University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2009 – PART I

Thursday, January 15, 2009 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Some physical constants and quantities in SI and natural units

Layout by Krister Henriksson, krister.henriksson@helsinki.fi (rev. April 7 2003)

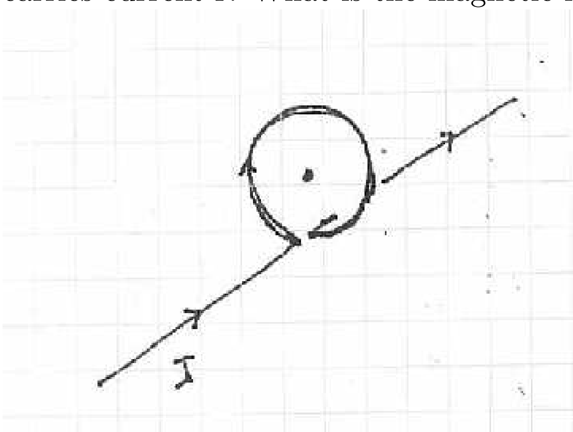
Physical constants in SI units

Constant	*Symbol*	*Value*
Pi	π	3.141592654
Universal gravitational constant	G	$6.67259 \cdot 10^{-11}$ Nm^2/kg^2
Standard acceleration of gravity	g_n	9.80665 m/s^2
Standard temperature	T_0	273.15 K
Standard pressure	p_0	1 atm = 760 Torr = 101325 Pa = 1.01325 bar
Molar volume at STP	V_m	$22.42420 \cdot 10^{-3}$ m^3/mol
Avogadro's constant	N_{A}	$6.0221367 \cdot 10^{23}$ 1/mol
Gas constant	R	8.314510 J/(mol·K)
Boltzmann's constant, R/N_{A}	k_B	$1.380658 \cdot 10^{-23}$ J/K
		$= 8.617385 \cdot 10^{-5}$ eV/K
Speed of light in vacuum	c	$2.99792458 \cdot 10^{8}$ m/s
Fundamental unit of electric charge	e	$1.60217733 \cdot 10^{-19}$ C
Mass of electron	m_e	$9.1093897 \cdot 10^{-31}$ kg
		= 0.51099906 MeV/c^2
Mass of proton	m_p	$1.6726231 \cdot 10^{-27}$ kg
		= 938.27231 MeV/c^2
Mass of neutron	m_n	$1.6749286 \cdot 10^{-27}$ kg
		= 939.56563 MeV/c^2
Universal atomic mass unit	u	$1.6605402 \cdot 10^{-27}$ kg
		= 931.49435 MeV/c^2
Permeability of vacuum	μ_0	$4\pi \cdot 10^{-7}$ N/A^2
		$= 1.256637061 \cdot 10^{-6}$ N/A^2
Permittivity of vacuum	$\varepsilon_0 = 1/(\mu_0 c^2)$	$8.85418782 \cdot 10^{-12}$ F/m
Coefficient in Coulomb's law	$1/(4\pi\varepsilon_0)$	$8.98755 \cdot 10^{9}$ Nm^2/C^2
Planck's constant	h	$6.6260755 \cdot 10^{-34}$ Js
		$= 4.1356692 \cdot 10^{-15}$ eVs
	$\hbar = h/(2\pi)$	$1.05457266 \cdot 10^{-34}$ Js
		$= 6.5821220 \cdot 10^{-16}$ eVs
Electromagnetic coupling constant	$\alpha = e^2/(4\pi\varepsilon_0 \hbar c)$	$7.29735308 \cdot 10^{-3}$
(fine structure constant)		= 1/137.0359895
Rydberg's constant	$R_\infty = m_e c \alpha^2/(2h)$	$1.0973731534 \cdot 10^{7}$ 1/m
Stefan & Boltzmann's constant	$\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$	$5.67051 \cdot 10^{-8}$ W/(m^2·K^4)
		$= 3.53925 \cdot 10^{11}$ eV/(m^2·K^4)

1. Consider a simple model for the thermal properties of the planets in which the planets act like black bodies. If, in such a model, the surface temperature of the earth is 278 K, what is the predicted surface temperature of Venus? Venus is about 0.72 AU from the sun, where 1 AU is the distance from the earth to the sun. (Ignore effects of the atmospheres of the planets.)

2. A straight cylindrical wire of radius a carries a current I and has resistance per unit length R/L. The wire is embedded in a cylindrical medium of thermal conductivity κ. If the embedding medium is in contact with a thermal bath at temperature T_b at a distance b from the center of the wire, what is the temperature at the surface of the wire?

3. A long straight wire is bent into a circular loop of radius R near its midpoint and this loop is twisted so that its plane is perpendicular to the wire. The wire carries current I. What is the magnetic field at the *center* of the loop in the wire?



4. Consider a ball rolling down an inclined plane with grade angle θ. If the coefficient of friction between the ball and the slope is μ, find the angle at which the surface of the ball starts to slip with respect to the surface.

5. A ball bouncing vertically off a smooth horizontal surface has a coefficient of restitution e_r, meaning that immediately after each bounce its speed is a fraction e_r of its speed immediately before that bounce. Find the time which elapses between the moment when it is dropped and when it finally comes to rest(no velocity or acceleration) .

6. In a recent report in Science, a planet was observed moving around a star in an orbit of radius 115 AU with an estimated speed, relative to the star, of 5.5 km/sec. If the orbit is circular, what is the mass of the star, in units of our sun's mass? 1AU=1.48 x 10^8 km.

7. A thin uniform circular ring of radius r, carrying uniformly distributed charge q and mass m, is placed in the xy-plane. The ring rotates with angular speed ω about its symmetry axis, parallel to the z-axis.

a. Find the ratio of its magnetic dipole moment to its angular momentum. This is called the gyromagnetic ratio g.

b. What is g for a uniform spinning charged sphere of radius R, charge Q, and mass M? Hint: Use the result from part a.

8. The 'contact term' in the hyperfine interaction between the magnetic moments of the electron and the nucleus of a hydrogen atom takes the form $H' = A\vec{I}\cdot\vec{S}$ where $\vec{I}$ is the angular momentum operator of the nucleus $\vec{S}$ is the spin angular momentum operator of the electron and A is a constant. Treating this as a perturbation, find the first order corrrections to the ground state energy of the hydrogen atom which it implies. Give the energies of the perturbed states and their degeneracies and describe their angular momentum properties.

9. An observer sees a supersonic airplane passing over a building 20 kilometers away and moving perpendicular to a line between himself and the building. A moment later the aircraft, still moving in a straight line, passes over another building which the observer knows to be 10 kilometers away from the first building and, at this moment, he hears the sound of the airplane *for the first time*. Find the ratio of the speed of the airplane to the speed of sound in the air.

10. Evidence has been reported of cosmic ray particles incident on the atmosphere of the earth having energies of 10^{20} eV.

a. If such particles were protons, what would their velocities be? (Give the difference between the speed of light and the speed of the particles.)

b. If the particles originated on the other side of our galaxy (about 10^4 light years away) how much time would have elapsed, in the rest frame of the particle, during its trip to the earth, assuming again that the particles are protons?

11. Write an expression for the vibrational specific heat per molecule of a gas of diatomic molecules modeled (quantum mechanically) as harmonic oscillators. Let the harmonic force be described by a spring constant K and the masses of the atoms in the molecules be M and m. Show that your result reduces to the expected equipartition result in the appropriate high temperature limit, which should be defined.

12. A permanent magnet of cylindrical shape is dropped through a vertically oriented cylindrical coil. The inside diameter of the coil is slightly larger than the diameter of the magnet and the magnet falls smoothly through. The coil is in series with an ammeter and a resistance R. Assume that the length of the cylindrical magnet is much less than the length of the coil and neglect any effect of eddy currents induced in the magnet.

a. Make a qualitative sketch of the emf $\mathcal{E}(t)$ across the coil as a function of time as the magnet passes into, through and out the bottom end of the coil. Make sketches indicating the direction of the induced currents assuming that the magnets north pole is facing down as it is dropped, as shown below.

b. Taking account of the self inductance L of the coil, write an integral which would permit the current in the circuit to be calculated as a function of time, given $\mathcal{E}(t)$.