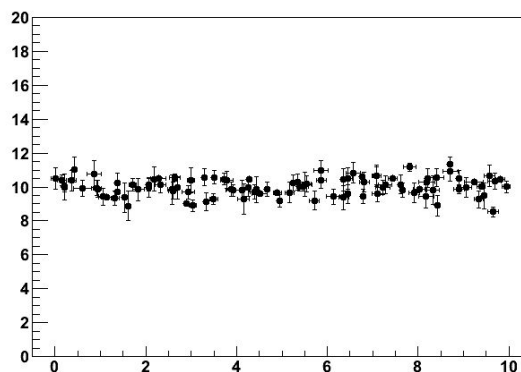
Every graph we make should also have some sort of title or caption. This helps the reader quickly to interpret the meaning of the graph without having to wonder what it's trying to say. It particularly helps in documents with lots of graphs. Typically, captions are more useful than just titles. If we have some commentary about a graph, then it is appropriate to put this in a caption, but not a title. Moreover, the first sentence in every caption should serve the same role as a title: to tell the reader what information the graph is trying to show. In fact, if we have an idea for the title of a graph, we can usually just put a period after it and let that be the first "sentence" in a caption. For this reason, it is typically redundant to include both a title and a caption. After the opening statement, the caption should add any information important to the interpretation of a graph that the graph itself does not communicate; this might be an approximation involved, an indication of the value of some quantity not depicted in the graph, the functional form of a fit line, a statement about the errors, etc. Lastly, it is also good explicitly to state any important conclusion that the graph is supposed to support but does not obviously demonstrate. For example, let's look at Figure 4 again. If we are trying to demonstrate that the acceleration is constant, then we would not need to point this out for a graph of the object's acceleration with respect to time. Since we did not do that, but apparently had some reason to plot position with respect to time instead, we wrote, "The acceleration is constant."

Lastly, we should choose the ranges of our axes so that our meaning is clear. Our axes do not always need to include the origin; this may just make the graph more difficult to interpret.

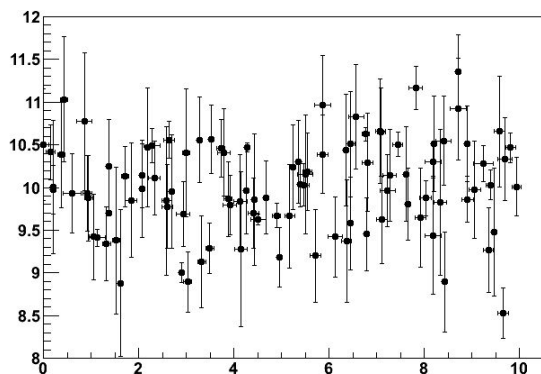
Our data should typically occupy most of the graph to make it easier to interpret; see Figure 5. However, if we are trying to demonstrate a functional form, some extra space beyond any statistical error helps to prove our point; in Figure 5(c), the variation of the dependent with respect to the independent variable is obscured by the random variation of the data. We must be careful not to abuse the power that comes from freedom in



(a)



(b)



(c)

**Figure 5:** Graphs with too much (a), just enough (b), and too little space (c) to be easy to interpret.

plotting our data, however. Graphs can be and frequently are drawn in ways intended to manipulate the perceptions of the audience, and this is a violation of scientific ethics. For example, consider Figure 6. It appears that Candidate B has double the approval of Candidate A, but a quick look at the vertical axis shows that the lead is actually less than one part in seventy. The moral of the story is that our graphs should always be designed to communicate our point, but not to create our point.

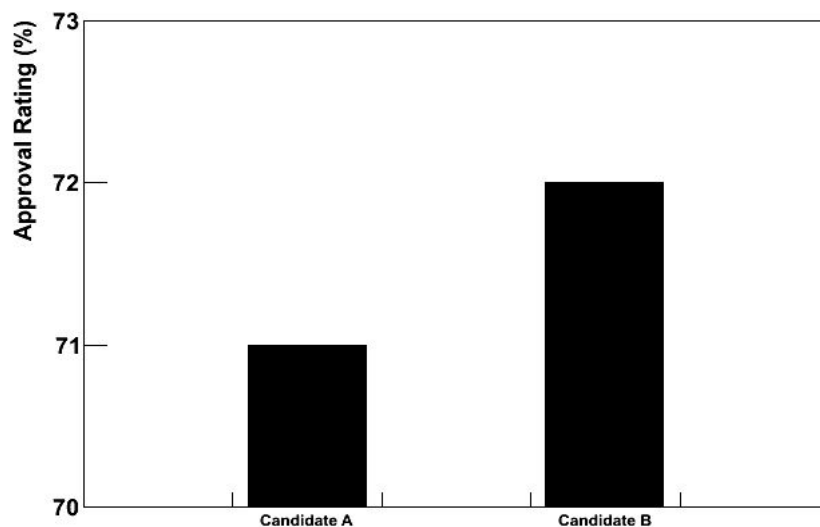
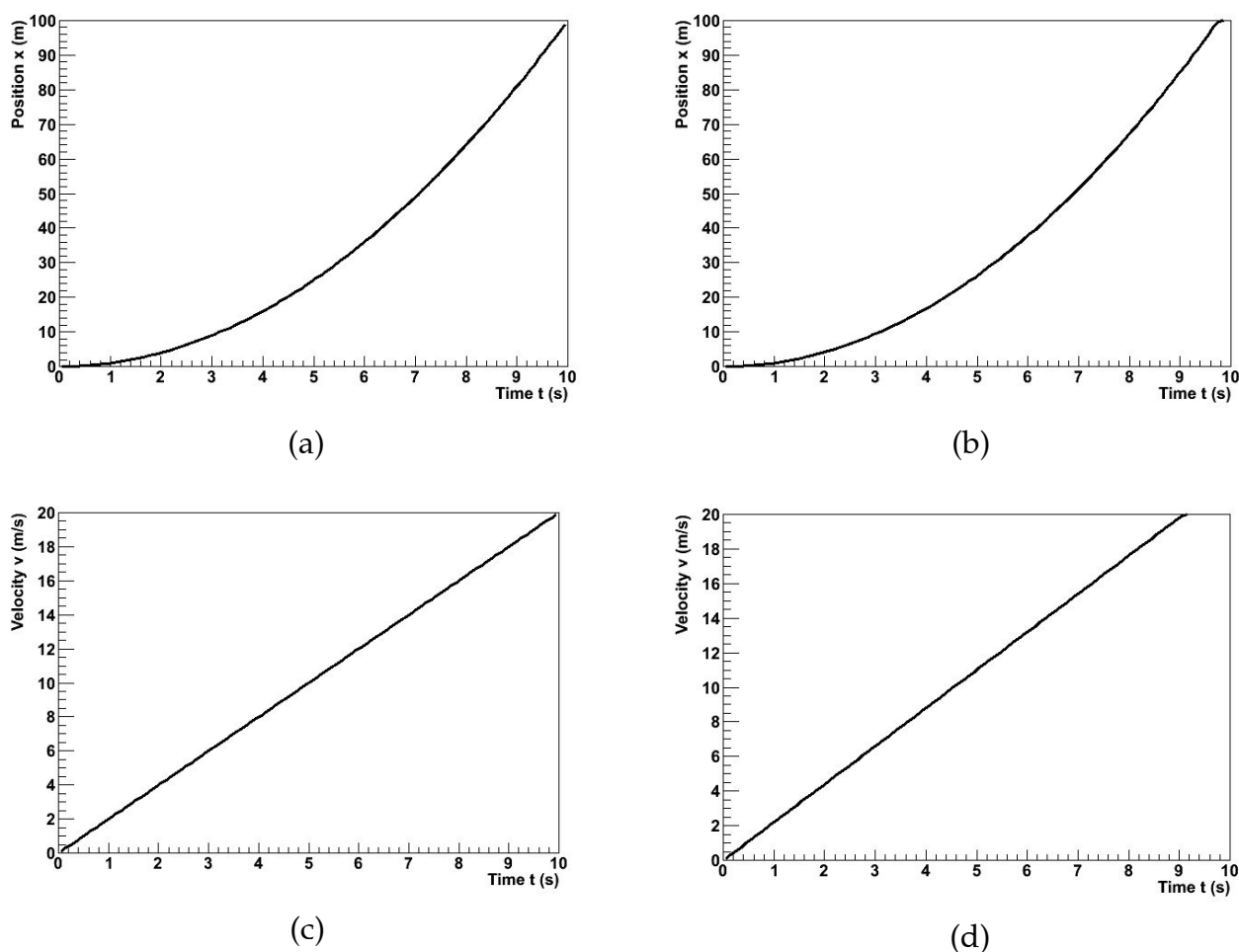


Figure 6: Approval ratings for two candidates in a mayoral race. This graph is designed to mislead the reader into believing that Candidate B has a much higher approval rating than Candidate A.

## Using Linear Relationships to Make Graphs Clear

The easiest kind of graph to interpret is often a line. Our minds are very good at interpreting lines. Unfortunately, data often follow nonlinear relationships, and our minds are not nearly as good at interpreting those. It is sometimes to our advantage to force data to be linear on our graph. There are two ways that we might want to do this in this class; one is with calculus, and the other is by cleverly choosing what quantities to graph.

The “calculus” method is the simpler of the two. Don't let its name fool you: it doesn't actually require any calculus. Let's say that we want to compare the constant accelerations of two objects, and we have data about their positions and velocities with respect to time. If the accelerations are very similar, then it might be difficult to decide the relationship from the position graphs because we have a hard time detecting fine variations in curvature. It is much easier to compare the accelerations from the velocity graphs because we then just have to look at the slopes of lines; see Figure 7. We call this the “calculus” method because velocity is the first derivative with respect to time of position; we have effectively chosen to plot the derivative of position rather than position itself. We can sometimes use these calculus-based relationships to graph more meaningful quantities than the obvious ones.



**Figure 7:** Position and velocity with respect to time for an objects with slightly different accelerations. The difference is easier to see in the velocity graphs.

The other method is creatively named “linearization.” Essentially, it amounts to choosing non-obvious quantities for the independent and/or dependent variables in a graph in such a way that the result graph will be a line. An easy example of this is, once again, an object moving with a constant acceleration, like one of those in Figure 7. Instead of taking the derivative and plotting the velocity, we might have chosen to graph the position with respect to  $\frac{t^2}{2}$ ; because the initial velocity for this object happened to be 0, this would also have produced a graph with a constant slope.

## The Bottom Line

Ultimately, graphs exist to communicate information. This is the objective that we should have in mind when we create them. If our graph can effectively communicate our point to our readers, then it has accomplished its purpose.



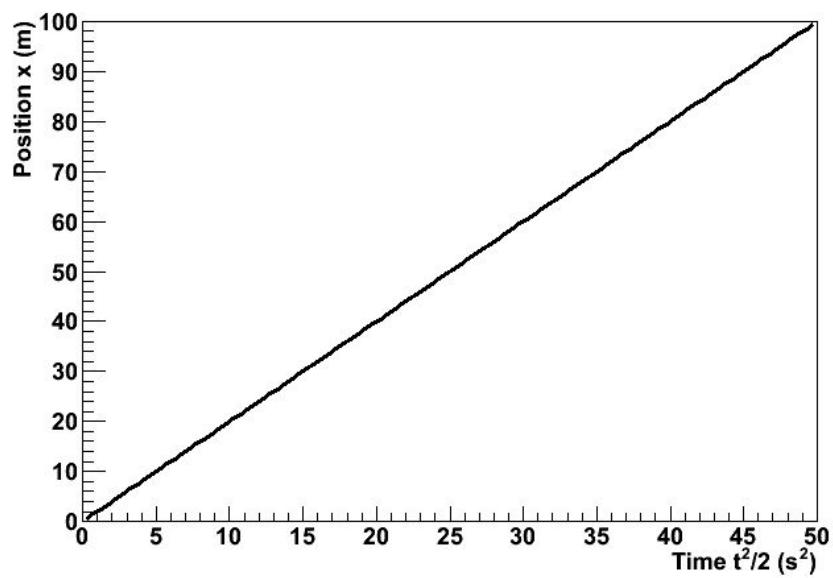


Figure 8: The position of the first object from Figure 7 plotted with respect to  $\frac{t^2}{2}$ . The relationship has been linearized.

## Appendix: Guide to Writing Lab Reports

Many students have a great deal of trouble writing lab reports. They don't know what a lab report is; they don't know how to write one; they don't know what to put in one. This document seeks to resolve those problems. We will address them in that order.

This manual includes examples of a good and of a bad lab report; examine them in conjunction with this document to aid your understanding.

### ***What Is a Lab Report?***

Everyone seems to understand that a lab report is a written document about an experiment performed in lab. Beyond that, a lab report's identity is less obvious and more disputed. Let's save ourselves some misery by first listing some things that a lab report is not. A lab report is not

- ... a worksheet; you may not simply use the example like a template, substituting what is relevant for your experiment.
- ... the story of your experiment; although a description of the experimental procedure is necessary and very story-like, this is only one part of the much greater analytical document that is the report.
- ... rigid; what is appropriate for a report about one experiment may not be appropriate for another.
- ... a set of independent sections; a lab report should be logically divided, but its structure should be natural, and its prose should flow.

So what, then, is a lab report? A lab report is a document beginning with the proposal of a question and then proceeding, using your experiment, to answer that question. It explains not only what was done, but why it was done and what it means. To try to specify the content in much more detail than this is too constraining; you must simply do whatever is necessary to accomplish these goals. However, a lab report usually accomplishes them in four phases. First, it introduces the experiment by placing it in context, usually the motivation for performing it and some question that it seeks to answer. Second, it describes the methods of the experiment. Third, it analyzes the data to yield some scientifically meaningful result. Fourth, it discusses the result, answering the original question and explaining what the result means.

There are, of course, other senses of what a lab report is — it is quantitative, it is persuasive, etcetera — but we will come to those along the way.

### ***How Do I Write a Lab Report?***

Now that we have a vague idea of what a lab report is, let's discuss how to write it. By this, we do not mean its content, but its audience, style, etcetera.

## **Making an Argument**

We already mentioned that a lab report uses an experiment to answer a question, but merely answering it isn't enough; your report must convince the reader that the answer is correct. This makes a lab report a persuasive document. Your persuasive argument is the single most important part of any lab report. You must be able to communicate and demonstrate a clear point. If you can do this well, your report will be a success; if you cannot, it will be a failure.

At some point, you have certainly written a traditional, five-paragraph essay. The first paragraph introduces a thesis, the second through fourth defend the thesis, and the fifth paragraph concludes by restating the thesis. This is a little too simple for a lab report, but the basic idea is the same; keep it in mind. Begin by introducing and stating your prediction in — logically enough — the Introduction and Prediction sections. Test your prediction in the Procedure, Data, and Analysis sections. Restate and critically evaluate both your prediction and your result in your Conclusion section.

## **Audience**

If you are successfully to persuade your audience, you must know something about her. What sorts of things does she know about physics, and what sorts of things does she find convincing? For your lab report, she is an arbitrary scientifically-literate person. She is not quite your professor, not quite your TA, and not quite your labmates, but she is this same sort of person. The biggest difference is that she doesn't know what your experiment is, why you are doing it, or what you hope to prove until you tell her. Use physics and mathematics freely in your report, but explain your experiment and analysis in detail.

## **Technical Style**

A lab report is a technical document. This means that it is stylistically quite different from other documents you may have written. What characterizes technical writing, at least as far as your lab report is concerned? Here are some of the most prominent features, but for a general idea, read the sample good lab report included in this manual.

A lab report does not entertain. When you read the sample reports, you may find them boring; that's OK. The science in your report should be able to stand for itself. If your report needs to be entertaining, then its science is lacking.

A lab report is a persuasive document, but it does not express opinions. Your prediction should be expressed as an objective hypothesis, and your experiment and analysis should be a disinterested effort to confirm or deny it. Your result may or may not coincide with your prediction, and your report should support that result objectively.

A lab report is divided into sections. Each section should clearly communicate one aspect of your experiment or analysis.

A lab report may use either the active or the passive voice. Use whichever feels natural and accomplishes your intent, but you should be consistent.

A lab report presents much of its information with media other than prose. Tables, graphs, diagrams, and equations frequently can communicate far more effectively than can words. Integrate them smoothly into your report.

A lab report is quantitative. If you don't have numbers to support what you say, you may as well not say it at all.

Some of these points are important and sophisticated enough to merit sections of their own, so let's discuss them some more.

### **Nonverbal Media**

A picture is worth a thousand words. Take this old sentiment to heart when you write your lab report, but do not limit yourself to pictures. Make your point as clearly and tersely as possible; if a graph will do this better than words will, use a graph.

When you incorporate these media, you must do so well, in a way that serves the fundamental purpose of clear communication. Label them "Figure 1" and "Table 2." Give them meaningful captions that inform the reader what information they are presenting. Give them context in the prose of your report. They need to be functional parts of your document's argument, and they need to be well-integrated into the discussion.

Students sometimes think that they are graded "for the graphs," and TAs sometimes over-emphasize the importance of these media. Avoid these pitfalls by keeping in mind that the purpose of these things is communication. If you can make your point more elegantly with these tools, then use them. If you cannot, then stick to tried-and-true prose. Use your best judgment.

### **Quantitativeness**

A lab report is quantitative. Quantitativeness is the power of scientific analysis. It is objective. It holds a special power lacking in all other forms of human endeavor: it allows us to know precisely how well we know something. Your report is scientifically valid only insofar as it is quantitative.

Give numbers for everything, and give the numerical errors in those numbers. If you find yourself using words like "big," "small," "close," "similar," etcetera, then you are probably not being sufficiently quantitative. Replace vague statements like these with precise, quantitative ones.

You will frequently need to give equations as well as numbers. If so, say something about whence the equation came and why it's there. You can't find the error in an equation, but you can propagate the error in the inputs to get the errors in the outputs. Do this.

Error analysis is a very important part of quantitiveness. This lab manual contains an appendix about error analysis; read it, understand it, and take it to heart.

To be quantitative, i.e., to give numbers and to analyze errors, you are going to need to do a lot of math. This is inescapable, but it's not so bad. For your purposes, you can think of mathematics as a particular language used to express ideas about physics. Think to yourself about what information you have and what information you want. Try (briefly!) to put into your own words, on a scrap piece of paper, how the information you have tells you the information you want, then use what you know about calculus and algebra to translate that idea into the math you need in your report. For example, "the vertical component of the acceleration ( $=a_x$ ) of the ball in free-fall due to gravity is the change ( $=\Delta$ ) in the vertical component of its velocity ( $=v_y$ ) during a particular change ( $=\Delta$ ) in time ( $=t$ )" would become

$$a_y = \frac{\Delta v_y}{\Delta t}$$

One way you could find this value is as the slope of  $V_y$  versus  $z$  in your MotionLab data.

### ***What Should I Put in My Lab Report?***

Structure your report like this.

#### **Introduction**

Do three things in your introduction. First, provide enough context so that your audience can understand the question that your report tries to answer. This typically involves a brief discussion of the hypothetical real-world scenario from the lab manual. Second, clearly state the question. Third, provide a brief statement of how you intend to answer it.

It can sometimes help students to think of the introduction as the part justifying your report to your company or funding agency. Leave your reader with an understanding of what your experiment is and why it is important.

#### **Predictions**

Include the same predictions in your report that you made prior to the beginning of the experiment. They do not need to be correct. You will do the same amount of work whether they are correct or incorrect, and you will receive far more credit for an incorrect, well-refuted prediction than for a correct, poorly-supported one.

Your prediction will often be an equation or a graph. If so, discuss it in prose.

#### **Procedure**

Explain what your actual experimental methodology was in the procedure section. Discuss the apparatus and techniques that you used to make your measurements.

Exercise a little conservatism and wisdom when deciding what to include in this section. Include all of the information necessary for someone else to repeat the experiment, but only in the important ways. It is important that you measured the time for a cart to roll down a

ramp through a length of one meter; it is not important who released the cart, how you chose to coordinate the person releasing it with the person timing it, or which one meter of the ramp you used. Omit any obvious steps. If you performed an experiment using some apparatus, it is obvious that you gathered the apparatus at some point. If you measured the current through a circuit, it is obvious that you hooked up the wires. One aspect of this which is frequently problematic for students is that a step is not necessarily important or non-obvious just because they find it difficult or time-consuming. Decide what is scientifically important, and then include only that in your report.

Students approach this section in more incorrect ways than any other. Do not provide a bulleted list of the equipment. Do not present the procedure as a series of numbered steps. Do not use the second person or the imperative mood. Do not treat this section as though it is more important than the rest of the report. You should rarely make this the longest, most involved section.

### **Data**

This should be your easiest section. Record your empirical measurements here: times, voltages, fits from MotionLab, etcetera.

Do not use this as the report's dumping ground for your raw data. Think about which measurements are important to your experiment and which ones are not. Only include data in processed form. Use tables, graphs, and etcetera, with helpful captions. Do not use long lists of measurements without logical grouping or order.

Give the units and uncertainties in all of your measurements.

This section is a bit of an exception to the "smoothly integrate figures and tables" rule. Include little to no prose here; most of the discussion belongs in the Analysis section. The distinction between the Data and Analysis sections exists mostly for your TA.

### **Analysis**

Do the heavy lifting of your lab report in the Analysis section. Take the data from the Data section, scientifically analyze it, and finally answer the question you posed in your Introduction. Do this quantitatively.

Your analysis will almost always amount to quantifying the errors in your measurements and in any theoretical calculations that you made in the Predictions section. Decide whether the error intervals in your measurements and predictions are compatible. This manual contains an appendix about error analysis; read it for a description of how to do this.

If your prediction turns out to be incorrect, then show that as the first part of your analysis. Propose the correct result and show that it is correct as the second part of your analysis.

Finally, discuss any shortcomings of your procedure or analysis, such as sources of systematic error for which you did not account, approximations that are not necessarily valid, etcetera. Decide how badly these shortcomings affect your result. If you cannot confirm your prediction, then estimate which are the most important.

### **Conclusion**

Consider your conclusion the wrapping paper and bow tie of your report. At this point, you should already have said most of the important things, but this is where you collect them in one place. Remind your audience what you did, what your result was, and how it compares to your prediction. Tell her what it means. Leave her with a sense of closure.

Quote your result from the Analysis section and interpret it in the context of the hypothetical scenario from the Introduction. If you determined that there were any major shortcomings in your experiment, you might also propose future work to overcome them.

If the Introduction was your attempt to justify your past funding, then the Conclusion is your attempt to justify your future funding.

### ***What Now?***

Read the sample reports included in this manual. There are two; one is an example of these instructions implemented well, and the other is an example of these instructions implemented poorly. Then, talk to your TA. He can answer any remaining questions that you might have.

There is a lot of information here, so using it and actually writing your lab report might seem a little overwhelming. A good technique for getting started is this: complete your analysis and answer your question before you ever sit down to write your report. At that point, the hard part of the writing should be done: you already know what the question was, what you did to answer it, and what the answer was. Then just put that down on paper.

## GOOD SAMPLE LAB

### Lab II, Problem 4: Projectile Motion and Mass

Athos

July 12, 2011

Physics 1101W, Professor: Porthos, TA: Aramis

#### Introduction

A group of medieval warfare enthusiasts is planning a reenactment and intends to build a trebuchet. If the reenactment is to be safe and realistic, the motion of the projectiles it launches must be well understood. The acceleration of the projectile is constant in time, as confirmed by a previous experiment. This experiment sought to understand the mass dependence of that constant acceleration. To do so, the projectiles were modeled using balls; the trebuchet, using an experimenter's arm. The hypothesis that the acceleration is mass-independent was confirmed.

#### Prediction

It is hypothesized that the acceleration of an object in projectile motion is mass-independent; this is depicted graphically in Figure 1.

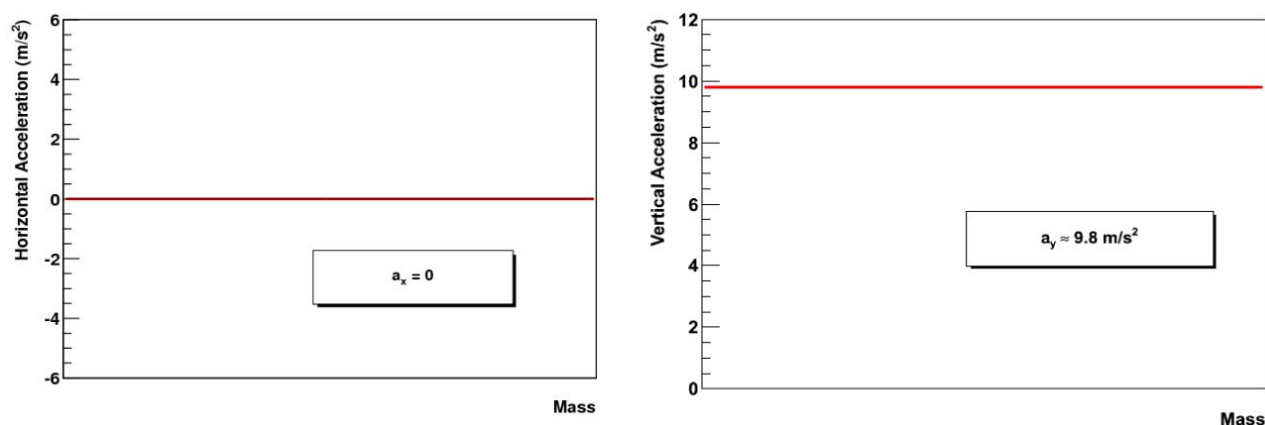


Figure 1: Horizontal and vertical components of acceleration of a projectile near Earth's surface.

The acceleration of all objects moving ballistically near the surface of Earth is downward and of a magnitude given by local  $g$ , approximately  $9.8 \text{ m/s}^2$ , i.e. is constant with respect to mass. Mathematically,

$$\frac{\Delta \vec{a}}{\Delta m} = \vec{0}$$



This is an assumption of our theory of kinematics.

## Procedure

Spherical balls, all of approximately the same size (in order to make approximately constant the effects of air resistance) but of varying masses, were used to model the projectiles. The force of the trebuchet was modeled by throwing by the experimenters. The resulting projectile motion was recorded with a video camera; MotionLab analysis software was used to generate (horizontal position, vertical position, time) triplets at each frame in the trajectories and, by linear interpolation, (horizontal velocity, vertical velocity, time) triplets between each pair of consecutive frames in the trajectories. A meter stick was placed less than 5cm behind the projectiles' plane of motion for calibration of this software. The position and velocity of each projectile as functions of time were fit

by eye as parabolas and lines, respectively. The acceleration of each projectile was then taken to be the slope of the velocity fit because this was deemed more reliable than the position fit and because it was easier to quantify the error in the velocity fit.

Two trajectories were analyzed in this fashion. Due to time constraints, the results of all the lab groups were combined to yield enough data for the analysis. The other groups' procedures were similar, but the details are unknown.

## Data

$M$ (g)	$a_y$ (m/s <sup>2</sup> ) (low)	$a_y$ (m/s <sup>2</sup> ) (best fit)	$a_y$ (m/s <sup>2</sup> ) (high)
48.8	9.7	10.0	10.7
51.4	9.3	9.5	11.1
57.3	9.0	10.0	10.6
75.0	9.0	9.7	10.0
141.2	9.1	9.8	10.5
148.6	9.3	9.9	10.8
165.5	9.4	10.0	10.5

Table 1: The vertical accelerations as measured by MotionLab fits of velocity and the associated masses. The uncertainty in all of the masses is 0.3g

## Analysis

The accelerations in the vertical ( $y$ ) direction as measured by the fits are given in Table 1 in the Data section. The accelerations in the horizontal ( $x$ ) direction are not given because they are all 0. Errors were assigned to the fits by finding the maximal and minimal values of the parameters which yield apparently valid fits. A constant, the average of the “best fit” accelerations listed in Table 1, was then taken as the single parameter in a 0-degree polynomial fit to the data. The error was taken to be the standard deviation from this parameter. The fit is depicted in Figure 2.

As Figure 2 illustrates, the fit falls within the error of all the data points, so it is valid to say that this has confirmed the prediction that the vertical acceleration is constant with respect to mass.

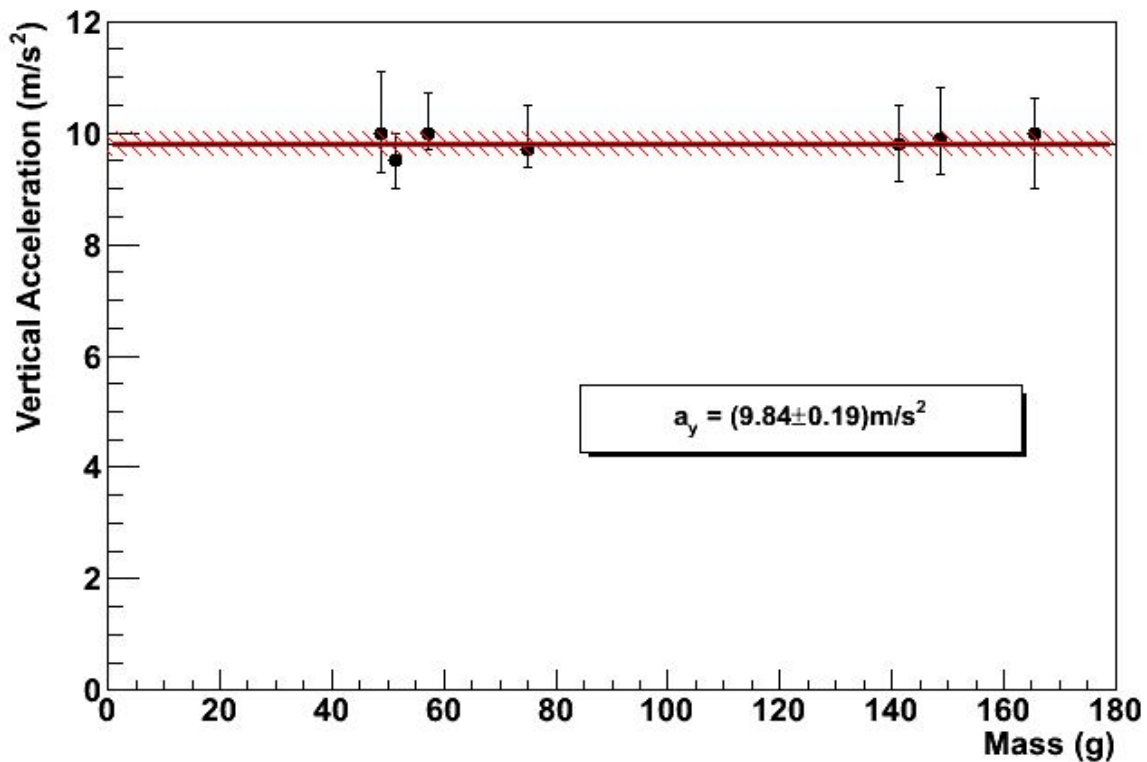


Figure 2: The measured vertical accelerations versus the respective projectile masses and the constant fit thereto. The errors in the masses are smaller than the markers.

Because all of the horizontal accelerations  $a_x = 0$ , the hypothesis that the horizontal acceleration of the projectiles is constant with respect to mass has been confirmed; although there exists a nonzero uncertainty in all of these measurements, 0 lies within all possible error intervals.

Possible sources of systematic error include air resistance, distortion due to the camera's optics, error in calibration due to the offset depth of the trajectories versus the meter stick, and the constraint that the first frame of the ball's motion was at time 0, which is accurate only to 0.016s. These, and any other systematics, are believed to be insignificant because the average and expected accelerations in both the horizontal and vertical directions are consistent with the individual measurements to within experimental error.

### Conclusion

The motion of projectiles launched by trebuchets was modeled by thrown balls. The hypothesis that the horizontal accelerations thereof are mass-independent was confirmed in that all were measured to be 0. The hypothesis that the vertical accelerations thereof are mass-independent was confirmed in that a single, constant acceleration of  $9.84\text{m/s}^2$  lay within the error intervals of all of the measured data points.

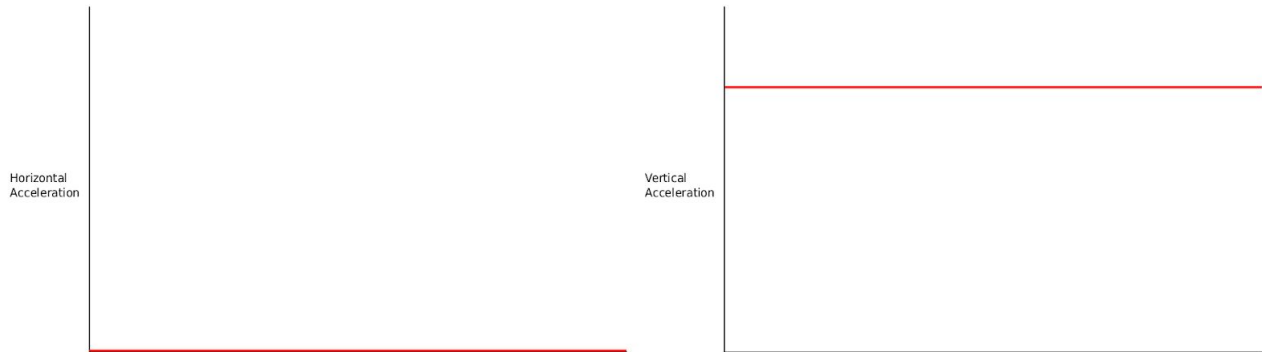
**BAD SAMPLE LAB****Lab II, Problem 4**

Comte de Rochefort

July 12, 2011

***Introduction***

We want to figure out how the trebuchet's projectiles will move if their mass is changed. A trebuchet is a kind of medieval catapult that uses gravity to launch rocks. First, we threw balls to simulate the rocks. We recorded them with a camera. Then, we analyzed the videos using MotionLab. Then, we decided that the acceleration does not change when the mass changes.

***Prediction******Procedure***

The procedure in this experiment began with setup. We collected the following materials:

- meter stick
- tennis ball
- baseball
- video camera on tripod
- computer with MotionLab software
- stopwatch

We then positioned the camera facing the wall. We taped the meter stick to the wall.

We next recorded the videos. We threw the tennis ball in a parabolic trajectory parallel to the wall and recorded a video of it with the camera and computer. We did the same for the baseball.

We then analyzed the videos with MotionLab. We began by setting  $t=0$  to the time when the ball left General Veers's hand. We then used the meter stick to calibrate the length in the video. We defined our coordinate system. It had the origin where the ball was at  $t=0$ ,  $x$  was horizontal, and  $y$  was vertical. We then had to make predictions about the position graphs. Since there is no acceleration in the  $x$  direction, we predicted it would be a straight, linear line. Since there is acceleration in the  $y$  direction, we predicted it would be quadratic. We derived the coefficients for the predictions by measuring how high and how far the ball went with the meter stick and how long it flew with the stopwatch. The first ball flew  $88 \pm 0.05\text{cm}$  in the  $x$  direction and  $90 \pm 0.05\text{cm}$  in the  $y$  direction, and took  $0.85 \pm 0.005\text{s}$  to complete its trajectory. The second ball flew  $110 \pm 0.05\text{cm}$  in the  $x$  direction and  $60 \pm 0.05\text{cm}$  in the  $y$  direction. It took  $0.86 \pm 0.005\text{s}$  to complete its trajectory. The predicted equations were  $x=0+1.054t$  and  $y=0+4.185t-4.9t^2$  for the first ball and  $x=0+0.694t$  and  $y=0+4.185t-4.9t^2$  for the second ball. We then added a data point at each frame in the ball's flight. We omitted some frames near the end of the video when the ball was in the distorted region. We took 24 data points for the first ball and 29 data points for the second ball. We fit graphs to the resulting data points. The fits were  $x=0+1.05t$  and  $y=0+3.47t-5t^2$  for the first ball and  $x=0+0.71t$  and  $y=0+4.37t-5t^2$  for the second ball. We then had to predict the velocity graphs of the balls. We did this by making the  $t$  coefficient in the position function the constant in the velocity function and the  $t^2$  coefficient in the position function the  $t$  coefficient in the velocity function. This made the  $xv$  graph a constant line and the  $yv$  graph a linear line. The predictions were  $xv=1.05+0t$  and  $yv=3.47-10t$  for the first ball and  $xv=0.71+0t$  and  $yv=4.37-10t$  for the second ball. After this, we had to fit the velocity graphs to the data points. The fits were  $xv=1.05+0t$  and  $yv=3.47-10t$  for the first ball and  $xv=0.71+0t$  and  $yv=4.37-10t$  for the second ball. The fits were the same as the predictions, so there were no errors in the predictions. We then got the accelerations from the coefficients of the fits. This was  $0.5$  of the  $t^2$  coefficient in the position fit and the same as the  $t$  coefficient in the velocity fit.

After analyzing the videos, we exchanged data with the other groups, left the lab, and analyzed the data.

## **Data**

### **Ball 1**

mass:  $57.3 \pm 0.05\text{g}$   
x distance:  $88 \pm 0.05\text{cm}$   
y distance:  $90 \pm 0.05\text{cm}$   
time:  $0.85 \pm 0.005\text{s}$   
x prediction:  $x=0+1.054t$   
x fit:  $x=0+1.05t$   
y prediction:  
 $y=0+4.185t-4.9t^2$   
y fit:  $y=0+3.47t-5t^2$   
xv prediction:  $xv=1.05+0t$   
xv fit:  $xv=1.05+0t$   
yv prediction:  $yv=3.47-10t$   
yv fit:  $yv=3.47-10t$

### **Ball 2**

mass:  $48.8 \pm 0.05\text{g}$   
x distance:  $110 \pm 0.05\text{cm}$   
y distance:  $60 \pm 0.05\text{cm}$   
time:  $0.86 \pm 0.005\text{s}$   
x prediction:  $x=0+0.694t$   
x fit:  $x=0+0.71t$   
y prediction:  
 $y=0+4.185t-4.9t^2$   
y fit:  $y=0+4.37t-5t^2$   
xv prediction:  $xv=0.71+0t$   
xv fit:  $xv=0.71+0t$   
yv prediction:  $yv=4.37-10t$   
yv fit:  $yv=4.37-10t$

### **Ball 3**

mass:  $165.5 \pm 0.05\text{g}$   
x prediction:  $x=0+1.126t$   
x fit:  $x=0+1.13t$   
y prediction:  
 $y=0+3.915t-4.9t^2$   
y fit:  $y=0+3.37t-4.9t^2$   
xv prediction:  $xv=1.13+0t$   
xv fit:  $xv=1.13+0t$   
yv prediction:  $yv=3.37-9.8t$   
yv fit:  $yv=3.37-10t$

**Ball 4**

mass: 51.4+/-0.05g  
x prediction:  $x=0+0.877t$   
x fit:  $x=0+0.82t$   
y prediction:  
 $y=0+4.469t-4.9t^2$   
y fit:  $y=0+3.8t-4.7t^2$   
xv prediction:  $xv=0.82+0t$   
xv fit:  $xv=0.82+0t$   
yv prediction:  $yv=3.8-9.4t$   
yv fit:  $yv=3.8-9.5t$

**Ball 5**

mass: 141.2+/-0.05g  
x prediction:  $x=0+1.203t$   
x fit:  $x=0+1.21t$   
y prediction:  
 $y=0+3.258t-4.9t^2$   
y fit:  $y=0+3.1t-4.9t^2$   
xv prediction:  $xv=1.21+0t$   
xv fit:  $xv=1.21+0t$   
yv prediction:  $yv=3.1-9.8t$   
yv fit:  $yv=3.1-9.8t$

**Ball 6**

mass: 148.6+/-0.05g  
x prediction:  $x=0+1.281t$   
x fit:  $x=0+1.4t$   
y prediction:  
 $y=0+3.258t-4.9t^2$   
y fit:  $y=0+4.1t-4.95t^2$   
xv prediction:  $xv=1.4+0t$   
xv fit:  $xv=1.4+0t$   
yv prediction:  $yv=4.1-9.9t$   
yv fit:  $yv=4.1-9.9t$

**Ball 7**

mass: 75.0+/-0.05g  
x prediction:  $x=0+0.943t$   
x fit:  $x=0+1.07t$   
y prediction:  
 $y=0+3.895t-4.9t^2$   
y fit:  $y=0+3.3t-4.85t^2$   
xv prediction:  $xv=1.07+0t$   
xv fit:  $xv=1.07+0t$   
yv prediction:  $yv=3.3-9.7t$   
yv fit:  $yv=3.3-9.7t$

***Analysis***

We calculate the accelerations from the fits because we know  $x = x_0 + v_0t + \frac{1}{2}at^2$ . All the accelerations in the x direction are therefore 0. The accelerations in the y direction are  $-10\text{m/s}^2$ ,  $-10\text{m/s}^2$ ,  $-9.8\text{m/s}^2$ ,  $-9.4\text{m/s}^2$ ,  $-9.8\text{m/s}^2$ ,  $-9.9\text{m/s}^2$ ,  $-9.7\text{m/s}^2$ .

We know that the x accelerations should be 0 because we are ignoring air resistance. We know that the y accelerations should be  $-9.8\text{m/s}^2$ . All of the y accelerations are close to this. They differ by  $0.2\text{m/s}^2$ ,  $0.2\text{m/s}^2$ ,  $0\text{m/s}^2$ ,  $4\text{m/s}^2$ ,  $0\text{m/s}^2$ ,  $0.1\text{m/s}^2$ , and  $0.1\text{m/s}^2$ ; these are all small.

There are several important sources of error in this lab. One is the fisheye effect of the camera lens. Another is the finite accuracy of the measuring devices. The stopwatch can only measure to 0.01s, and the meter stick can only measure to 0.001m, so these measurements are only accurate to half of those values. There is error in MotionLab, too, as can be seen in the differences between some of the position and velocity fits. There was error in that we couldn't throw the balls exactly the same every time. Finally, there could have been human error. We know that all of these errors were not significant, though, because all of the measurements of acceleration were so close to the known right values.

## ***Conclusion***

We measured the acceleration of seven balls in projectile motion and got things very close to the right values every time. We can therefore say that the mass dependence of the accelerations in the x and y directions are both constant. In the x direction, it is  $0\text{m/s}^2$ , and in the y direction, it is  $-9.8\text{m/s}^2$ . This was true for all the masses. This is the same as our original prediction. We can therefore say that this experiment was a success.